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OF THE
ROYAL METEOROLOGICAL SOCIETY

Editor: P. A. SHEPPARD

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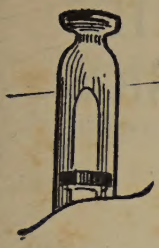

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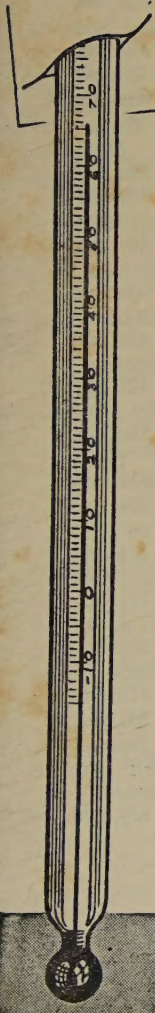

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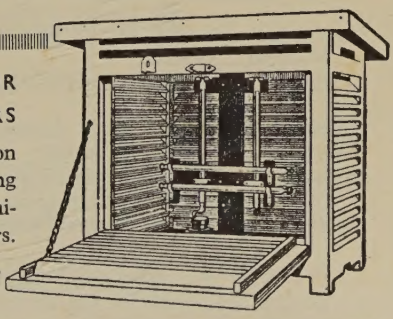

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QUARTERLY JOURNAL

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EDITORIAL

We take meteorology to be the study of the physics of the atmosphere – and its chemistry too, if the distinction is a profitable one today. Now, someone has said that meteorology properly begins some distance below the earth's sea or land surface, and our atmosphere reaches to 1,000 km or more above sea level. Yet hardly a paper appearing in this *Journal* deals with physical processes outside the layer from sea level to 30 km or so and some kinds of physical phenomena *within* that layer are little discussed in our pages. (The subject of our current Review is an example of the latter.) Moreover, the reason for this lack of balance is certainly not to be sought in any dearth of original research on natural phenomena below and above the limits we have just mentioned. Is this a desirable state of affairs, and if not what might we do to encourage the *Journal* of the Royal Meteorological Society to become a journal for the whole of meteorology ?

We personally regret the separation, as far as publication in this country is concerned, of physical oceanography and the physics of the upper atmosphere from the physics of the first 30 km. These branches of geophysics are surely not yet so specialized as to merit treatment in water-tight or air-tight compartments. Indeed at this very stage of their individual growths they appear to be posing problems which have much in common with each other. Thus we have recently been reminded, by Rossby and by Eady, that the general circulation of the oceans has striking similarities with the general circulation of the atmosphere and, by Ratcliffe, Ryle and others, that wind systems and turbulence are as much a feature of the 100 km and higher levels of the atmosphere as of the lower levels to which this *Journal* has paid more attention. But, in this country at least, it is a matter of some chance whether these similarities are appreciated by the workers to whom they may be of concern; a common meeting ground, while available, is hardly used, progress is as a result probably slower than it would otherwise be, and some research effort is possibly wasted. Moreover, how much more stimulating do our studies become when we see their scope expanding and reaching into realms undreamt of earlier. Nor is it only the similarities between one branch of geophysics and another that may be rewarding; the differences in behaviour of systems only partly alike may prove equally instructive and productive of original ideas to the discerning mind.

Benefits of a different kind might also accrue. The troposphere and lower stratosphere, to which the pages of our *Journal* are now largely devoted, are the realm of ' weather,' a study of primary concern to the official meteorological services and to their professional members. Researches on the upper reaches of the atmosphere and on the ocean are, on the other hand, pursued elsewhere in the main, in university departments, observatories and special research institutes. Might it not inspire in the worker from these institutions a greater appreciation of the kind of work with which the official meteorologist is necessarily concerned and of his techniques – they are

of no mean order – if the *Journal* and Society were the common meeting ground for both ? We can imagine official service and university or similar department both the richer for the liaison on the independent ground of the Society. Furthermore, a certain tendency to suppose that ' atmospheric physics ' can profitably be distinguished from meteorology, a tendency we regret as of no scientific value and possibly conducive to prejudice, might be reversed.

Ours is the only learned society in this country whose primary aim it is to encourage the study of meteorology – the whole of it – and it seems proper therefore for us to foster the collaboration between physicists and others, whose realm may be any part of the atmosphere or even the ocean, through the wider use of the pages of our *Journal* and by means of the Society's meetings. This need conflict in no essential respect with the legitimate needs of workers in one specialist field to write on and discuss the techniques peculiar to their study in their own specialist journal, if it exists. Technique is not lightly to be divorced from results, but results and their integration are the major concern. Whence, a step would have been taken in the direction we recommend, if, for example, the ionospheric physicist told us in our own pages what temperatures he infers aloft, what winds and what turbulence.

There is no single step which will take us to this, in our view, desirable goal. Should, however, these lines reach the eyes of any non-Members who explore the high atmosphere or the ocean, they may be assured that a warm welcome awaits them in our Fellowship, and that we shall be pleased to consider very sympathetically any communication they may submit for our *Journal*.

551.594

Atmospheric electricity; some recent trends and problems

By T. W. WORMELL
Cavendish Laboratory, Cambridge

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1. Introduction

It is traditional to apply the title, Atmospheric Electricity, primarily, to the electrical phenomena of the lower atmosphere. Such a restriction, although arbitrary, is a convenient one; it limits us to an interconnected series of phenomena, whose experimental investigation requires a reasonably homogeneous series of techniques. The high conductivity and general properties of the ionosphere, concerning which a wealth of detailed information has been won by the power of radio methods of investigation, will thus only be relevant in so far as they react on the electrical phenomena of the lower atmosphere.

Although the spectacular phenomena of the lightning discharge attracted the attention, in the 18th century, of a group of pioneers, of whom Franklin was the most outstanding, the early history of the subject consists mainly of the investigation of effects observable in fine weather. The existence of these has been known for at least two hundred years and they have been intensively observed for nearly a century. With the completion of the main outlines of the picture, although certain perplexities still remain, and with the new possibilities opened by modern experimental techniques, the emphasis has tended to shift more and more towards the study of the complicated electrical effects associated with disturbed weather conditions. A great and ever growing effort has been put into these investigations during the past twenty years, and while our knowledge of the facts has expanded enormously, our understanding of the basic physical processes behind them has hardly kept pace. It has become progressively

more probable that in these disturbed weather effects, and supremely in the electrification of cumulonimbus clouds, lies the source of the electric field of fine weather, but an adequate understanding of the mechanism whereby a thundercloud acquires its electrical charges still eludes us.

This review will be concerned mainly with the electrical effects associated with, or originating from, precipitation clouds, and, in particular, cumulonimbus clouds. Reasons of space would, in themselves, forbid any attempt to make either text or bibliography really comprehensive, and attention has been concentrated on an attempt to tell a connected story by selecting what seemed most relevant or most suggestive. All discussion of what might be called the fine structure of the lightning discharge process has perforce been omitted.

In making such a selection, some degree of personal prejudice is almost unavoidable; perhaps this may serve as a general apologia.

2. The vertical electric field and current of the atmosphere in fine weather

The fundamental electrical phenomenon of the lower atmosphere is a vertical electric field which, with a clear sky, is almost invariably directed downwards. The magnitude, in such conditions, of the vertical gradient of potential, near to the ground, is of the order of $+ 100$ volt/metre (v/m); it involves the existence of a negative charge of about $- 3$ esu/m^2 on the surface of the conducting earth. These effects, which have been studied in many places and over long periods of time, are of the same sign and order of magnitude, in fine weather, over the whole earth. The field is somewhat enhanced near to human habitation and particularly in industrial areas; thus, at Kew, the mean value of the field in fine weather is $+ 363$ v/m. Over the oceans the mean value is $+ 130$ v/m and over the whole earth perhaps $+ 120$ v/m.

Balloon observations have shown that, as one rises from the ground, the intensity of the field soon diminishes and has fallen, at a height of 10 km, to a few per cent of its value at the surface. The lower atmosphere thus normally contains a positive space charge.

Ordinary air is a slight conductor of electricity due to the existence in it of ions, which are produced by the radiations from traces of radioactive material in the ground and in the air, and also by phenomena induced by the penetrating cosmic radiation which originates outside the terrestrial atmosphere. Over land the main sources of ionization in the lower layers are the radioactive substances derived from the ground, but the contribution from the cosmic radiation increases steadily with increasing height. Over the oceans, the cosmic rays are the main contributor even in the surface layers. The ionic equilibrium, which has been studied in detail by numerous workers, is a complicated matter. The primary ionization process consists in the ejection from a molecule of an electron; this at once attaches itself to another neutral molecule and very soon there exists a small ion of each sign, consisting of a positively or negatively charged molecule surrounded by a cluster of a few neutral molecules, and driven at a velocity of about 1·5 cm/sec in a field of 1 v/cm. The small ions disappear, in part by recombination, but also, effectively, by becoming attached to much larger neutral particles, such as condensation nuclei or pollution particles, a process which reduces their mobility to a small fraction of its original value. While there will thus exist, normally, ions with a whole range of mobilities, it is sufficient for many purposes to consider them as made up of a group of small ions with mobilities of the order of 1·5 cm^2 sec^{-1} v^{-1}, and of a group of large ions with mobilities of the order of

10^{-4} cm^2 sec^{-1} v^{-1}. The precise numbers in the equilibrium state are very variable, being dependent on local conditions; the figures listed in Table 1 are not untypical for the surface layers. It is noteworthy that the number of small ions present and the conductivity, over the ocean, are rather larger than over land despite the much greater rate of ion production in the latter case.

TABLE 1. NUMBERS OF IONS OF EACH SIGN

	Rate of production (cm^{-3} sec^{-1})		Number present (cm^{-3})	
	Due to radioactivity	Due to cosmic rays	Small ions	Large ions
Oceanic air	0	2	700	200
Country air	8	2	600	2,000
City air	8	2	100	20,000

The numbers of positive and negative ions are not very different but there is necessarily a small excess of positive ions to make up the positive space charge of the lower atmosphere.

In consequence of the vertical field and of the conductivity of the air, an electric current flows downward, in fine weather through the lower atmosphere into the earth, the positive and negative ions moving downwards and upwards respectively. The magnitude of the conduction current, in the free air, is given by

$$i = \mathbf{e}\, X \sum_r ({}_r n_1\, {}_r k_1 + {}_r n_2\, {}_r k_2)$$

where i = current intensity
$\mathbf{e}$ = magnitude of the electronic charge
X = intensity of the vertical field
${}_r n_{1,2}$ = number of positive (or negative) ions of type r per unit volume
${}_r k_{1,2}$ = mobility of positive (negative) ions of type r.

If there is a vertical gradient of space-charge density, turbulent transfer will cause a vertical convection current and, indeed, careful consideration is necessary in attempting to derive the total vertical current into the ground from measurements of the field and of the polar conductivities of the air. It is possible however to measure the current directly; its mean value over the oceans is nearly 4×10^{-16} a/cm^2; over land the value is more variable and 2×10^{-16} a/cm^2 may perhaps be taken as a representative value.

The conductivity of the air increases fairly rapidly with increasing height above the ground, for two reasons; ionic mobility is greater at lower air densities and the number of ions present is also greater owing to the enhanced effect of the downcoming cosmic radiation. A vertical current, independent of height, is thus maintained through a considerable vertical thickness despite the decrease in the vertical field at higher levels.

Our picture of these fundamental processes, and our understanding of the conditions which must be fulfilled by any mechanism which maintains the fine-weather field and current, are incomplete and imprecise unless we include the effects of the very high conductivity of the upper atmosphere (Wilson 1906). Almost the whole of the difference of potential between the ionosphere and the earth occurs across the layers accessible to balloon observations and its magnitude can thus be estimated reliably

from direct measurements, in fine weather, and is of the order of 4×10^5 v. The ionosphere can be regarded for our present purpose as an equipotential region and hence this figure is true for the whole earth, including, for example, any regions affected by thunderstorms. We are thus led to picture a leaking spherical condenser with a negative charge, of the order of 5×10^5 C, on the inner sphere (the earth), the corresponding positive charge being distributed through the slightly conducting space between the electrodes, and the current into the inner electrode being of the order of 1800 a. In the absence of a balancing process most of the charge would disappear in 5 min.

Wilson (1920) suggested that the generators, which maintain the current and the potential gradient observed in fine-weather regions, are thunderstorms and showers. The negative charge, which predominates in the lower part of a thundercloud, induces a positive charge on the neighbouring part of the earth's surface and hence an equal negative charge appears on remote parts of the earth, but this effect appears to give only a small fraction of the negative charge observed in fine-weather regions. The thunderstorm, however, in which the differences of potential are enormous compared with the potential of the ionosphere, also drives a vertical current upwards from the earth, through the cloud and on into the highly conducting regions above. This current is carried, between the ground and the cloud, in part discontinuously by lightning discharges, in part continuously by convection on precipitation and by ionization currents, a plentiful supply of ions originating from point discharge at the earth's surface. Above the cloud the current appears to be carried mainly by continuous ionization currents. The sum of the upward currents, maintained in this way by the earth's thunderstorms, seems to be of the same order as the total downward current observed in fine-weather regions. With the steady extension of our detailed and reliable knowledge of the magnitude of these various effects, the correctness of this theory has become increasingly probable and it now seems to be fairly generally accepted. It would perhaps be more logical, in our present state of knowledge, to approach the matter from the other direction. We know that, between the conducting earth and ionosphere, thunderstorms drive negative charge to the earth and positive charge into the upper atmosphere. We thus expect a potential difference to develop and a leakage current in the undisturbed parts of the earth; both should increase until the current leaking away just balances that supplying the charge in thundery regions. Such a current is, in fact, found in fine-weather areas; it is in the predicted direction and, as far as can be estimated, of the right order of magnitude. Until some discrepancy is proved, it would seem unnecessary to look for other answers to the classical problem of the maintenance of the earth's negative charge.

The electric field near the ground is in a state of continuous fluctuation about its mean value. Fairly rapid variations, with periods of the order of seconds, are usually of small amplitude and can be ascribed to turbulence and an uneven space-charge distribution in the vicinity. Again, with a sufficiently sensitive and rapid indicating system, disturbances due to distant lightning discharges may always be detected; the brief disturbance, ' atmospheric,' radiated by a lightning discharges several thousand kilometres distant, frequently exceeds 0·1 v/m in amplitude. On a rather different time-scale, the fine-weather field shows, on the average, a well-marked regular diurnal variation, with an amplitude of perhaps 30 per cent, which at most ordinary observing stations, shows a characteristic dependence on local time and is hence controlled primarily by local causes. A very close correlation has been demonstrated between the diurnal variation of potential gradient and the diurnal variation of atmospheric pollution (Chree and Watson 1924, Whipple 1929b).

A measurement of the potential gradient near the earth's surface may be considered as a large-scale potentiometer experiment. The difference of potential, v, between a point at a height of 1 m, say, and the ground, is given by the equation

$$v = Vr/R$$

where V is the total potential difference between the conducting upper atmosphere and the earth, R is the total resistance of a vertical column of air, of unit cross-section, extending from the ground to the ionosphere, and r is the resistance of the first metre of that column. Ignoring, for the moment, possible variations in V, a diurnal variation of v, dependent on local time, must depend on variations in the ratio r/R. The close correlation with the degree of local pollution demonstrates that the variations in conductivity are to be ascribed, mainly, to the immobilizing of small ions by their attachment to pollution particles. The exceptionally large mean value of the potential gradient at such a station as Kew finds a similar explanation.

A significant advance was made by Mauchly (1921, 1923) with the discovery that the diurnal variation of the potential gradient over the oceans, with an amplitude of the order of $\pm$ 15 per cent, was of a simple type, and, over a large range of longitudes, dependent on universal time, and was, moreover, in phase with the similar variations at seven places in the Arctic and Antarctic regions, as indicated in data assembled by Hoffmann (1924). It thus appeared that there was a world-wide variation of V, the potential of the conducting ionosphere, during a Greenwich day, although its effects might be masked by local causes except in favourable circumstances.

Wilson (1922) pointed out that the theory which sought the origin of the fine-weather potential gradient in thunderstorms would predict a world-wide maximum when the most thundery regions of the earth attained their maximum activity, and suggested tentatively that Mauchly's results were significant in this connection. Whipple (1929a) investigated the matter in some detail, and showed that the results assembled by Mauchly, Hoffmann, and others were in excellent agreement with the theory. This conclusion is illustrated by Fig. 1, reproduced from Whipple's paper.

3. The electric fields produced at the earth's surface by thunderstorms and lightning discharges

The first reliable determinations of the magnitudes of the charges involved in thunderstorm processes were due to the investigations of Wilson (1916, 1920). His experimental technique abandoned the classical potential collector and consisted in recording continuously the electric charge on a conductor of simple geometrical form, which was maintained automatically at earth potential. This was achieved by the ingenious use of a form of capillary electrometer whose indications were recorded photographically. The time of response of the whole system, to a sudden change in the vertical field, was less than 0·1 sec. After applying a small correction for the distortion due to the small hut, which protected the recording apparatus, the method yields absolute values of the vertical field over level ground, and of the sudden changes which accompany lightning discharges, and delineates reliably the form of the variation of field which follows a discharge (the 'recovery curve'). If $M = 2\Sigma h\,\delta q$ be the electric moment of the charge distribution in a thundercloud, where δq is the charge in a small volume element at a height h, then the field-change F observed at the ground at a distance L, which is large compared with the dimensions of the charged region involved, is given by

$$F = \delta M/L^3 ;$$

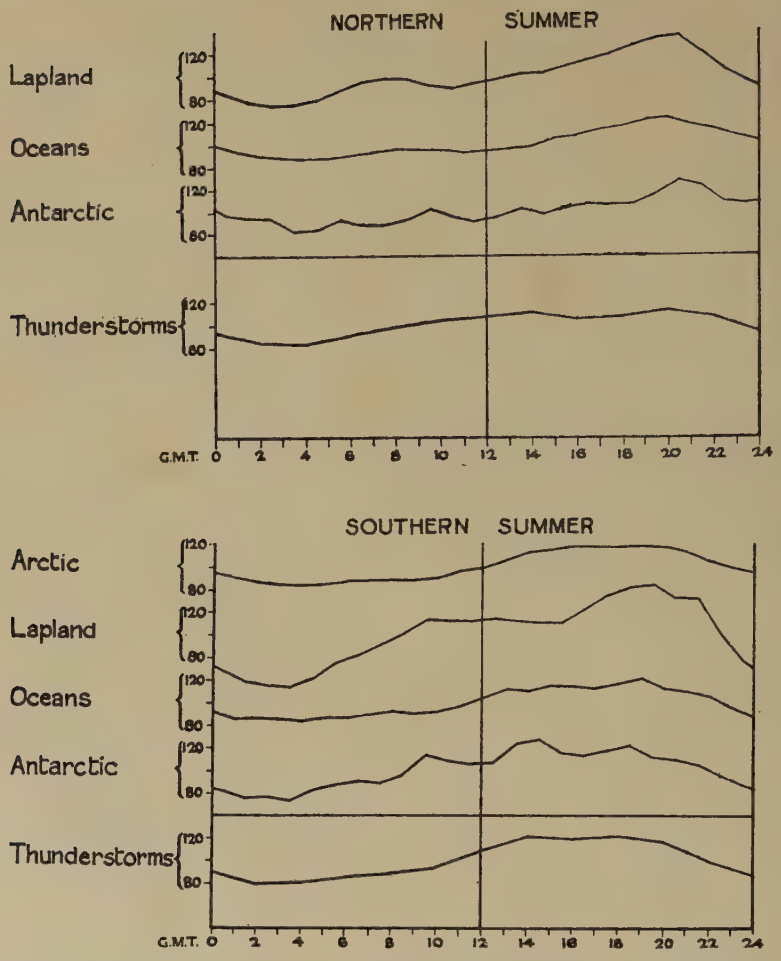

Figure 1. Diurnal variation of potential gradient and of thunderstorm frequency. Percentages of mean values (Whipple).

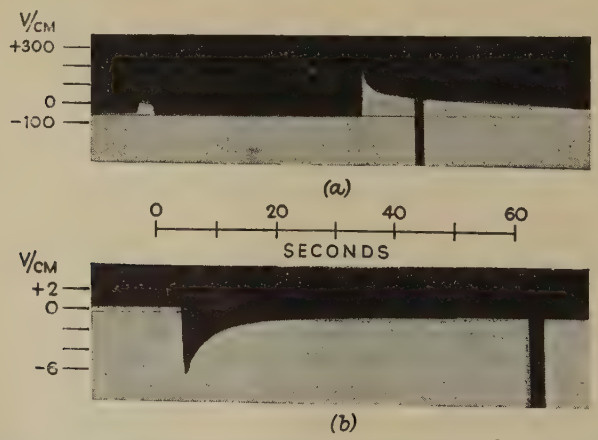

Figure 2. Field-changes and recovery curves associated with : (a) a close discharge (3 km) and (b) a distant cloud discharge (19 km). The vertical black lines indicate the commencement of thunder.

(a) Pre-discharge field — 55 v/cm; field-change + 375 v/cm. Note the rapid decay of the intense field after the discharge.

(b) Pre-discharge field + 0·1 v/cm; field-change — 6·6 v/cm. Smooth recovery curve.

δM is the electric moment destroyed by the discharge. This relation has been verified for the electrostatic effect of an average lightning discharge for values of L from 20 to 200 km (Pierce 1950). The median value for δM is about 110 C km (Wormell 1939, Pierce 1950). Values for individual flashes may vary over a range exceeding 10 to 1; there are no marked differences in magnitude between positive and negative changes, or between those due to flashes to earth and those entirely in the cloud. The charge transported in an average flash is of the order of 20 C to 30 C. If a charge of this order be distributed throughout a spherical volume, of a size such as to produce the breakdown field at its surface, the potential differences produced are of the order of 10^9 v. This should perhaps be regarded as an upper limit, but the actual differences of potential existing between the poles of a thundercloud are likely to surpass 10^8 v.

Before proceeding to discuss the signs of the observed field-changes it is convenient to recall the usual sign convention. A vertical field will be termed positive if it exerts a downward force on a positive charge; a field-change is positive if it increases a positive field, or decreases the magnitude of a negative field. A bipolar cloud is said to be of positive polarity if the mean height of the positive charge exceeds that of the negative charge. The field-change, due to a discharge which conveys negative charges only to earth, is positive at all distances. That due to discharge in the cloud, between charges at significantly different heights, reverses in sign at a certain distance from the discharge (Wilson 1920). In particular, the field-change, due to a flash which neutralizes a charge distribution of positive polarity, is positive at short distances, negative at great distances, and vanishes at a distance which, if the axis of the bipole is vertical, is of the order of $2^{\frac{1}{2}}$ times the mean height of the two charges.

The effects of such a reversal are very clear when field-change records at various distances are examined, and they provided, in the hands of Wilson (1920) and of Appleton, Watson-Watt and Herd (1926), the first clear indication that the charge distribution in a thundercloud, as evidence by lightning discharges, is of positive polarity. The effect is particularly clear-cut in South African storms, where the proportion of cloud discharges is higher. Schonland (1928a) reported, as a result of observations on some 50 storms, that, for discharges within 7 km, positive field-changes were 21 times as frequent as negative; for discharges beyond 15 km, negative discharges were nearly 10 times as frequent as positive. About 10 per cent of the flashes were discharges bringing a negative charge to earth; most of the remainder being cloud discharges in a cloud of positive polarity. A prolonged series of observations at Cambridge (Wormell 1939, Pierce 1950) has shown that, in English storms, the ratio of the numbers of positive and negative field-changes falls steadily from about 4·4, for flashes within 5 km, to about 1·1 at 30 km, and remains constant beyond that distance. While the proportions of flashes of different types are thus quite different in the two series of observations, in both cases the typical flash to earth brings down negative charge and the typical cloud discharge destroys a bipolar distribution of positive polarity. The figures given are averages over a long period; great differences occur in the behaviour of individual storms, in England, and also at different stages in the development of a storm.

Studies of the actual fields due to a thunderstorm, as distinct from the field-changes, yield results which, while less clear-cut, are in general agreement with the field-change data. Beneath the storm, the intense fields are predominantly negative (Wormell 1927, 1930, 1939), but periods of equally intense positive field also occur, often associated with very heavy rainfall. When the storm is moderately distant, there is, on the average, a definite, although not very marked, enhancement of the normal positive field.

The fields observed at the earth's surface beneath a thundercloud are considerably less than would be expected from the value of the electric moment destroyed by a lightning discharge, and are, indeed, less than the field-changes produced by close discharges (Wormell 1939). A flash within 3 km, say, may cause a field-change exceeding 500 v/cm, while the quasi-steady pre-discharge field is not often appreciably greater than 100 v/cm. In such cases the very intense field after the discharge, which is usually of the opposite sign to the pre-discharge field, rapidly dies away, (Fig. 2 (a)). The effect is usually ascribed to a space-charge blanket, the quasi-steady pre-discharge field being limited, by space charge, to the value at which the point-discharge currents, from prominent objects such as trees on the earth's surface, reach the maximum value which the cloud can maintain. The very strong fields existing temporarily after a flash are then due directly to the space charge. While this seems a very satisfying explanation, it must be admitted that the marked increase of the field intensity with height, which is implied by the presence of such a space charge, has not as yet been directly observed.

The actual form of the variation of field during the discharge process has been studied extensively at Cambridge (Wormell and Pierce 1948, Pierce 1950 and 1952, Wormell 1952), using oscillographic equipment, and taking special precautions to delineate faithfully the effects of the slower processes in the discharge which have been largely overlooked in the past (Fig. 3). For the distances at which these observations were made (30 km to 150 km), the field at any moment is proportional to the instantaneous value of the varying electric moment of the cloud. The field-change due to a flash to earth, consisting of one stroke, and carrying negative charge to the ground, is found to consist of a slow *L* portion, of mean duration 0·1 sec, followed by a rapid *R* step, with a duration of the order of 1 m sec, which may be followed again by a slow *S* portion, of mean duration 0·12 sec. A discharge to earth, consisting of several strokes, causes a field-change which again starts and ends with slow *L* and *S*

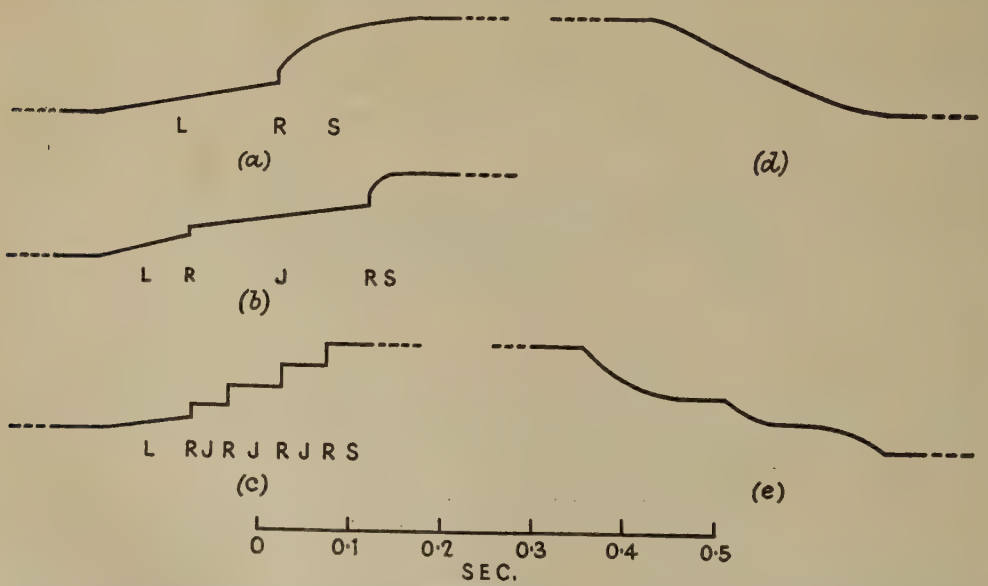

Figure 3. Examples of field-changes due to distant flashes. (a), (b) and (c) are due to discharges to earth with 1, 2, and 4 strokes respectively, all bringing down negative charge. (d) and (e) are cloud discharges which destroyed positive electric moment. The rapid steps, marked *R* in (a), (b), and (c), are characteristic of flashes to earth, although they may only make up a fraction of the total change of field due to the discharge (Pierce).

portions, possesses an *R* step corresponding to each stroke, and, joining the *R* steps, slow *J* portions during which the change in field may be considerable (Fig. 3(b)) or may be inappreciable (Fig. 3(c)). The average magnitudes of the change in electric moment associated with the various portions of a field-change bringing negative charge to earth and consisting of two strokes are given in Table 2. The *L*, *R*, *J*, *S* notation is a modification and extension of that introduced by Malan and Schonland (1951a).

TABLE 2. AVERAGE CHANGE OF ELECTRIC MOMENT (C Km) ACCOMPANYING VARIOUS STAGES OF A DISCHARGE TO EARTH WITH 2 STROKES

L portion	1st *R* step	*J* portion	2nd *R* step	*S* portion
31	28	Zero in 75 per cent of cases 50 (mean of the other 25 per cent)	34	19 (if present)

The interpretation of these observations is fairly clear, starting from our knowledge of the lightning-discharge process, as disclosed, primarily, by the work of Schonland and his collaborators (Schonland *et al.* 1935, 1938), and of Appleton and Chapman (1937). The *L* portion is to be associated with a downward developing leader, although its duration is very considerably greater than previous work had suggested. The *R* step is the result of the violent return stroke, which occurs when a leader reaches the ground. The duration of these rapid steps, to which the changes in electric moment given in Table 2 correspond, is normally 1 to 2 m sec; the initial portion of an *R* step is very fast and perhaps corresponds more closely to what has been called a return stroke; the change of electric moment associated with it is only a fraction of that given in the table. The absence of any appreciable change of field during the *J* portion, in 75 per cent of examples, is interesting. During these intervals a positively charged leader streamer is believed to be travelling from the top of the channel, blazed by the first return stroke, towards the next volume of negative charge to be tapped. Zero change of field, during this transport of charge, indicates that the electric moment is not changing, i.e., that the leader is travelling horizontally. It can therefore be concluded that in 75 per cent of multi-stroke flashes to earth, in this country, the negatively charged regions in the cloud, which are discharged to earth by the successive strokes of the flash, are all at sensibly the same height. In the other 25 per cent of cases the form of the field-change indicates that the heights of these regions steadily increase for the successive strokes. This appears to be the typical case for South African storms which are of great vertical dimensions (Malan and Schonland 1951b).

Another result of some practical importance comes directly out of these observations. Since the presence of an *R* step is an unambiguous indication of a flash to earth, it is possible to deduce the proportion of flashes of the various possible types, which occurred in the storms recorded, without any risk of bias or selection by the apparatus. The results of an examination of some 1,500 field-changes are summarized in Table 3.

TABLE 3. PROPORTION OF FLASHES OF VARIOUS TYPES IN THUNDERSTORMS OBSERVED AT CAMBRIDGE

Flashes to earth (per cent)		Discharges not reaching the earth (per cent)	
Bringing down negative charge	31	Moving negative charge upwards	45
Bringing down positive charge	4	Moving negative charge downwards	10
Bringing down charge of both signs	2	Moving negative charge in both directions	8
Total	37	Total	63

As was to be expected, the commonest types of discharge are, first, a cloud discharge which moves negative charge upwards and presumably occurs between the two main charges of the thundercloud, and secondly, a discharge to earth from the negative charge in the lower part of the cloud.

The field-change due to a discharge is followed by a recovery of the field. When observations are made at moderate distance, 15 km to 20 km say, from the discharge, the form of this recovery frequently approximates closely to an exponential curve, Fig. 2 (b), with a time-constant of about 7 sec (Wormell 1939). The interpretation of these ' recovery-curves ' has been discussed by Wilson (1929) and Wormell (1939). Let us consider the not unusual case where a cloud is producing internal discharges, each destroying a positive bipolar charge distribution, of electric moment 110 C km, after which the charges and, in particular, the electric moment are regenerated. Beneath the cloud, immediately before the discharge, there will be a negative field of the order of 100 v/cm, and between the ground and the cloud base a positive space charge. This space charge will be mainly in the lower layers near the ground (Wilson 1925). Immediately after the discharge in the cloud a very intense field exists at the ground beneath. This is due to the positive space charge and most of it will disappear very rapidly (Fig. 2 (a)) driven into the ground by the strong field or neutralized by a rush of negative ions from point discharge at the surface. At a distant observing point there are no complications due to local point discharges or space charge. Just before the flash, the field here is proportional to the electric moment of the cloud and of the space charge beneath it. The magnitude of the total space charge is of the same order as the total negative charge in the cloud but since it resides mainly in the lower layers its electric moment will be small (less probably than 20 per cent of that of the cloud). The cloud discharge causes a negative field change at the distant station, and the subsequent rapid disappearance of the space charge a further small change in the same direction; this could scarcely be distinguished from the effects of the discharge itself, which may well have a duration exceeding 0·2 sec. Thereafter, the exponential variation of field (Fig. 2 (b)), effectively pictures the regrowth of electric moment in the cloud. Its initial rate is the rate at which the cloud can regenerate electric moment when the internal field is small; the diminution of rate, which follows, is explained by the increasing electrostatic forces on the charged elements, as the internal field grows, and by the increased rate of dissipation, by continuous currents, of the growing free charges as they develop. It is difficult to estimate the relative importance of these two effects but the observed charges on individual drops (Gunn 1950) show that the former cannot be neglected; both would tend to cause the recovery to approximate to the exponential form. The field observed at a distant station often becomes nearly constant for some time (Fig. 2 (b)) before the next discharge occurs; the diminished rate of generation then nearly balances the dissipation. The charge neutralized in an average flash is of the order of 20 C and the initial rate of regeneration about $1/7$ sec^{-1}; the vertical current inside the cloud is thus about 3 a, immediately after the internal discharge.

There is, however, a more fundamental aspect of the matter. In order to generate a large electric moment, it is not sufficient simply to attach opposite charges to neighbouring particles of different sizes; the charges must also be separated through a large vertical distance. Further, the rate at which the vertical separation occurs cannot exceed the rate of fall of the precipitation particles in still air.

Let us consider a volume of cloud, throughout which, after a discharge, the field is small, and let us assume that there exist, distributed through this volume, a net charge Q, attached to cloud particles or in the form of large ions, and a charge $-Q$ on precipitation elements. The initial rate of regeneration of moment would, if it

remained constant, replace the whole moment destroyed by the discharge ($2\Sigma h\,\delta q = 110$ C km), in 7 sec. The maximum vertical separation which the charges Q can acquire in 7 sec is about 40 m. It follows that the magnitude of the positive and negative charges which are separating, in the first second or two after the discharge, is about 1,400 C. These charges would necessarily be distributed through a very considerable volume which was, on the whole, electrically neutral, but as the separation proceeds throughout the volume, a positively charged region would appear near the top, and a negatively charged region near the base of the cloud.

A further condition must be satisfied. The vertical separation of the charges proceeds until the field builds up to a value at which another discharge is initiated. Just before this occurs, the precipitation particles must still be falling, since the electric charge being lost by continuous dissipation currents, from above and below the cloud, must be continually replaced. In other words, the total weight of the precipitation elements in the active region must exceed the upward force they experience, due to their electric charge Q, in the large-scale field. Recent observations, to be considered in Section 4, suggest that this field, over a considerable region, is of the order of 1,000 v/cm. The weight of precipitation must thus exceed $1{\cdot}4 \times 10^{10}$ g, and probably by a very substantial margin.

The magnitudes of the charges deduced is rather startling; they would be sufficient without replenishment, to supply the charges removed in a considerable number of lightning discharges. It would thus seem that the primary charge-generating mechanism, whatever its nature, need not necessarily supply new charge to the region as rapidly as it is being dissipated, during a brief spell of intense lightning activity.

It has sometimes been objected that discussion based on a simple bipolar model for a thundercloud, or thunderstorm cell, is an unrealistic oversimplification of the facts. The simple picture has been used in this section because it is all that is indicated by a study of the principal effects of lightning discharges, and all that is needed in their interpretation. It is not intended to imply that the structure of an actual cloud is necessarily as simple as this.

4. The detailed electrical structure of a cumulonimbus cloud

The pioneer investigations in this field were carried out from Kew Observatory (Simpson and Scrase 1937, Simpson and Robinson 1941). A point-discharging apparatus was attached to a sounding balloon, the current passing between two steel points pressing on a special type of pole-finding paper. With this device, termed an alti-electrograph, a reliable record of the sign of the vertical component of the field was obtained at all heights during the flight of the balloon, released under an active thunderstorm; a rough indication of the intensity of the field was also given by the width of the trace. The signs of the fields consistently observed indicated that the upper part of a thunderstorm is positively charged and most of the lower portion negatively charged; in other words, the observed field was negative beneath the cloud, positive within the cloud, and negative again above the cloud top. In nearly all the thunderstorms investigated at Kew, there was also evidence of one or more localized regions, in the base of the cloud, with positive charge, usually associated with very heavy rain. Simpson and Robinson indicated that their results could be represented schematically by a thunderstorm model with an upper charge of $+24$ C, distributed in a spherical volume of 2 km radius, centred at a height of about 6 km. Beneath this was a negative charge of -20 C, in a sphere of 1 km radius, centred at a height of 3 km, and beneath this, again, a charge of $+4$ C, in a sphere of radius 0·5 km, centred at a height of 1·5 km. They emphasized that the upper positive charge and the bulk of the negative charge

were in regions where the temperature was below 0°C; the lower positive charge was at a temperature above the freezing point. The lower positive charge appeared usually to be absent in a shower, as distinct from a thundercloud.

These results gave precision to our picture of the charge distribution in a thundercloud. The actual field intensities suggested by the records, both beneath and within the cloud, were surprisingly small and barely exceeded 100 v/cm. These observations have often been quoted as evidence that the general field intensity inside a cloud is not very great, and that regions of intense field are rare and of very limited extent. The authors themselves, however, expressed caution as to such an interpretation of the records, and Robinson (1951) has pointed out that 15 per cent of the useful ascents came to a violent end in a rapidly increasing field.

Similar observations and results regarding the charge distribution have been reported by Chapman (1950), who used a radiosonde device, the observations being telemetered to the ground. The strongest field indicated was 210 v/cm. Kuettner (1950) has studied records of point-discharge current obtained from the Observatory roof on the Zugspitze (10,000 ft), the apparatus at this altitude being effectively situated within the thundercloud. The same structure was again indicated; the lower positive charge centred at about 0°C, in a region with horizontal dimensions less than 1 km, the main negative charge in a larger region, centred near — 8°C, and the main positive charge higher still and yet more extended in space. Finally, observations of the currents flowing down a barrage balloon cable in stormy weather, made by Davis and Standring (1947), in this country, are also in agreement with the same general structure.

Gunn (1947, 1948, 1949a, 1950) has reported some extremely interesting results of measurements, from an aircraft, of the electric field, and also of the charges on individual precipitation elements, inside thunderclouds and other precipitating clouds. The field measurements involved simultaneous observations with field meters (of the type of a ' field mill ' or generating electrostatic voltmeter), situated above and below the plane fuselage, in order to eliminate the effects of any net charge acquired by the aircraft. The charge on the individual precipitation particles inside the cloud was studied by observing the momentary pulse, induced in a metal ring as they shot through without touching it. The apparatus was calibrated by firing bullets through the ring and catching them in a Faraday cylinder!

The mean of the maximum field intensities encountered in each of nine storms was 1,300 v/cm; on one occasion a field of 3,400 v/cm was measured just before the aircraft was struck by lightning. (The figures given represent the vertical field actually observed at the surface of the aircraft body, as deduced from the readings of the two field meters; the true field, undistorted by the presence of the aircraft, will be smaller than the quoted value if its direction is nearly vertical, but it may be larger if has a large horizontal component, as was sometimes the case, when a current was observed to be flowing into one wing-tip and out from the other.) The most intense fields were usually found near the 0°C level. Frequent reversals of sign were encountered.

A series of interesting observations of the charge on the precipitation particles was made in clouds at a weak cold front which exhibited no thunderstorm activity. With the freezing level at 11,000 ft, all the precipitation particles between 20,000 and 26,000 ft carried positive charges; below 10,000 ft they were all negative. Between 10,000 and 20,000 ft both signs were encountered. The mean magnitude of the charge per particle was about 0·036 ESU. The charge per unit volume, carried on the precipitation particles, was sufficient to cause very intense large-scale fields. As such were not observed, it was concluded that a compensating charge was present, carried presumably on cloud particles. Such charges could not, of course, be detected by the technique employed.

A similar investigation was also carried through, inside an active thunderstorm, at a series of different levels; the results were extremely complex. Particles of both signs were observed at nearly all levels, although regions were found where only one sign was present. The most intense electrification, 0·27 ESU per drop, was found at 7,500 ft where the temperature was + 10°C; the freezing level was at 14,000 ft. The charge per particle was less than one quarter of the maximum value at both 5,000 and 20,000 ft. The large-scale fields observed had intensities ranging up to 2,000 v/cm.

The apparent space-charge distribution, on the precipitation, was very complicated. The charges on the precipitation elements were very high, sometimes near the maximum physically possible, and, in the regions of uniform charge, would have produced impossibly high fields unless compensating charges were present, presumably, again, on cloud particles. It may be noted that the mechanical force on a drop carrying a charge of 0·27 ESU in a field of 2,000 v/cm is 1·8 dyne; this is equal to the weight of a drop of about 1·5 mm diameter. The electrical forces on the precipitation elements may thus profoundly affect their rates of fall and hence their general history.

Gunn's observations have shown that intense fields are not uncommon inside a thundercloud. They have demonstrated, further, that highly-charged precipitation elements occur over a great range of heights in the cloud. These charges are effectively neutralized, to a considerable extent by an opposite charge on smaller particles. While the earlier balloon work performed the invaluable function of establishing a typical and reasonably simple structure for a thundercloud, it now seems probable that balloons fail systematically to reach the regions of intense field and to survive. Their failure to indicate any considerable increase of field intensity as they rise from the ground to the cloud base remains puzzling.

5. Vertical electric currents due to thunderclouds

The observations so far discussed have established that the electrical structure of a thunderstorm is such as to maintain a vertical current from the earth upwards into the cloud base, and also to drive a current upward from the top of the cloud; they show further that the potential of the upper atmosphere does, in fact, vary in phase with the rise and fall of the total thundery activity on the earth's surface. We must now consider the various processes which convey the current, and whether its magnitude is adequate to balance the charge which leaks away from the upper atmosphere in fine-weather areas.

Electric charge is transported between the earth, and the base of a thundercloud by lightning discharges, by convection on charged raindrops, and by ionization currents; these three processes will be considered in turn.

The great majority of lightning discharges to earth bring down negative charge, the average magnitude being of the order of 20 C; this is true wherever systematic observations have been made. In England, flashes to earth comprise about 40 per cent of all discharges (cf. Section 3, Table 3); in lower latitudes the proportion tends to be considerably smaller. The typical thunderstorm cell produces, during its period of maximum activity, several flashes per minute; such activity does not usually persist for more than, say, half an hour. The effective current carried in this way, beneath a single cell, may thus reach values of a few amperes for brief periods. In an extensive storm successive cells develop and several may be active simultaneously. The total thunderstorm activity over the earth has been estimated roughly (Brooks 1925) to be equivalent, on the average, to 100 flashes/sec. Most of these storms are in tropical regions and the proportion of discharges to earth is perhaps 10 per cent. Unless, therefore, Brooks's figure is a marked underestimate, the total negative charge transported

to earth by lightning is appreciably less than the fine-weather current of 1800 a.

The charge carried to the ground by thunderstorm rain bears no simple relation to the charge on the raindrops when they leave the cloud, since they are falling, usually, through an upward stream of ions, of charge opposite to that in the cloud base (cf. Section 7). The net charge brought to the ground by thunderstorm rain is, indeed, positive, despite the predominance of negative charge in the cloud base and of negative fields at the ground. Simpson (1949) finds, for rain in general, that in intense fields the charge brought down is of the opposite sign to that of the field. While a reliable estimate of the average effect is difficult, the mean downward current carried on the rain must be only a fraction of the upward ionization current, except possibly in extremely heavy precipitation.

The ionization currents in the intense fields beneath thunderstorms are very considerable, since a plentiful supply of ions originates from the brush discharges from prominent objects on the earth's surface, such as trees and other vegetation. Experiments by Wilson (1923) showed that appreciable discharge currents occur from a grass-covered surface in fields not greatly exceeding those found beneath thunderstorms, and it is not improbable that it is this factor which determines the actual values attained by the field, at the ground, during a storm over land. Wilson emphasized the importance of the point discharge and, indeed, suggested that it probably played the major role in the vertical transport of charge from the earth in stormy conditions.

Wormell (1927, 1930) made a systematic investigation of the discharge from an elevated metal point, 12·3 m high, in the centre of a field fringed by fairly high trees, the nearest of which was 70 m from the point. The discharge current, during a storm or period of rain, was integrated by means of a special form of gas micro-voltameter, which indicated the charge of each sign which had been lost from the point, as well as the net effect. Some observations were also made of the actual behaviour of the current during a storm, supplemented by observations of the vertical field. This investigation, which extended over about four years, demonstrated that, during periods when the field was sufficiently intense to cause appreciable discharge from the point (fields exceeding about 8 v/cm), there was a marked predominance of negative fields. Over a lengthy period, the total loss of positive charge from the point was about twice the negative charge dissipated. The currents reached maximum values of about 15 μa, and the net positive charge lost from the point while an active thunderstorm or shower passed over, might be 15 mC. The average net annual loss of positive charge from the point was 0·12 C. Experiments with various pointed objects, in an artificial field, showed that the current from a point of the type employed was not very markedly different, in magnitude, from that from a twig, leaf, blade of grass, etc. at the same height. It therefore seemed possible that the observed currents might be of the same order as those from a tree of the same height as the artificial point. After making a census of trees in the surrounding countryside, Wormell (1930) estimated tentatively that the point-discharge current/km^2 should be at least as great as that from 800 points of the type which had been used. This would entail an ionization current of at least 0·012 a/km^2 beneath the central part of a thunderstorm. Similar observations have been made with artificial points by various workers, in particular by Whipple and Scrase (1936), at Kew, who recorded the current continuously, and by Chiplonkar (1940) in India. They confirm the predominance of negative fields, when the field is large; in this connection, Chiplonkar's work is specially valuable in that it gives information concerning tropical storms. The magnitudes of the currents observed, from an artificial point, in all three investigations are in reasonable agreement when some allowance is made for differences in exposure of the points.

Schonland (1928b) made observations on the discharge current from a small tree, 4 m in height, and sufficiently small to be uprooted and insulated. He estimated a current of 0·16 a/km² due to point discharge beneath the central region of a thunderstorm, in a field of 160 v/cm. Simpson (1949) has estimated, from a study of simultaneous records of the charge on rain, the point-discharge current, and the field, obtained at Kew, that, in that neighbourhood, the total point-discharge current/km² from the surrounding country is 2,000 times that observed from the Kew artificial point; this gives a current of the order of 0·02 a/km² under the central part of a storm. Again, Smith (1951a) has estimated the magnitude of this current, at Cambridge, from a study of the manner in which the very intense fields, which follow close lightning discharges, die away (Fig. 2a). Using Wormell's observations (1939), Smith deduced a value of 0·018 a/km². Currents of this order are also suggested if, following Wilson (1925), we consider the simple model of a uniform cloud base, and uniform point-discharge current from the earth, ignore the effects of charges on the precipitation, and assume a field of 100 v/cm at the ground and 3,000 v/cm at the cloud base.

To sum up, it seems very probable that, for some reason, Schonland's figure is a considerable overestimate, even after allowing for the rather high fields he reported. Wormell's early figure was intended to be a conservative estimate and may well be a little low. A value of 0·02 a/km² in the regions of strongest field now seems about the best estimate we can make. Chalmers (1952) has given reasons for believing that the current per unit area is rather insensitive to the nature of the earth's surface covering. With this figure, the net upward ionization current beneath an active thunderstorm may well be of the order of 0·5 a.

An interesting new piece of evidence comes from the work of Gish and Wait (1950), who have succeeded in measuring, from an aircraft, the vertical current upwards from the summit of a thundercloud towards the upper atmosphere. Observations were made over 21 different storms; in all cases an upward current was observed. One storm gave an exceptionally large current of 6·5 a; the mean of the others was 0·5 a. These results fill an obvious gap in the observational evidence for our picture of the circulation of electric charge in the atmosphere, and the magnitude of the observed currents is in remarkably good agreement with the estimates of point-discharge current beneath a storm. It may be recalled that the usually quoted figure for the probable average number of storms active at any moment, due to Brooks (1925), is 1,800 and our rough estimate of the total ionization current they maintain thus about 1,000 a. Brooks's figure is, admittedly, very uncertain, and probably an underestimate and the total contribution from precipitation clouds which produce no lightning discharges may be far from negligible.

The question has also been approached from a rather different angle by Wormell (1930), who estimated a rough electrical balance sheet for a specified small area of the earth's surface, at Cambridge. There was no suggestion that the figures were representative of the whole earth. The items as listed are given in Table 4.

TABLE 4. Estimated annual loss and gain of charge by the earth. (Wormell 1930)

	coulomb yr^{-1} km^{-2}
1. Fine-weather current	+ 60
2. Precipitation of all types	+ 30
3. Lightning discharges	− 20
4. Point-discharge currents	− 100

Item 1, which was based on an assumed fine-weather current of 2×10^{-16} a/cm^2, has been criticized by Whipple and Scrase (1936) as being about twice the true value, and by Gish and Wait (1950) as being about one-half the true value! For this rough calculation, the original figure will serve. The figure for Item 3 is now more reliable; we know that the proportion of all flashes which reach the earth, in this region, is appreciably higher than was assumed in 1930. According to Table 3 the net transfer of negative charge to the earth is equivalent to the effects of 28 per cent of all discharges. Wormell (1939) has estimated, from field-change records, that the mean annual rate of occurrence of flashes, of all types, near Cambridge is about 1 km^{-2} and hence of flashes to earth 0·4 km^{-2}. Golde, on the other hand (1945), as the result of a survey of experiences of 'near misses' from lightning strokes, concludes that there are 2 flashes to earth, km^{-2} yr^{-1}, in England. There is an appreciable discrepancy between these two figures. Golde's result, which is probably rather high, would (cf. Table 3) make item 3 about -30; the other figure would make it nearer -6 C km^{-2} yr^{-1}. Finally, item 4 was deduced by multiplying the observed discharge from a single artificial point by a factor of 800. The discussion given earlier suggests that this factor is almost certainly too small. The best fit with all the available evidence would be obtained by increasing the magnitude of item 4, by between 50 and 100 per cent. The original conclusion still stands, and is indeed strengthened; it would appear to be possible that the four processes balance one another approximately or even that, in this locality, the earth on the whole gains a negative charge.

It must be noted that a very considerable part of the point-discharge contribution comes from occasions when there is no thunderstorm. It seems possible that the currents from showers and heavy frontal rain may make a contribution, by no means negligible, to the total interchange of electricity between earth and atmosphere.

6. The mechanism of generation of electric charge in a cumulonimbus cloud

6.1. *The meteorological structure of a thundercloud*

The typical thunderstorm contains one or more centres of activity or cells. If one watches a large thunderstorm, as can be done most readily at night, electrical activity, judged by the production of lightning discharges, is confined to one or more centres of rather restricted size. In one of these, lightning commences to appear, the frequency of occurrence increases to a maximum rate, which may have any value from one in several minutes to ten, or more, per minute, and later, diminishes again. The whole rise and fall of activity may last, in a typical case, some 10 to 30 min, but the final stage of moderate steady rain with occasional discharges may be rather persistent. Successive centres develop in turn, and more than one may be active simultaneously. An extended storm system may thus be producing lightning discharges for many hours.

The intensive investigations of recent years, carried out principally in the United States (Byers and Braham 1949), into the physical structure of cumulonimbus, have disclosed a life-cycle which fits in with these familiar facts. Three main stages are distinguished in the life-story of a single cell. In the first, the cell is a swelling cumulus, growing rapidly in vertical extent and characterized, almost throughout the cloud-filled volume by updraughts, whose intensity is a maximum in the upper levels, where vertical velocities of 15 m/s are not unusual and considerably higher values may occur. The cloud top may reach far above the 0°C level but, in the early stages, the particles present are all of liquid water and all comparatively small. Later, the top of the cloud becomes glaciated and larger particles of snow, graupel and rain appear, although owing to the updraughts, they are carried upwards or fall very slowly. The number and

size of the precipitation particles continue to increase, and at length appreciable rain falls to the earth; at about the same time, strong downdraughts appear in the lower part of the cell, more particularly in that part of it from which rain is falling. This marks the second, or mature stage of the cell, during which it attains its maximum vertical extent, and may reach the tropopause; the horizontal extent may now be 8 km and is rather variable. There are still very strong local updraughts, particularly in the upper part of the cloud. Precipitation particles consist mainly of rain below and snow or graupel above, but hail may also occur in restricted regions. Owing to the strong updraughts, quite large particles may be carried to great heights. The mature stage is gradually transformed into the final dissipating stage, in which the whole cell contains air moving downwards or almost at rest. Rain continues to fall for some time but gradually diminishes in intensity and ceases.

The appearance of visible signs of glaciation at the top of the cloud is, in this country, a rather reliable indication that intense electric fields are developing, although not necessarily that lightning will occur. The first few flashes may occur before any appreciable precipitation reaches the ground; they are usually discharges within the cloud (Wormell 1939). According to Workman and Reynolds (1949), the first appearance of precipitation beneath the cloud base occurs, usually, about 10 min after a radar echo can first be detected, i.e., after an appreciable number of particles, of what might be called precipitation size, are present in the cloud. They also trace a sequence of behaviour, with regard to the production of lightning, very similar to that outlined by Wormell. The way in which the electrical structure of the cloud develops and changes during the cycle of development is not known in any detail. The main positive and negative charges are presumably present from the latter part of the first stage until the dissipation of the cell, although in the last stages the electrical structure is probably very confused and charges of both sign present in parts of the base. It would be of much interest to know at what stage positively charged regions first appear there.

The location of the main region of charge separation (near the — 10°C level), and the fairly close association in time of the appearance of intense fields and of visible signs of glaciation at the cloud top, suggest that the presence of ice, and perhaps of ice and supercooled water in juxtaposition, is an essential to the main process of charge generation, or at any rate to its final stages. Findeisen (1940) reported that the disturbances in aircraft radio-equipment are much more noticeable when the plane is in or near the part of the cloud containing ice. Smith (1951a, b) has produced some negative evidence pointing to the same conclusion. Simpson (1949), in a study of rain electricity, lists a limited number of occasions of fairly heavy rain without any pronounced electrical effects; Smith reports that the estimated height of the cloud top, on most of these occasions, was below the 0°C level. Rain under such conditions is believed to be rare, in this country, and the absence of any considerable charges on these occasions is suggestive.

The discussion of recovery curves, in Section 3, required the presence in the cloud of a considerable volume in which there existed negatively charged precipitation elements, the corresponding positive charge being either attached to cloud particles or, conceivably, existing in the form of large ions. Gunn (1947) found in a shower cloud, in the lower portion, negatively charged precipitation with positive charge either on cloud particles or as ions; in the middle regions precipitation particles of both signs, and at the top, positively charged precipitation, associated apparently with negative cloud particles or ions. His results in an actual thunderstorm were so complicated that little clear-cut information can be deduced from them.

We proceed to consider some of the numerous mechanisms which have been suggested; in order to be of any value a process must charge the precipitation particles with charge of the right sign and must produce electrification in adequate amount and with sufficient speed; the time available, from quite early in the cumulus stage will hardly exceed an hour or two. The interval from the first occurrence of an appreciable concentration of precipitation elements to the development of strong fields is considerably smaller.

6.2. *Influence theories*

We consider, here, processes which only occur in the presence of an electric field, their sign being determined by that of the field with which we start, and the charge acquired by the larger particles present having such a sign that subsequent gravitational separation of particles of different sizes causes an enhancement of the original field.

In a vertical electric field, raindrops are electrically polarized, the upper and lower hemispheres carrying equal and opposite surface charges, if the drop has no net charge. Elster and Geitel (1913) considered the effects which may arise from the collisions of small cloud particles with electrically polarized raindrops falling through them. If the drastic assumption be made that, at such a collision, the small particle may make electrical contact with the raindrop and yet rebound without coalescence, then an influence process is possible. A particle rebounding from the lower half of the falling drop would remove some of the charge residing on that half and the large drops would tend to be left with net charges of the sign of the polarization charges on their upper hemisphere. With further gravitational separation, the initial field would be enhanced. It has long seemed extremely improbable that the fundamental physical assumption on which this theory is based is tenable (see, e.g., Simpson 1942). If, however, the raindrop be replaced by a hailstone, it may still be necessary to consider the effects of such a process, although they may be modified or masked by other effects when the ice and water phases are simultaneously present. In the growth of a hailstone, as described for example by Ludlam (1950), there may be a stage where cloud particles are being captured, but much of the captured material is shed again as liquid water before it can freeze. Regarded purely as an electrostatic problem, such a process will, in the presence of an electric field, charge the growing hailstone. Shedding of water from the upper half of the hailstone will tend to destroy the existing field, rebound of water from the lower hemisphere to enhance it.

Wilson (1929) pointed out that if the effect of the electrical forces on the motion of an ion or charged cloud particle in the neighbourhood of a falling raindrop be considered in detail, and if particles which strike the drop are captured, there is indeed, if certain conditions are satisfied, a selective charging process which, with gravitational separation, enhances the field. This process is almost unique amongst theories of thunderstorm electrification in that it is based on fundamental and familiar properties of ions and, in consequence, the rate of acquisition of charge may be formally written down in terms of the various physical parameters involved. Since the process almost certainly does occur, substantially as described by these equations, an attempt will be made to summarize, in some detail, and in a more general form than has hitherto been done explicitly, the results to be expected from it, as a first step towards an assessment of the conditions under which it may be of importance.

The following symbols are used in the discussion :

X = the vertical electric field, i.e., the large-scale field
Q = the net charge on the raindrop
Q_0 = the value of Q at $t = 0$
a = the radius of the drop
v = the velocity of fall of the drop in still air
k_1, k_2 = the mobilities of positive and negative ions respectively
λ_1, λ_2 = the polar conductivities due to positive and negative ' fast ' ions, defined as ions for which $v < kX$
Λ_1, Λ_2 = the polar conductivities due to positive and negative ' slow ' ions, for which $v > kX$
ϵ = dielectric constant
τ = the time required for a drop, initially uncharged, to reach half its final equilibrium charge in a constant field,

Symbols A, B, C, B', C', to represent certain combinations of some of the above quantities, are introduced and defined in the text.

For the sake of clarity of exposition, it will be arbitrarily assumed for the remainder of this section that the field X is positive, i.e., directed downwards. A reversal of this sign simply changes the sign of the charge to which every step of the analysis refers.

A conducting sphere in a vertical field carries a polarization charge $+ 3Xa^2/4$ on its lower hemisphere. If a drop is held at rest relative to the air and carries no net charge, and if all the ions present are of uniform mobility, a current, carried by positive ions, of amount $3\pi a^2 \lambda_1 X$, flows into the upper part and a current $3\pi a^2 \lambda_2 X$, carried by negative ions moving upwards, flows out of the lower portion; the sphere will thus commence to acquire a net charge at an initial rate $3\pi a^2 (\lambda_1 - \lambda_2) X$. These statements remain true if the air is moving slowly upwards relative to the drop. If however $v > k X$, ions of both signs are moving upwards relative to the drop and, if it has no net charge, no positive ions can reach it owing to the positive polarization charge on its lower surface. The initial charging rate is now $- 3\pi a^2 \Lambda_2 X$; this is the process of selective ion capture envisaged by Wilson. Laboratory experiments by Gott (1933, 1935) demonstrated that, when a water drop fell through a region containing ions of both signs, or a cloud of small droplets of both signs, in an electric field, selective capture did occur if, and only if, $v > kX$. The sign of the charge acquired was as expected and its magnitude of the order predicted.

The condition $v > k X$ is fundamental for selective capture to occur. For a raindrop, v will not exceed about 6 m/s, and with ordinary small ions, ($k \simeq 1{\cdot}5$), selective capture is impossible if the field exceeds about 400 v/cm. Selective capture of large ions or charged cloud particles is possible, however, in any field up to the point where a discharge occurs. It is thus necessary, in general, to specify the portion of the air's conductivity due to ' fast ions ' for which $v < k X$ and ' slow ions ' for which $v > k X$; the latter will be assumed to include all particles for which the forces, due to their charge and the electrical field, are large compared with their weight and inertial effects, which will be neglected.

The detailed equations describing this charging process were deduced by Whipple and Chalmers (1944), for the case of a sphere in a constant field and when the air flow corresponds to the case of viscous motion; their analysis is thus appropriate to small water drops whose rate of fall is accurately described by Stokes's Law, but is not accurately true for raindrops. The same equations can, however, as Chalmers (1947) has pointed out, be deduced more simply, if less rigorously, by making two assumptions: first, that the current intensity into each element of surface of the drop is given by the product of the field at the surface and the polar conductivity of the air, due to ions of

opposite sign to that of the surface element considered, as in the classical method of determining the air's conductivity, and secondly that any positive charge present on the surface of the lower hemisphere 'screens' an equal amount of negative charge from the impact of upward moving positive ions. Whipple and Chalmers also pointed out that the turbulent wake left by a falling raindrop is unlikely to affect the capture of ions very greatly and there seems no reason to doubt that the equations describe the process of acquisition of charge by a raindrop to a fair approximation.

Neither the spherical shape, nor the conductive property of the raindrop is essential to the selective capture process (Wormell 1939, 1948; Chalmers 1947, 1948). The field at an external point due to a conducting sphere in a uniform vertical field is the field of a vertical dipole of moment Xa^3; that due to an uncharged dielectric sphere is the field of a dipole of moment $Xa^3(\epsilon - 1)/(\epsilon + 2)$. The dielectric constant of ice is very high, of the order of 80, and there is thus no significant difference between the motion of ions in the neighbourhood of a raindrop or of a sphere of hard-frozen ice of the same size. Wall's theory of thunderstorm electrification (1948), which has received considerable attention from continental writers, is essentially a detailed but qualitative discussion of the selective capture process when the precipitation particle is a non-conductor (ice).

We proceed to consider the general case, where both 'fast' and 'slow' ions of each sign are present in the space surrounding a spherical particle, which is falling freely relative to the air.

(i) $Q_0 > 3Xa^2$

If the net charge on the drop exceeds $3Xa^2$, the sign of the surface charge is everywhere positive and we have simply

$$\frac{dQ}{dt} = -4\pi(\lambda_2 + \Lambda_2)\,Q \quad . \quad . \quad . \quad . \quad . \quad (1)$$

whence

$$Q = Q_0 \exp\{-4\pi(\lambda_2 + \Lambda_2)\,t\} \quad . \quad . \quad . \quad . \quad (2)$$

i.e., Q diminishes according to an exponential law until $Q = 3Xa^2$, after which a more complicated picture arises. (The case $Q_0 < (-3Xa^2)$ is similar.)

(ii) $(-3Xa^2) < Q_0 < 3Xa^2$

If the magnitude of the net charge on the drop is less than $3Xa^2$, it is readily shown that the total positive charge on the drop's surface is $(Q + 3Xa^2)^2/(12Xa^2)$ and the total negative charge, $(3Xa^2 - Q)^2/(12Xa^2)$. We have therefore the following equations:

The current carried to the drop by negative ions is

$$\left[\frac{dQ}{dt}\right]_1 = -\pi(\lambda_2 + \Lambda_2)(Q + 3Xa^2)^2/(3Xa^2) \quad . \quad . \quad (3)$$

The current reaching the drop by fast positive ions is

$$\left[\frac{dQ}{dt}\right]_2 = \pi\lambda_1(3Xa^2 - Q)^2/(3Xa^2) \quad . \quad . \quad . \quad (4)$$

If $Q > 0$, no slow positive ions can reach the drop.

If $Q < 0$, the current reaching the drop by slow positive ions is

$$\left[\frac{dQ}{dt}\right]_3 = -4\pi\Lambda_1 Q \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The total current into the drop is thus given by an equation of the form

$$\frac{dQ}{dt} = -A\{(Q+B)^2 - C^2\} \quad . \quad . \quad . \quad . \quad (6)$$

where, if $Q < 0$,

$$A = \frac{\pi(\Lambda_2 + \lambda_2 - \lambda_1)}{3Xa^2}, \quad B = \frac{3Xa^2(\lambda_1 + \lambda_2 + \Lambda_2 + 2\Lambda_1)}{\Lambda_2 + \lambda_2 - \lambda_1}$$

$$C = \frac{6Xa^2\{(\lambda_1 + \Lambda_1)(\lambda_2 + \Lambda_2 + \Lambda_1)\}^{\frac{1}{2}}}{\Lambda_2 + \lambda_2 - \lambda_1}$$

For $Q > 0$, a similar equation holds, but with B and C replaced by B' and C' where

$$B' = \frac{3Xa^2(\Lambda_2 + \lambda_2 + \lambda_1)}{\Lambda_2 + \lambda_2 - \lambda_1} \quad \text{and} \quad C' = \frac{6Xa^2\{\lambda_1(\lambda_2 + \Lambda_2)\}^{\frac{1}{2}}}{\Lambda_2 + \lambda_2 - \lambda_1}$$

Q thus tends to the limiting value $Q = -(B - C)$. If its initial value lies between 0 and $-(B - C)$, its variation with time is described by the equation

$$Q + B = C \coth(ACt + \delta) \quad . \quad . \quad . \quad . \quad (7)$$

provided $C \neq 0$; δ is a constant of integration.

(If initially $-(B - C) > Q > (-3Xa^2)$, then $Q + B = C \tanh(A\,C\,t + \delta')$)

If Q is initially positive, but less than $3Xa^2$, an equation of the same form as (7) holds, but with B' and C' replacing B and C, until $Q = 0$.

A drop having initially a positive charge Q_0, $(3Xa^2 < Q_0 < 0)$ thus loses its charge in a time t where $t = \{\coth^{-1}(B'/C') - \coth^{-1}[(Q_0 + B')/C']\}/AC'$ and its behaviour is thenceforth described by Eq. (7).

These results which are complete, may be summarized as follows :

(α) With ions of both signs present, the drop will ultimately acquire a charge $-(B - C)$ whatever its initial charge. $-(B - C)$ is negative if the conductivity due to all the negative ions exceeds the conductivity due to ' fast ' positive ions alone. If the initial charge is greater than $3Xa^2$ or less than $(-3Xa^2)$, its magnitude decreases exponentially to one of these values and its magnitude is henceforth described by Eq. (6), with appropriate values of the constants A, B, C or A, B', C'.

(β) With negative ions only present, a case more likely to occur beneath rather than within the cloud, a highly negatively charged drop, $Q < (-3Xa^2)$ will retain its charge definitely. A large positive charge will first decrease exponentially to $+3Xa^2$. The behaviour of a drop with an initial charge between $\pm 3Xa^2$ is described by Eq. (6) which now however takes a simpler form since C and C' vanish and $B = B' = 3Xa^2$ while $A = \pi(\lambda_2 + \Lambda_2)/(3Xa^2)$. The solution is

$$\frac{1}{Q + 3Xa^2} - \frac{1}{Q_0 + 3Xa^2} = \frac{\pi(\lambda_2 + \Lambda_2)t}{3Xa^2} \quad . \quad . \quad (8)$$

and the final charge is $-3Xa^2$.

(γ) With positive ions alone present, any negative charge will disappear. If the ions are all slow, any charge $Q \geqslant 0$ is retained indefinitely, since no ions can reach the drop. If fast positive ions are present the final charge is $+3Xa^2$ unless its initial value exceeded this value when it would be retained indefinitely.

In all cases the equilibrium charge acquired by the drop is proportional to a^2 and the charge per unit mass of water to $1/a$. The rate at which the final charge is approached depends in a complicated manner, not only on the conductivity but also on the relative contributions by fast and slow ions. The time required to reach a definite

fraction of the final charge is independent of the drop size and of the field. The behaviour will be illustrated by considering a very few special cases.

We return to Case (α), and take the initial charge to be zero. The initial rate of charging is then $dQ/dt = -3\pi Xa^2(\Lambda_2 + \lambda_2 - \lambda_1)$; the final charge is $-(B - C)$ and the time τ required to acquire half the final charge is $\frac{1}{AC}\coth^{-1}(2 + B/C)$.

Suppose, first that all ions are ' slow ' (this is necessarily the case if X does not exceed a few hundred v/cm), $\lambda_1 = \lambda_2 = 0$. Values are given in Table 5 of the final charge Q_1 and τ for various assumed values of the ratio Λ_1/Λ_2.

TABLE 5. VALUES OF THE FINAL CHARGE Q_1 AND THE TIME τ REQUIRED BY AN INITIALLY UNCHARGED DROP TO REACH $\frac{1}{2}Q_1$

Λ_1/Λ_2	$Q_1/(Xa^2)$	$\tau\Lambda_2$
0	− 3	0·32
0·2	− 1·26	0·098
1	− 0·52	0·039
5	− 0·14	0·010
∞	0	..

Suppose next that some ' fast ' ions are also present. The mobility of the negative ions is irrelevant but the existence of an appreciable number of fast positive ions modifies the results appreciably. Taking Q_0 zero, as before, and the particular case $\Lambda_1 = \Lambda_2 + \lambda_2$, values of the final charge Q_1 and the time to reach half that value are given in Table 6.

TABLE 6. VALUES OF THE FINAL CHARGE Q_1 AND THE TIME τ REQUIRED BY AN INITIALLY UNCHARGED DROP TO REACH $Q_1/2$, IN THE PRESENCE OF SOME ' FAST ' POSITIVE IONS, AND WITH THE PARTICULAR ASSUMPTION $\Lambda_1 = \Lambda_2 + \lambda_2$

$\lambda_1/(\Lambda_2 + \lambda_2)$	$Q_1/(Xa^2)$	$\tau(\Lambda_2 + \lambda_2)$
0	− 0·52	0·039
0·5	− 0·22	0·032
1	0	..
2	+ 0·52	0·038

The condition that the field should build up as the result of selective ion capture is, with the initial field we have assumed, that Q_1 should be negative, i.e., $\lambda_1 < (\lambda_2 + \Lambda_2)$. Provided that this is well satisfied, and provided also that Λ_1 is not much greater than Λ_2, the equilibrium charge on a raindrop is in round figures $-Xa^2/2$ and the time required by an uncharged drop to attain half its final charge of the order $0{\cdot}05/(\lambda_2 + \Lambda_2)$. The force on a drop due to its equilibrium charge is about $\frac{1}{2}X^2a^2$; this equals the weight of the drop, when $a \simeq 0{\cdot}1$ mm, in a field of 3,000 v/cm.

The discussion so far, has been concerned solely with the charge on a single drop, the external field and the conductivity remaining constant. Beneath the cloud the field may be varying with height and the problem of the variation of the charge on a falling rain drop, correspondingly more complicated. Within the cloud, gravitational separation of the drops charged by the Wilson capture process and the ions, now left with an excess of charge of opposite sign to that of the drops, may increase the large-scale field X. The equilibrium charge towards which the drops are tending is itself proportional to X and thus changing with time. In small fields all ions are

to be classed as ' slow,' in the sense we have used the term, but when the field exceeds a few hundred v/cm small ions must be transferred from the slow to the fast class. Beyond this point, any excess of small positive ions acts in such a way as to tend to diminish rather than to enhance the existing field, the growth of the charges depending on the quantity $(\Lambda_2 + \lambda_2 - \lambda_1)$.

Consider a region in the middle levels of a developing cell, containing precipitation particles and cloud particles, the latter having radii up to perhaps 15 μ. Suppose that all these are initially uncharged and that there exists a positive field of the order of 1 v/cm. Selective capture of negative small ions will occur for all particles whose velocity of fall exceeds about 1·5 cm/sec, i.e., precipitation and the larger cloud particles. As a result of this capture the positive ions are present in excess and the smaller cloud particles will capture positive charge at a rate proportional to the difference in the two polar conductivities. The fall of the larger particles under gravity causes an enhancement of the field; by the time it has reached a few volts per cm selective capture is confined to precipitation particles, all the cloud particles tending to acquire positive charges. It would now be possible for the process to continue smoothly, the original field being multiplied many times, until the rate at which small ions are driven by the field exceeds the rate of fall of the larger particles as diminished by the electrical forces on those particles. From this point small ions are ' fast ' ions and the difficulties arise which were mentioned in the preceding paragraph.

Once the process of charge generation is well under way, there will exist inside the active region a growing positive field; above and below the region, negative fields will develop and if precipitation particles and a suitable supply of ions find their way into these regions a capture process of the opposite sign to that in the main body of the cloud may develop. Herein may lie the explanation of Gunn's observations (1947) that in a shower cloud there was, near the base, negatively charged precipitation embedded in positive charge on much smaller particles, and near the top, positively charged precipitation surrounded apparently by negatively charged ions or cloud particles. Wilson capture very near the base of the cloud may also be part of the explanation of the regions of positive charge with positive field beneath them which are found there.

We have still to consider whether the capture process is sufficiently rapid to be considered an important mechanism in a thunderstorm. The maximum possible rate at which it could proceed, unaided, cannot exceed the rate at which ions are being produced in the lower atmosphere. If the process were perfectly efficient, i.e., if every ion were used to charge the right sort of particle, the rate of charge production would correspond to the 10 ions cm^{-3} sec^{-1} which normally appear in the lower atmosphere. This is equivalent to about 6 C km^{-3} hr^{-1}. Since, further, the process is far from being of such perfect efficiency, it is impossible therefore that of itself it can separate the observed charges during the life-time of a typical cell. There are obvious possible sources of greatly enhanced ionization in the later stages, when intense fields are present, but it must not be forgotten that at this stage the positive ions must be ' slow ' ions, or must be very rapidly transformed effectively into slow ions. Whether the selective capture process can, unaided, build up the field through the earlier stages with sufficient speed may perhaps be still considered an undecided question and it is necessary to consider the effects of other processes occurring in a thunderstorm. The function of these other processes may well consist, at least in part, in producing a supply of ions of the right type at the right stage of the developing field, the actual separation of charge remaining dependent on an influence mechanism, that is on the polarization of the larger particles in the existing field.

It has been tacitly assumed, so far, that the direction in which the field builds up is controlled by the normal fine-weather field. This was part of Wilson's original suggestion. Some have thought that, if the fine-weather field is maintained by thunderstorms, this is dangerously near an argument in a circle. Wilson (1920) pointed out, however, that other things being equal, a cloud of positive polarity should maintain a larger current between the earth and upper atmosphere than one of the opposite polarity. The electrical resistances between the clouds and the earth are not markedly different, but the resistance between the positive cloud and the upper atmosphere is likely to be the lesser, since in this case free electrons can be dragged downwards from the plenciful supply in the high atmosphere. If then we consider a regime with no fine-weather field and clouds of both polarities equally probable, there will be a small net current upwards from the earth to the upper atmosphere. The potential of the latter will gradually rise and it will tend, more and more, to control the polarity of subsequent storms.

Wall (1948) considers control by the fine-weather field incredible and has put forward another suggestion which is of the nature of an *ad hoc* assumption concerning the properties of ice crystals in the atmosphere and has little to recommend it (see Simpson 1951).

It is convenient, at this stage, to consider briefly a theory due to Frenkel (1944, 1946, 1947). The basis of this theory is the assumption that water drops in ionized air acquire a negative charge spontaneously by preferential absorption of negative ions, due to a contact difference of potential between water and air. This belief is based on, first, the familiar phenomenon of condensation in an expansion chamber, on negative ions, now interpreted as the capture of negative ions by an embryo droplet which is thereby stabilized, and secondly, on certain experiments which suggest a potential drop across a plane surface of water, in bulk, in contact with air. Frenkel assumes therefore that the equilibrium state of a water drop is one in which it carries a negative charge, $-aV$, where V is about 0·25 volt; the corresponding positive charge is left in the air in the form of ions (presumably large ions) near the drop. Gravitational separation of negative drops and positive ions then produces a positive field inside a cloud, but it must be emphasized that there is no direct experimental evidence that a drop is charged in this manner. Frenkel points out further that except for very small drops in a small field, the capture of ions due to the polarization charges on the drop is a more important phenomenon. He considers the acquisition of charge by the latter mechanism to depend merely on the difference between the conductivities due to positive and negative ions, rejecting the selective effect due to the motion of the drop (the essential point in the Wilson mechanism), because, to quote his own words, 'the air surrounding the drop must move along with it (according to the usual laws of viscous motion).' This criticism is the more surprising in that the case of viscous motion is the one case for which the capture process has been worked out in complete detail (Whipple and Chalmers 1944), the detail including the actual trajectories of the ions near the drop. Frenkel estimates that, as a result of his fundamental charging process, any cloud of small drops will develop a field proportional to its liquid-water content per unit volume and finds, for a water content of 1 g/m^3 a field of several hundred v/cm.

It must be emphasized that the 'Frenkel' charge (a/1,200 ESU, where a is in cm), if it exists at all, is only important for very small drops. In the cases which Frenkel considers in detail he always envisages negatively charged cloud particles ($a \simeq 10\ \mu$) and positively charged ions. The charge acquired by the Wilson capture process is about $\frac{1}{2} Xa^2$. It exceeds the Frenkel charge for all drops whose radii exceed

50 μ, in a field of 100 v/cm, or 0·5 mm in a field of 10 v/cm. Again, the charges are still derived from the ions present in the air and Frenkel's picture avoids none of the difficulties, considered above, of the slowness of the build-up of the field.

It would thus seem that Frenkel's mechanism, if it occurs at all, is only of importance in the very early stages of growth of the field and primarily in determining the sign of field which is going to develop.

6.3. *The rupture of water drops*

The process of electrification by the breaking of water drops, which causes, for example, pronounced electrical effects in the vicinity of a waterfall, was first investigated in detail by Lenard (1892) who was originally of the opinion that rupture on a flat, solid obstacle was an essential condition for the production of appreciable charges. Simpson (1909) carried through a careful series of experiments which appeared to establish that the breaking of a drop in a violent air-blast could produce a considerable electrification. The results were found to be very sensitive to traces of impurity in the water, but, for distilled water, the fragments of the broken drop carried a positive charge; the surrounding air was shown, with an Ebert apparatus, to contain ' ions ' of both signs, with an excess of negative charge. Lenard, later (1921), reporting on work by Hochschwender, accepted the reality of the effect but insisted that it was essential that the drop be ruptured with great violence so that small fragments are torn from the surface layers. Many subsequent workers have made related observations and a large volume of work emphasizes that the effect is very sensitive to impurity and to the precise manner in which the drop is broken, and suggests that the normal sign is as found by Simpson. Estimates of the magnitude of the charge produced are variable; according to Simpson the rupture of a large drop (4 mm radius), produces about 6×10^{-3} ESU of charge or about $2{\cdot}3 \times 10^{-2}$ ESU per gram of water. In a recent careful investigation, Chapman (1950) finds charges about ten times as large as Simpson observed; ' ions ' were again found in the air, with an excess of negative ions, their mobilities being of the order of the mobilities of small ions. This effect has long held a prominent place in the literature of thunderstorm electricity; since it would, with gravitational separation of the drop fragments and ions, produce a charge distribution of negative polarity, it is of interest mainly as an explanation or partial explanation of the positive charge near the base of a storm or possibly as a source of ionization for other processes. It is clear that repeated rupture of the same mass of water would be necessary for this process to yield, of itself, the charge per unit mass found on precipitation inside a thunderstorm which may approach the maximum physically possible.

Gill and Alfrey (1952) in a paper describing experiments on the splashing of water on various surfaces make two devastating suggestions; first, that there is no reliable evidence that any ions are produced when water splashes on a solid surface, the effects being completely explicable in electrostatic terms when contact potential differences are considered; secondly, that there is no evidence that either ions or charge are produced when a drop is broken in the air ! The charges observed in the latter case are ascribed to spurious effects due to impact of the fragments of the drop with the walls of the experimental vessel; the ' ions ' which many workers claim to have investigated are claimed to be very highly charged small drop fragments. While one can hardly dismiss thus cursorily a great volume of careful earlier work, a very careful scrutiny of the evidence is now necessary before one can, with any confidence, erect a thunderstorm theory on the charges produced by rupture of a drop in air.

6.4. *The friction of ice crystals*

Simpson and Scrase (1937) suggested that the main charge structure of a thundercloud could be accounted for by the results of the impact of ice crystals on one another. Data are lacking for making this into a quantitative theory of thunderstorms and indeed there is no conclusive laboratory evidence as to the sign of the effect.

Recent work on the electrification in a cloud of solid particles suggests (Kunkel 1950) that there is no pronounced tendency for particles of a particular size to have charge of a single sign, both signs being present on the particles which may be highly charged. Chapman (1950) finds that when a snowflake strikes a snow surface, collected from the same storm, there is a small release of negative charge to the air. This is of the wrong sign to explain the thunderstorm structure.

6.5. *Effects arising from the co-existence of the ice and water phases*

It has been known since the early experiments of Faraday, and later of Sohncke, that pronounced electrical effects occur when water in the form either of a stream of liquid or of a cloud of small drops impinges on a surface of ice and, in part, rebounds from it. Most workers have agreed with these early observations in reporting that the ice is left with a strong positive charge and a negative charge is carried away by the rebounding water. Findeisen (1940, 1943) carried through a detailed study of the phenomena occurring when a deposit of hoar frost is growing or evaporating and also when a layer of rime is growing by deposition from a cloud of water drops. The majority of his investigations were carried through in the laboratory, using a closed system and air of controlled humidity or an artificial cloud, but a few were done with a portable apparatus in the open, and these included occasions when a natural cloud of supercooled water drops was blowing through the apparatus and depositing rime on the collecting electrode. In all his experiments the receiving surface was very cold, being kept at a temperature of the order of — 60°C during the periods when the deposit was growing.

When a deposit of frost was growing direct from the vapour phase (sublimation), the deposit was found to acquire a positive charge, the mean rate of charging over a period of 20 min from the start being about $0{\cdot}5 \times 10^{-6}$ ESU cm^{-2} sec^{-1}, if the air velocity over the surface was about 0·35 m/sec. The rate of accumulation of charge was rather smaller with higher wind speeds. The rate of acquisition of charge tended to fall off with time, an effect ascribed by Findeisen to a change in the nature of the ice surface, but the effects of the charge already present may well be another cause.

When a frost deposit was evaporating a similar effect was observed but now the frost-covered body acquired a negative charge. The charging was reported to be a little more vigorous in this case than during sublimation, the rate of charging depending on the nature of the evaporating surface, but being of the order of 10^{-6} ESU cm^{-2} sec^{-1}.

When a rime layer was growing by deposition from a cloud of water drops, a much more active separation of charges was observed, the rime acquiring a positive charge. The charging ceased if the surface became smooth and glassy, as is the case if the drops freeze slowly, or if it became wet. The effects observed with a natural supercooled cloud were of the same sign, but exceeded in magnitude those found with artificial clouds in the laboratory; the difference was ascribed to the smaller size of the drops in the natural cloud and their consequent more rapid freezing. The magnitude of the rate of charging associated with riming, to determine which it was necessary to measure and correct for the charges already present on the cloud particles, was of the order of 10^{-3} ESU cm^{-2} sec^{-1}.

Findeisen was able to show that the electrical effects were associated with the

production of numerous small ice splinters which were thrown off from the growing or evaporating deposits. Electrical effects were only present when the exposed surface was opaque and polycrystalline; they were absent if the ice was in the form of a smooth glassy layer and are not to be expected in the case of the growth or evaporation of a single crystal. The ice splinters covered a range of sizes; the larger ones were visible under suitable illumination and their size and electrical properties could be determined directly. The visible ones had dimensions of the order of 40 μ and charges of the order of 10^{-5} ESU. The splinters were observed directly during sublimation or evaporation of a deposit; their direct observation during riming is more difficult and their presence had to be inferred. The electrical properties of the splinters were investigated by observing their zig-zag paths as they fell under gravity in a horizontal alternating electric field. The majority were uncharged; among the remainder charges of both signs were found, with an excess of sign opposite to that accumulating on the surface from which the splinters were derived. It was implied that this excess charge, in fact, balanced the charge left on the surface.

Findeisen stated, and indeed emphasized, that these electrical effects are considerably modified and may even be reversed in sign if an electric field is present. With a positive field, i.e., with the lines of force directed inwards on to the surface, the splinters carry away an excess of negative charge, by whatever process they are being formed. A field intensity of 3 v/cm, or more, suffices to control the sign; under these circumstances the charge separation may be thought of as a simple electrostatic process.

Findeisen sought a physical explanation of these effects, in part, in contact differences of potential (Volta effect), investigated very carefully by Lange (1940), between water and ice, and even between rime and ice; between, that is, a complex and a single crystal. He considered, however, that other factors, in particular the temperature gradient across the layer, were involved.

Owing to the seemingly great significance of these results, a very detailed and careful re-investigation of these phenomena has been made by Kramer (Clay and Kramer 1947; Kramer 1948). The existence of pronounced electrical effects was confirmed but there were very marked differences in detail from Findeisen's results. During the growth of a frost deposit by sublimation, a positive charge first appeared on the surface, but later this decreased and the charge ultimately became negative. During evaporation of the deposit, no charging could be detected. During riming by accretion of liquid water a considerable charge appeared, but its magnitude was rather less than found by Findeisen and its sign the reverse of that found by him. The charge carried by the splinters was also investigated; at certain stages the net charge on the splinters was claimed to be of the same sign as that appearing on the frost layer, an observation whose interpretation is very obscure. Kumm (1951) has also investigated the charge on the splinters during sublimation and evaporation of a layer of frost; his measuring device was considerably more sensitive than had been used previously. He found no significant differences between the properties of the splinters thrown off during growth or evaporation of the deposit; in either case positive splinters were about seven times as numerous as negative ones, throughout the range of charges found, and Kumm concluded that the frost deposit acquires a negative charge whether it is growing or evaporating. The charges found on individual splinters ranged from 8×10^{-5} ESU to 7×10^{-7} ESU, the latter being the least charge he could detect. The mean size corresponded to a sphere of radius 6 μ. During riming Kumm failed to detect any charging at all! While some of the differences between the results of these three investigations are due to different interpretations of the observations, there are some puzzling contradictions in the experimental results.

Dinger and Gunn (1946), while looking for electrical effects associated with the freezing of water, found that when distilled water is frozen in an atmosphere of air, the resulting ice, on melting, possesses a positive charge, which may be of the order of 1 ESU/g, and an equal negative charge is released to the air on bubbles which have been trapped in the ice. The effect does not occur with air-free water.

Workman and Reynolds (1948, 1950) have carried out an extensive investigation of the effect of impurities, dissolved in the water, on the electrical phenomena associated with the change of state from water to ice. A minute trace of certain ionic solutes was found to have profound effects on both the observed contact differences of potential between ice and water, and also on the charges produced by the impact of supercooled water drops on a moderately cold ice surface. With a trace of ammonia in solution, the ice develops a positive charge; small amounts of sodium chloride or of calcium carbonate, soluble in the presence of carbon dioxide, gave the opposite sign to ammonia. The amounts of solute required were no greater than are commonly found in rain.

Lueder (1951) has made an interesting investigation of the electrical effects accompanying riming. The experiments were made in natural supercooled clouds at a mountain observatory in order that the contaminants present in the water should be those occurring in nature. The rime was deposited on a slowly rotating metal rod, similar to an aircraft propeller, the motion being intended to simulate the capture of supercooled drops by a falling hailstone or pellet of graupel. The rod was enclosed, for part of each revolution, by a Faraday cage and its charge caused a periodic variation in the potential of an electrode. It was necessary to interpret the effects observed : (a) while the deposit is steadily increasing in thickness, (b) when a large part of the deposit suddenly breaks away, (c) as the deposit renews itself again. Lueder claims that his results require that a growing deposit of rime carries a very strong negative charge, an equal positive charge being carried away on the parts of the water drops which are flung off again without freezing. The charge per unit volume of ice appeared, initially, to be enormous, but was soon diminished by the effects of the field at the surface due to the charge ; it was estimated that the charging stopped when that field reached 700 v/cm. In consequence, the maximum charge to be expected on a spherical ice pellet of radius a turned out to be 2·4 a ESU. Lueder points out that, in a large-scale field, the polarization of a falling ice pellet will, if the field is positive, permit the negative charging of the pellet, associated with riming at its lower surface, to continue until thunderstorm fields are attained. While Lueder's results are stimulating, it must be admitted that the deduction of the magnitude of the charge is very indirect; it seems, in particular, very dangerous to base the argument, in part, on the effects observed when a large part of the rime deposit suddenly breaks away violently from the rod.

The apparent contradictions and inconsistencies between the results of these various workers give an impression of great confusion in our knowledge of the fundamental physical facts. While some of the differences are differences of interpretation, it is clear that the results of such experiments are greatly influenced by traces of impurity and may be grossly distorted by unrecognized spurious effects. There is a direct and unexplained contradiction between the observations of Findeisen and Kramer with respect to the sign of the charge acquired during riming, while Kumm, in this case, finds no charge at all, or rather no charged splinters. There is also a discrepancy between the results of Lueder and Findeisen with regard to the sign of the effect when the water drops come from a natural supercooled cloud. Lueder suggests that Findeisen's results were vitiated by traces of contaminants, such as ammonia, in his water supply,

but this ignores the results of his work with natural clouds. A difference in the experimental conditions may well be significant in that Lueder worked with his riming surface at the air temperature of the natural cloud, — 2°C to — 8°C, and speaks of some of the water rebounding as liquid. The other observers used receiving surfaces which were strongly cooled, to — 60°C in Findeisen's case, and it may be presumed that nothing, other than ice splinters, escaped. Findeisen himself believed that the temperature gradient across the rime layer was of crucial importance.

There appears to be no reason to doubt the result that, in the presence of an electric field of sufficient intensity, a considerable separation of charge takes place during the riming process and that its sign is controlled by the direction of the field. According to Findeisen, a field of 3 v/cm suffices to take control; according to Lueder, perhaps owing to different experimental conditions, the rime becomes negatively charged unless a very strong field (exceeding 700 v/cm), is tending to stop the process. The experimental investigation of this effect was always made with a field normal to the rime layer and would correspond, in the case of a growing pellet of graupel, to the radial field due to its charge, the only effect being ultimately to stop further growth of the charge. If however there exists a large-scale vertical field, the pellet is polarized. Most of the riming is to be expected at the lower surface, as it collides with cloud particles in its fall, and if the net charge is small, the direction in which further charging occurs will be controlled by the polarization charge on the lower part of the surface. The charge thus acquired by the pellet, as it falls, is indeed of the correct sign for the large-scale field to build up. There will be a maximum net charge proportional to Xa^2. We have indeed return to a generalized version of the Elster and Geitel process. The charge liberated by the rupture of a drop is also likely to be controlled by a large-scale field, if present, but the effects produced will depend on the manner in which the drop breaks, relative to the axis of polarization, and such a process seems more likely to work towards the destruction of the field than its building up.

6.6. *Summing up*

At the present juncture, the most probable solution of the problem of the main charges of a thunderstorm would seem to be an influence mechanism in this generalized sense, namely selective ion capture, reinforced, at some stages, by the various processes which occur around an ice particle in an electric field. Of the latter, the process of riming associated with the formation of hail or graupel seems likely to be the most efficient. It may be noted, further, that the production of charged splinters effectively increases the supply of large ions; the splinters investigated by Kumm had mobilities intermediate between those of small and large ions and may thus increase the efficiency of the Wilson capture process in intense fields. It is doubtful, however, if their number, concerning which we have no reliable knowledge, for the conditions in a thundercloud, is sufficient to produce an appreciable effect in this way; it is also not obvious what would happen at a collision if both the 'ion' and the particle are of ice.

Findeisen (1940), Coulomb (1943), Bricard (1946), Wichmann (1948) and others have attempted to combine some of the various processes which have been discussed in a model of a thundercloud, and to assess their relative importance in different parts and at different stages in the development of the cloud. In the present state of the subject, when our knowledge of most of these processes is considerably less certain than it seemed five years ago, it does not seem worth while to attempt anything of this nature even qualitatively.

The positive charge found in the base of the thunderstorm, at any rate at certain stages of its life history, is a separate problem. A number of examples of this phenomenon, recognized by exceptionally heavy rain and an intense positive field at the ground, have been examined by Browne (1952) at Cambridge, using radar equipment with the beam directed vertically into the cloud, Under such circumstances, a most unusual type of radar echo, of great intensity, is observed from low levels in the cloud, from a region, that is, whose temperature is well above the freezing point. The radar-echo intensity from a given height is proportional to $\Sigma N_a a^6$, where N_a is the number of particles of radius a per unit volume. A typical observation is that in descending from 5,000 ft to 3,000 ft (the lower limit for reliable observations by this method), the echo intensity increases by a factor of 100, increasing with decreasing height according to an exponential law. Echoes of this type were observed on several occasions immediately after the start of the period of very heavy rain in a thunderstorm, and usually persisted at a constant height for about 2 min after which they became transformed into a more normal type of echo from a precipitation cloud. The simplest interpretation of the observed echo is to assume that it indicates a rapid vertical gradient in the concentration of large raindrops, that, in fact, drops are breaking, growing again to maximum size and rebreaking repeatedly. If it be assumed that a drop on breaking, forms a definite number of equal fragments (the results are not sensitive to the precise number of fragments assumed), the radar observations determine how far these fragments fall before, having again grown to the critical size, they break once more. The liquid-water content of the cloud can be estimated from meteorological data available and hence one can estimate how far a drop fragment must fall, relative to the air, before it reaches the size at which it breaks. The two estimates can only be reconciled if there is a considerable updraught in the region concerned. Again, the radar observations yield not only relative, but absolute values of the quantity $N_a a^6$, and these, in combination with the observed rate of rainfall at the ground, fix the size at which the drops must be assumed to break. A typical example gave with a rainfall rate of 20 mm/hr, an updraught of 4·8 m/sec and the diameter at which the drops broke 3·0 mm. The last figure is smaller than might be expected, but according to Hochschwender (1919), such a drop is broken by a sudden gust of 10 m/sec; whether the conditions in the cloud could be so violent as to do this frequently will not be further discussed. The observations also give the frequency with which drops are being ruptured per unit horizontal area of the cloud and this leads to an estimate of the charge, present in the region as a result of drop breaking, of 5×10^{-2} C/km^3 (in this estimate Simpson's figure (1909) has been used for the charge produced per drop ruptured, and the doubt which has been cast on the process recently has been ignored). This charge is only a few per cent of the charge which Simpson and Robinson (1941) estimate to be present in a restricted volume. It would thus seem that some other mechanism must be invoked to account for the major part of this positive charge, and indeed it has not proved possible to fit all the facts and observations smoothly into a self-consistent picture. Moreover the radar observations are not, unfortunately, conclusive evidence that drop rupture is occurring at all. Once the existence of an updraught is admitted, the observations can be interpreted almost equally well by the rapid growth of raindrops of moderate size by capture of cloud particles, without any increase in their number.

Malan and Schonland (1951b) and Smith (1951a) have independently suggested that a major part of the low positive charge is derived from positive ions, originating from point discharge at the ground, carried by converging air currents to a region of updraughts, and there trapped on raindrops.

The suggestion of updraughts over a region of very heavy rain, which comes from these various lines of approach, may seem disconcerting. The radar observations quoted provide, however, very strong evidence for the occurrence of such updraughts, at levels of say 5,000 ft, over the leading edge of the region of very heavy rain and extending about half a mile into that region. After that the typical echo disappears and a general downdraught presumably supervenes.

7. The charge on rain in strong electric fields

The electric charge brought down per unit volume of rainfall has been studied for more than a generation by numerous observers. We shall only consider, briefly, the implications of some of the most recent work. The classical methods of investigation of rain charges may perhaps be considered to have reached their climax in a memoir by Simpson (1949), discussing observations made at Kew. For occasions of rainfall accompanied by a strong electric field at the ground (sufficient to cause appreciable point-discharge current), he finds a very strong tendency for the charge on the rain, observed in bulk, to be of the opposite sign to the sign of the field. This he calls the ' mirror-image effect.' It is by no means clear why this effect has been overlooked or even denied by so many previous workers. Simpson concluded that, if the field intensity at the ground exceeds 20 v/cm, the rain derives the charge it possesses, when it reaches the ground, from the natural point-discharge current by capture of some of the ions through which the drops have fallen. The form of the relation between the observed rain current, the rate of rainfall, and the point-discharge current from an artificial point, leads to the conclusion that, in very heavy rain, nearly all the ions are caught and returned to the ground on the raindrops, and hence to an estimate of the point-discharge current, per unit surrounding Kew, which is the more convincing for being so direct. The estimate is that the current is about 0·019 a/km^2 in a field of 100 v/cm. The charge found on the rain in bulk frequently exceeds that which could be acquired by Wilson capture, with ions of one sign only present (amounting to $3Xa^2$ for a drop of radius a), in the field existing at the ground. This is strong evidence that the field is indeed stronger at some height above the ground, as is necessitated by the space-charge derived from the point-discharge ions. If a drop approaches its equilibrium charge in the enhanced field at some considerable height, there is no way in which the charge can be reduced to the equilibrium value, appropriate to the smaller field near the ground, unless there is a considerable supply of ions of the opposite sign also present.

Chalmers (1951) has also discussed these results, together with some others obtained at Durham, in more detail, in terms of the very idealized picture of rain composed of drops of uniform size all carrying the same charge at a given height. He finds the results are best fitted by assuming that the primary charging process in the cloud is active, on the average, down to about 800 m from the ground, which is surprisingly low, and estimates the point discharge current at Kew at about 2·3 times the value which Simpson gave.

Further advance in our understanding of these problems is more likely to come from a study of the charges carried by individual precipitation elements. Such observations, at the ground, were made by Gschwend (1922), Chalmers and Pasquill (1938) and Gunn (1949b). Gschwend and Gunn measured also the sizes of the individual drops by catching them on dyed filter paper and measuring the stains. These observations served principally to indicate the great complexity of the phenomena, frequent reversals of sign being observed on successive particles, and no obvious correlation

being apparent with the size of the particles. Hutchinson and Chalmers (1951) report a further series of observations, in which charge and size were observed simultaneously, again using the filter-paper technique, and point discharge and field were also recorded. The results of observations on 173 drops show statistically a correlation between drop charge and point-discharge current and also between the ratio of drop charge to discharge current and drop size.

In an endeavour to speed up the rate at which observations could be taken, so that an adequate sampling of the properties of the rain could be made very rapidly, while conditions might be presumed to be constant, Smith (1951a) devised a new technique and with it made an extensive series of observations at Cambridge, investigating some 9,000 individual drops. Charge was measured by the induction effect as the drop fell, in succession, through two insulated metal rings, and the time interval between the two induced pulses gave the rate of fall of the drop, which is a sensitive indication of size for the smaller drops only. A similar arrangement was used by Gunn (1949a). Smith also made an independent determination of drop size, by allowing the drop to fall between the vertical plates of a condenser and determining the temporary change in capacity from the momentary deviation in frequency of a high-frequency oscillator, of which the condenser formed part. The properties of the drop were displayed on an oscillograph, which was photographed on continuously moving film; continuous records of field and point-discharge current, synchronized with the drop recordings were also made. For each drop, the display consisted of two pulses, whose height indicated the charge, and whose separation measured the time taken to fall a known distance; the height of a third pulse was proportional to the mass of the drop. The speed of recording was limited solely by the rapidity with which drops could be persuaded to fall into the receiver.

If two-minute periods are selected, in each of which the electric field is of uniform sign, not varying greatly in magnitude, and sufficiently strong to cause appreciable point-discharge current, then the properties of all the drops show, in 80 per cent of such occasions, a very characteristic distribution. Four examples are reproduced in Fig. 4, in which each dot represents the relation between charge and diameter for a single drop. Each of these diagrams is a plot of all the drops observed in a period of about two minutes; all refer to occasions of negative potential gradient, and the whole series were in fact recorded within a single 15-minute period. During periods of high positive potential gradient the typical distribution diagram is simply the mirror image of one of these figures.

It will be noted, first, that even with such rapid sampling, the most striking feature of the results is the large range of charges present in the rain on drops of similar size; the range may include charges of both signs. The mean value of the charge on drops of a given sign (Fig. 4) shows, however, a systematic behaviour, being of opposite sign to the potential gradient for small drops, and of the same sign as the field for large drops. The positive and negative limits of the distribution also show a systematic behaviour. In a negative field, the negative limit of charge is roughly proportional to drop diameter, while the positive limit rises to a maximum, at a diameter of 1 mm say, and then decreases and changes sign at a diameter of, say, 2 mm (these figures are of course dependent on various parameters and in particular on the field). The variation of the mean charge with drop size is qualitatively just what would be expected as a result of Wilson capture of ions. The rate at which a drop approaches its equilibrium charge by this process depends only on the conductivity and is independent of drop size and of the field, except in so far as the field determines the conductivity, by point discharge. The slower rate of fall of a small drop thus ensures that its charge is nearer

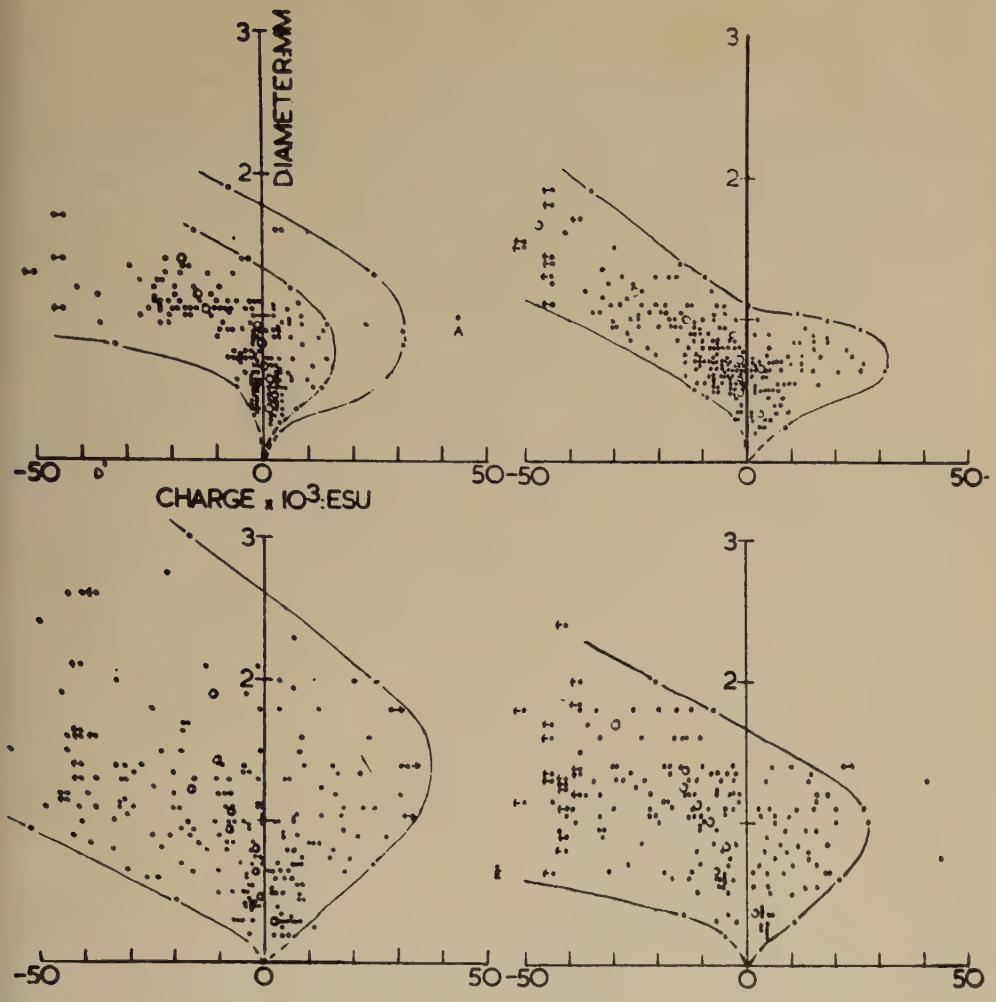

Figure 4. Four typical examples of the distribution of charges on the raindrops collected in a period of about 2 min with a large and nearly steady negative potential gradient. Each dot represents a drop, charge being plotted against diameter. The circles represent mean charge for a given size. The mean charge is positive for the smaller drops, but increasingly negative with increasing size (Smith).

to the equilibrium value, $3Xa^2$, at all stages of the descent. Smith discusses this matter in detail and concludes that a quite self-consistent picture can be built up for the way in which the mean charge depends on drop size; it is necessary, of course to assume that the field is considerably greater at higher levels than at the ground, in order to explain the magnitude of the observed charges.

It remains to consider the large range of charges observed. We now know that two drops of sensibly the same size can fall nearly simultaneously into a small receiving vessel and yet possess charges which are totally different in magnitude and may be of opposite signs. It is not possible to explain this by assuming that the drops started from the cloud with different charges. For the smaller drops, less than, say, $\frac{1}{2}$ mm diameter, the charge with which they reach the ground is controlled solely by ion capture on the descent and is practically independent of the charge they possessed on leaving the cloud. We are therefore forced to conclude, either that some other unrecognized process is at work, which affects the charge on the drops on their way

from the cloud to the ground, or alternatively that two drops of similar size, which reach the ground together, may have started out from the cloud while separated by a considerable horizontal distance, and further that there are noteworthy variations in the intensity of the field near the cloud-base, where the bulk of the ion capture occurs, over horizontal distances of say 100 m. The last suggestion is perhaps intelligible if there are local fluctuations in the intensity of updraughts. While the suggestion with regard to the horizontal diffusion of raindrops is, at first sight, surprising, there is some evidence of a very appreciable lateral spread of drops of uniform size released at, say, 500 m from the ground (Davies 1950).

8. Conclusion

One is tempted, in summing up, to declare that this is an unfortunate period at which to attempt to survey the subject of atmospheric electricity, but a similar remark could probably have been made, for different reasons, at any time during the last twenty years. Our conception of the ' general circulation ' of electric charge has seemed to grow more secure with the filling in of more details. We know the outline of the electrical structure of a thundercloud, but much more detailed knowledge is required as to how it varies during the life-cycle of a cumulonimbus cell. This is a field of some activity and information is coming both from direct aircraft observations and from the detailed study of the lightning discharge. The latter is again a field in which much work is being done but upon which it has scarcely been possible to touch in this article. The main blot in our survey concerns the fundamental problem of the source of thunderstorm electrification. Our understanding of the selective ion capture process is now fairly complete and we can express the rate of growth of the charge on a particle in terms of the various parameters which control it. In order to use the resulting equations, however, we require to know not only the conductivity of the air, but the mobility spectrum of the ions present. The data concerning the process of riming, and other processes requiring the presence of ice, which seemed to be so promising a few years ago, are in a state of considerable confusion. Elucidation of these matters and of the slight differences in physical conditions which are the probable cause of the confusion, while most urgent, is likely to be a long and laborious process. Until it is achieved, however, a quantitative theory of thunderstorm electrification can hardly be begun. When this theory does come, and is properly integrated with the rest of cloud physics, it is likely to react on certain aspects of that subject. The charging processes which a raindrop undergoes as it falls from the cloud-base to the ground are reasonably clear, although the considerable increase in field intensity from the ground to the cloud, for which there is very strong indirect evidence, has not yet been directly observed. The reasons for the surprisingly large range of charges on the drops in a sample of rain are also imperfectly understood.

9. Acknowledgments

If it is not inappropriate, I should like here to acknowledge two debts. First, to Professor C. T. R. Wilson, C.H., F.R.S., Hon.Mem.R.Met.S., who first introduced me to the study of Atmospheric Electricity and with whom I had the great privilege of working for several years; and, secondly, to the group of young men at Cambridge, with whom I have had the pleasure of discussing and investigating some of these problems during the past few years. Any merits this review may possess are due in large part to them; the faults are mine.

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Atmospheric ozone and the upper-air conditions

By Sir CHARLES NORMAND, C.I.E., D.Sc.

Presidential Address delivered before Royal Meteorological Society on 23 April 1952

1. Introduction

The ozone amount over Oxford this morning was 0·290 cm. By this we mean that if all the ozone in the vertical atmospheric column over Oxford were collected together in a layer at the bottom of the column and reduced there to standard pressure and temperature, its thickness would be slightly less than 0·3 cm or 3 mm. For comparison, we may recall that if the atmosphere itself could be made homogeneous, all of it being reduced to the same standard pressure and temperature throughout, its height would be about 8 km. So today's ozone accounts for about 3 mm in a column of 8 km or amounts to but 3 molecules of ozone in 8 million molecules of air. One's first reaction is – why bother about it ?

As is so often the case, interest in the subject increased with the growth of knowledge about it. One might say it became respectable in scientific meteorological circles when the results of Dobson's masterly campaign of the years 1925-9 became known. The data then published on the seasonal and geographical distribution of ozone have remained a fundamental source of facts and a challenging problem for the theorist. In other respects of the ozone problem, e.g., in its vertical distribution and in its photochemistry, considerable advances in knowledge have been made in the intervening years. Accounts of the present state of the ozone problem as a whole have appeared recently in the excellent monograph by R. A. Craig (1950), in the clear authoritative paper by F. W. P. Götz (1951) and in the larger posthumous book by the pioneer, Professor Ch. Fabry (1950), which is particularly lucid in its accounts of the earlier work.

Tonight we are to consider one aspect of the ozone problem, the relation of ozone with weather. Dobson and his colleagues (1929) were the first to notice the important association between day-to-day variations in ozone and surface weather, finding a tendency to high ozone values on the west side of cyclonic centres and low values within anticyclones; these results referred to the neighbourhood of the British Isles. Later studies of the ozone-weather relationship have included examination of the ozone values near frontal systems by Dobson (1942, 1946) and by Tonsberg and Langlo (1944). Near the equator, at Kodaikanal, the daily variations in ozone amount are barely significant and of no recognizable association with weather. Of special interest were Lejay's (1937) results in Shanghai which indicated an ozone increase when the Siberian anticyclone protruded towards Shanghai and seemed opposed to Dobson's early result. The discrepancy could be reconciled, suggested Haurwitz (1938), because the anticyclone extension over China and the rear of a cyclone over W. Europe have each a low-pressure trough above them at high levels. It is clear enough that several workers at that time had the idea that the upper-air changes would give the clue to those of ozone, but in the absence of daily upper-air charts no intensive study could be made along these lines. Since the war Nicolet (1945), Dütsch (1946) and Reed (1950) have suggested mechanisms for the increase of ozone in upper-air troughs. Reed, in particular, has offered a solution of the problem in considerable detail and along acceptable lines, to which reference will be made later. Meanwhile let us first get a picture of the actual ozone changes associated with synoptic patterns on the upper-air charts now published daily by the London Meteorological Office and other national services.

2. The ozone values

The ozone values are, in the main, those obtained at Oxford between Nov. 1950 and Jan. 1952 with the standard Dobson spectrophotometer. A daily record of ozone values demands some justification. For, the basic ozone observations are made on the direct rays from the sun and Oxford has many sunless days. In the 1930's Dobson developed a method of computing the ozone amount from observations of zenith cloud. His dot diagram (Dobson 1942) showing the correlation between sun and cloud measurements is not well-known and is therefore reproduced here in Fig. 1. The scatter of the dots about the diagonal straight line, though arising partly from real rapid variations in the ozone amount, is in the main a measure of the error in the cloud readings. Taking into account the position of the origin on this diagram, one sees that the scatter is small compared with the absolute value of the ozone amount; it is also considerably less than the amplitudes of the ozone fluctuations with which we shall be dealing. The cloud observations, needless to say, play a valuable part in maintaining continuity in the Oxford record.

In a few of the subsequent diagrams, those in which the method of curve-parallels is used in order principally to examine fluctuations of a size corresponding to the ridges and troughs in the long-wave pattern of the upper-air flow, some smoothing of the individual data is desirable to eliminate the very short-period fluctuations. The smoothing adopted here is of the $(a + 2b + c)/4$ type where a, b, c are the means of successive days. The seasonal trend has also been eliminated by subtracting the smooth annual curve drawn through the 12 monthly means.

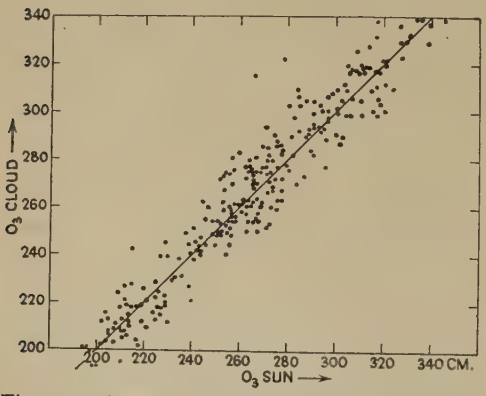

Figure 1. Ozone estimates from zenith cloud observations compared with those from the standard sun observations (Dobson).

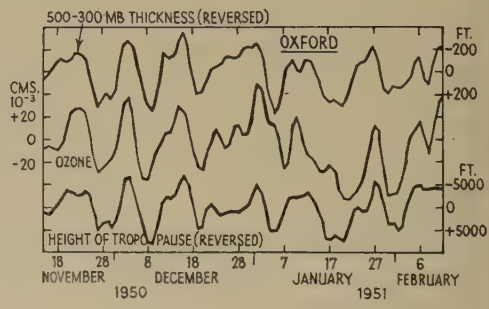

Figure 2. Smoothed daily observations of ozone, tropopause height (H_c) and 500-300 mb thickness (Th_{500}) at Oxford, Nov. 1950-Feb. 1951.

3. Long-wave fluctuations : winter 1950-1

In Fig. 2 the smoothed ozone values for three months in the winter 1950-1 are compared with the simultaneous smoothed values over Oxford of height of tropopause and thickness of the 500-300 mb layer. These are the two highest elements whose values over Oxford can readily be obtained from the daily upper-air charts published by the London Meteorological Office; their vertical scales are inverted in this and later diagrams on account of their inverse correlation with ozone.

Clearly a significant and considerable correspondence exists among the three curves. When the fluctuation is marked in periods of pronounced long waves of the upper flow, as from 22 Nov. 1950 to the end of the year and again from 20 Jan. to 10 Feb. 1951, the ozone fluctuates simultaneously with the other two elements. There is no

obvious lag between one element and another. The correlation is not perfect. Occasionally it is not at all close; and indeed it is almost reversed between 5 and 12 Jan. 1951. A few of the minor waves also appear unrelated. If one reflects however that the ozone is determined from readings of a spectrometer at ground level and the other two curves from radiosondes and if for the moment the week around 9 Jan. may be neglected, the outstanding impression is one of remarkable closeness of fit. The curves can be slid vertically closer to one another, interfolding well without intersecting at any point. As, however, one of my first lessons in meteorology from Sir Gilbert Walker was a warning about the dangers inherent in the method of curve-parallels and about the ease with which one's judgment may be tricked into seeing apparent parallelisms, when few or none exist, it is necessary to consider the matter in greater detail. So some days of high and low ozone are chosen from Fig. 2 and their upper-air charts examined, namely 4 Dec. 1950, a day of high ozone; the extremes in one oscillation 8 to 19 Dec. (Fig. 3); and the double oscillation 21 Jan. to 6 Feb. 1951 (Fig. 4). In each of the three charts shown for 4 Dec. (not reproduced here), (i) the 300-mb contours, (ii) the thickness contours of the 500-300 mb layer and (iii) the contours of tropopause height, the major feature was a trough stretching NE. from England to Scandinavia. Oxford with its high ozone was on the trough line. The upper wind was part of a strong solid current from the north, the trajectory at the 300-mb level starting well within the Arctic circle.

The upper-air charts associated with the single large oscillation in ozone starting on 8 Dec. 1951 are illustrated in Fig. 3 by the 300-mb contours on 8, 14, 15 and 19 Dec., days of low, high, high and low ozone respectively; as 14 Dec. was cloudy all day, the chart of the following day when the sun could be observed for an ozone value is shown as well. The low ozones coincide with the passage of high-level ridges over Oxford and the high ozone with a trough and the whole exemplifies one complete long wave in

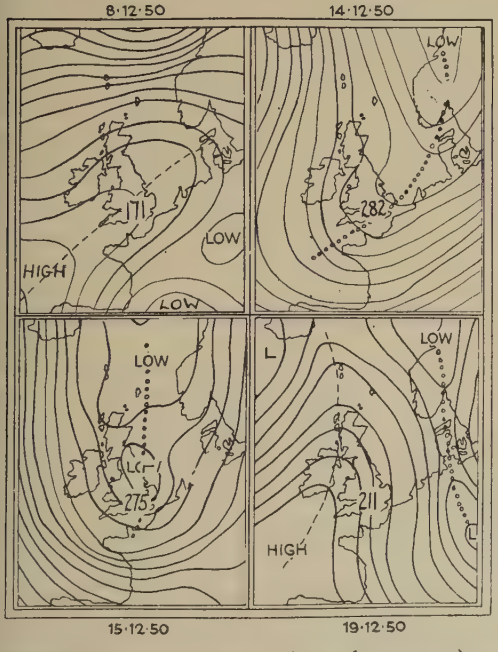

Figure 3. Upper-air charts (300 mb contours) on days of high and low ozone at Oxford, corresponding to one long-wave oscillation in the upper air, Dec. 1950 (values of ozone at Oxford entered on the charts).

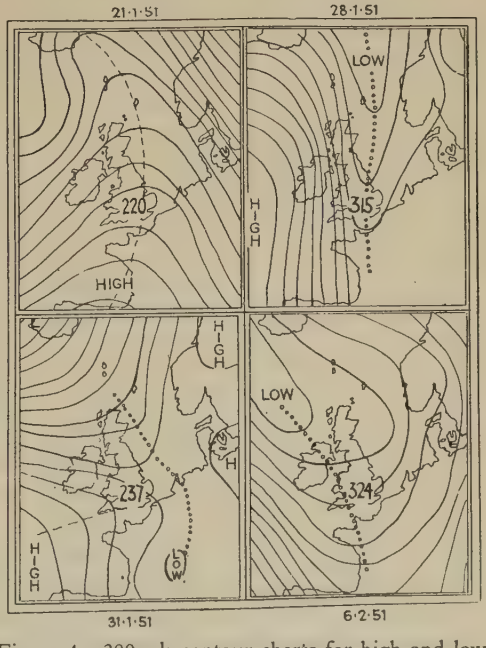

Figure 4. 300 mb contour charts for high and low ozone at Oxford in a double long-wave oscillation, Jan.-Feb. 1951.

the upper-air flow. The source of the air of high ozone value on 14 and 15 Dec. is in the region of the Arctic winter as it was on 4 Dec., a point to which we shall return when discussing the so-called ozone ' gap ' of the Arctic regions in mid-winter.

The other illustration (Fig. 4) of upper-air conditions for the winter period contains the charts of 21, 28 and 31 Jan. and 6 Feb. 1951, days of low, high, low, high ozone respectively and covering two major oscillations on the curves of Fig. 2. They tell much the same story as Fig. 3, the high ozone values of 28 Jan. and 6 Feb. being associated with high-level troughs and the low ozone of 21 Jan. with a high-level ridge over S. England. The contour pattern accompanying the low ozone of 31 Jan. shows a high-level ridge pushing into a trough; or it might be regarded as the right-hand edge of the exit of a jet stream. Though not far from the trough, Oxford is in the ridge, the isobars as drawn in the published chart being strongly and unmistakably anticyclonic there. The difference in air source in the top two charts of Fig. 4 is again quite marked, the higher ozone coming by much the more northerly route. Between the lower two charts the difference in origin of the Oxford air is much less marked and the outstanding alteration is the change from anticyclonic to cyclonic curvature in the 300-mb contour lines.

4. Anomalous and short-period ozone changes

The ozone changes around 10 Jan. on Fig. 2 are quite anomalous. They fail to show the associations of the other twelve weeks. This anomalous week was a period of minor systems rather than of long-wave predominance. The *Daily Weather Reports* mark the rapid passage of an occlusion over England on the 10 Jan., with shallow trough. The tropopause rose rapidly in the afternoon after the passage of the trough and there was every sign of considerable vertical movement in the upper air. That some minor systems affect the ozone at this and other seasons of the year can be seen by referring to the four days 10 to 13 June 1951 (charts not reproduced here). The ozone values at Oxford on these successive days were 273, 239, 280, 243. The contour lines of the 500-300 mb thickness changed twice from trough to ridge in the same period. Such daily changes are smoothed out in Fig. 2 and the subsequent similar figures. This particular example tells us that short-period changes can occur with the same general associations as those of the larger-wave patterns. It does not follow that this is typical of the short-period changes, which have not been systematically examined and would require consultation of the detailed working charts of several high levels for adequate investigation.

5. Long-wave fluctuations : spring to end of year

The diagram of curve parallels for the period 11 Feb. to 30 May 1951 (Fig. 5) was shown at the meeting of the International Meteorological Association in Brussels in Aug. 1951. In spring, as is well known, the amplitude of the ozone fluctuations reaches its maximum, corresponding both to the seasonal spring maximum in ozone content and to the maximum gradient of ozone across the latitudes. Again there are a few anomalies, e.g., near the end of February and around 18 May; but the major and minor maxima and minima on the ozone curve have simultaneous counterparts on one or both of the other curves. The changes correspond well in direction, if not always in amplitude. As in Fig. 2, the three curves can be slid vertically nearer to interfold without intersection. When the days of increasing and diminishing ozone on Fig. 5 were examined to test whether they were associated with winds of more northerly or more southerly source respectively, the answers seemed almost equally ' yes ' and ' no.' On the other hand, when the choice is restricted to the days of very high and very low ozone, the source

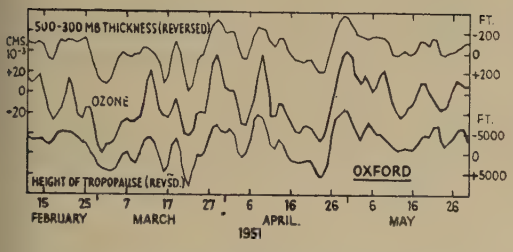

Figure 5. Smoothed daily observations, O_3, H_c and Th_{500} at Oxford, Feb. to May 1951.

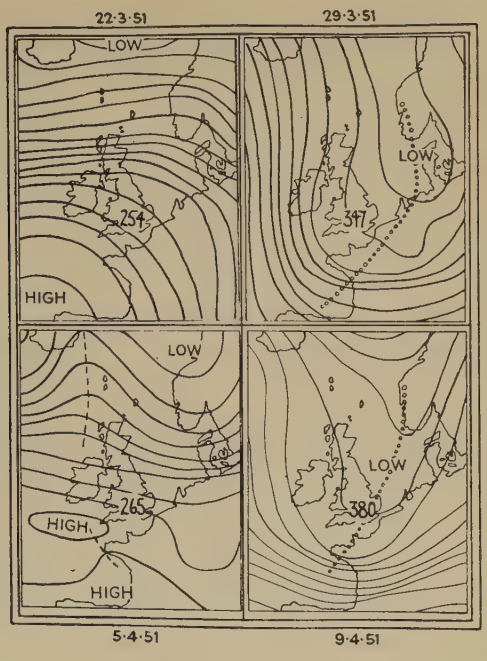

Figure 6. 300 mb contour charts for high and low ozone at Oxford in a double long-wave oscillation, Mar.-Apr. 1951.

of the upper air is in general, as in winter, more northerly in the former than in the latter and the associated air-flow patterns aloft are those of trough and ridge respectively. Take for example the double oscillation in ozone between Mar. 22 and Apr. 9. The four charts (Fig. 6) are for days of low, high, low, high ozone and the association of low ozone with anticyclone and high ozone with trough again stands out clearly.

To complete a full year's comparison of ozone with upper-air data by the parallel-curve method, Figs. 7 and 8 are included, taking the data up to the end of 1951. In summer (Fig. 7) the amplitude of the ozone fluctuations is, as usual, less than in spring and, although there are short intervals from 20 June and around 11 Sept. of little or no correlation between ozone and the other curves, the main maxima and minima again coincide fairly closely with those of the other curves, the marked double oscillation between 7 and 27 July in particular being conspicuous and coincident on all three curves. Even the smaller oscillations of slightly shorter period in August, which was a cyclonic month, are remarkably coincident. The general trend in the ozone curve in August is a rise relative to the curve of annual variation, not an absolute rise, the cyclonic weather of August having apparently kept up the ozone level when it normally falls. On the change over to an anticyclonic regime at the beginning of September the ozone values fell, and, as it were, changed to a lower level. In October (Fig. 8) the major oscillations are again evident on all three curves and the weather charts of 22, 25, 30 Oct. and 14 Nov. (not reproduced here) exhibit the similar trough-ridge sequence of the earlier diagrams. There followed a relatively quiet November and December, when the flow in the upper air did not oscillate in the large long-wave patterns of the previous year and the association between ozone and upper-air data seemed correspondingly less marked. For example, the tropopause maximum on 23 Nov. is not reproduced on the ozone curve; the ozone maximum and minimum on 20 and 23 Dec. do not have their counterparts

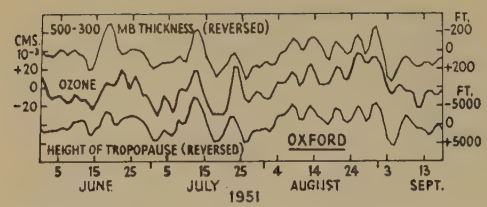

Figure 7. Smoothed daily observations, O_3, H_c and Th_{400} at Oxford, June-Sept. 1951.

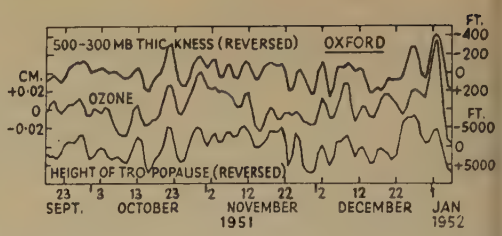

Figure 8. Smoothed daily observations, O_3, H_c and Th_{400} at Oxford, Sept.-Dec. 1951.

on the other curves. After 25 Dec. when a large oscillation sets in, all three curves agree in the timing of the changes, if not in the magnitude of the amplitudes.

When the 300-mb contours at Oxford on days of maximum and minimum ozone successively were drawn tangential to a fixed vector AB with AB pointing in the direction of the air flow, the diagram of Fig. 9 was obtained, cyclonic contours at maxima of ozone and anticyclonic at the minima.

On the whole the outstanding feature of the year is the correspondence of the main ozone fluctuations of from 6 to 14 days' period with coincident oscillations of opposite phase in the heights of the tropopause and of the 300-mb contours. Correspondingly the low ozone values in any month tend to be in the ridges or anticyclonic parts of the long-wave pattern of the upper-air flow and the high ozone in the troughs. This must mean that in the upper levels an ozone ridge is generally a pressure trough and *vice versa*.

6. Latitudinal distribution of ozone and advection

To the question – why does the ozone amount vary with weather? – one of the earliest answers was – by advection, i.e., by horizontal transport. On the average, ozone increases with latitude. So a wind from the north, or a polar air mass, should bring more ozone with it than a tropical air mass from the south. Many years ago Dobson pointed out the main objection to this theory – namely that in autumn the north-south gradient of ozone is small – and that the percentage decrease of this gradient from spring to autumn is much greater than the decrease in amplitude of the ozone fluctuations. Fig. 10, which is a form of diagram first given by Götz, provides a picture of the normal variation of

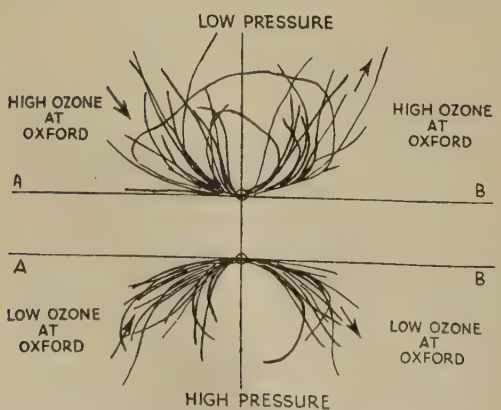

Figure 9. Upper-air isobars are cyclonic and anticyclonic respectively at times of high and low ozone. Lines AB were placed tangential to 300 mb contours at Oxford with AB pointing in direction of wind.

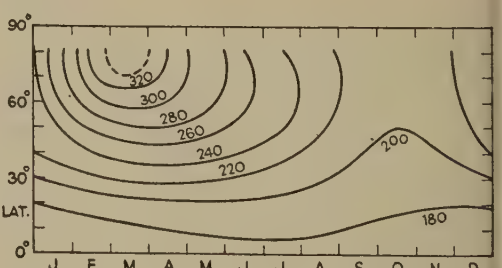

Figure 10. Seasonal and latitudinal distribution of ozone (after Götz, but without the Arctic winter 'ozone-gap').

ozone during the year at different latitudes. Not only in autumn, but in almost all the six months from July to December, the normal gradient of ozone from Lat. 30° to 80° does not at all support any simple advection theory of ozone changes; some other means of changing the ozone amounts must be found. In the spring months it is a different story. Then the north-south gradient is strong and may well be an effective contributing factor.

May I digress for a moment to say a little more about Fig. 10 ? Götz was the first to point out the tendency to a maximum of ozone near Lat. 60° in the summer and autumn, what he has called the ' ozone belt,' a tendency which recent observations support. The less convincing feature in Götz's diagram was the ' ozone-gap,' a short-lived but intense minimum of ozone in the Arctic regions in December and January. It is true that during the 1940's the Tromsø data did seem to show a strong deficit of ozone in the winter months. There was always the possibility however, that the apparent results arose largely from using an instrument at the limit of its sensitivity. For the past 18 months, two instruments which were readjusted at Oxford and fitted with photo-multipliers, have been in use at Tromsø and Spitsbergen. They are much more sensitive to weak ultra-violet light than any previous ozone instrument. Dr. E. Tönsberg in a recent private communication (received a fortnight ago) writes about the Tromsø winter results as follows :

> A rough examination of the blue-sky observations for 1951/52 seems to give the same conclusion as for the winter 1950/51, namely, no monthly mean values of ozone amount lower than 0·020 cm or, in other words, no definite minimum in November-December in either of the past two years . . . . Today I am forced to consider my earlier ozone values in winter as very doubtful.

Moon observations, made at Tromsø and Spitsbergen with the aid of the modernized instrument in the winter months, support the same conclusion. Further, though indirect, evidence comes from the days of ozone maxima at Oxford in Dec. 1950 and Jan. 1951, mentioned above, when the upper winds were from the Arctic. No matter what other factors were at work to increase the ozone, it is difficult to believe that they would so completely swamp the advective effect, if the Arctic ozone values were very low. Until suitable confirmation of low winter ozone values is received, it is advisable to discount the previous evidence for the ozone-gap and so it is excluded from Fig. 10.

7. The vertical pattern in the ozone relationships

We have been concerned in the preceding paragraphs with what may be called the horizontal pattern in the ozone relationships. The vertical pattern was outlined statistically by Meetham (1937), following the earlier work of Dobson and colleagues on the correlation of ozone with certain characteristics of the free air. Briefly, the result was that high ozone was associated with low temperature in the upper troposphere, low tropopause, high temperature in the stratosphere, low pressure in the upper troposphere and stratosphere. A repetition of these calculations for the year 1951 at Oxford produced the same results, the signs of the correlations being consistently the same, though the magnitudes varied considerably from month to month. The one significant change was a decrease in correlation between the levels of 100 mb and 70 mb, whereas Meetham, whose temperature data had been obtained by means of the old ballons-sondes and Dines meteorographs, found it increasing up to his highest available level of 18 km.

The classical investigations of W. H. Dines on the characteristics of the free air revealed many years ago that correlations of a similar nature prevailed among the weather elements themselves. We might now say that they indicated the atmosphere's preference

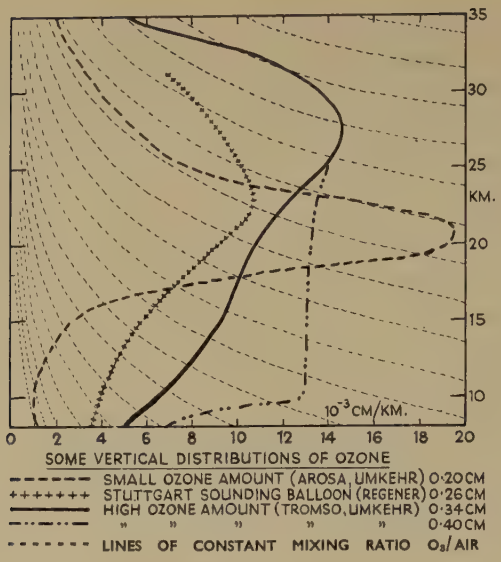

Figure 11.

for a particular pattern of vertical motion in linkage with the long-wave sinusoidal upper-air flow. Fleagle (1947) is one of several authors who have written on this subject and his idealized vertical cross section shows the stratospheric air descending with vertical divergence, or being subjected to lateral squeeze and downward stretch, in the interval between ridge and trough, the reverse holding good between trough and ridge. As the mixing ratio of ozone varies greatly with height, any air motion involving convergence and divergence must alter the total ozone amount/cm^2 in a vertical column. Let us consider this point in somewhat more detail. The ozone is nearly all in the stratosphere below 40 km, with centre of gravity at 22 km and according to latest photochemical theory the equilibrium state holds strictly only *above* 35 km. Below 35 km the time of restoration to equilibrium from any disturbed state increases quite rapidly, becoming a matter of a few weeks at 27 km. Most of the ozone, therefore, can be considered ' conservative ' in the sense that its mixing ratio in a parcel of air remains unchanged during the period of a long-wave oscillation in the upper-air flow. Now, Fig. 11 includes four different vertical distributions of ozone, selected by Götz as typical of small, medium, large and very large amounts. Lines of constant mixing ratio have been added to the diagram, which may be considered as analogous to humidity mixing-ratio lines on a tephigram. At the heights at which ozone is conservative a non-turbulent vertical displacement must be represented by movement along the suitable line of constant mixing ratio. We can readily picture how by vertical motion the transformation from one distribution to the other can be effected. For example, the change from the high (0·34 cm) to the very high ozone curve (0·40 cm) requires no vertical motion above 25 km, descent of 1 km at 20 km and of 2 km at 15 km and no vertical motion again at 8 km. This is the kind of flow just mentioned with horizontal convergence or lateral squeeze and downward stretch, the column between 25 km and 15 km being stretched to occupy the height from 25 km to 13 km. In one of the very large fluctuations of ozone in spring we may therefore picture a change of the following type occurring : on a particular day the distribution at Oxford is that of the medium amount (0·26 cm), while simultaneously over Iceland it is of the high amount (0·34 cm). The air flows from Iceland south-eastwards and arrives a few days later at Oxford in an upper-air trough, so that in the journey from Iceland the distribution is transformed by vertical divergence or stretch to the very high amount

(0·40 cm). Oxford would then experience a total change 0·40 − 0·26 = 0·14 cm, of which 0·34 − 0·26 = 0·08 cm would be by advection and 0·06 by vertical divergence. Reed (1950) has discussed this type of explanation very fully and satisfactorily. It is to be noted that the region in the stratosphere about which we are certain of the occurrence of vertical motion is in the few kilometres above the tropopause. Fleagle's diagram agrees with generally accepted opinion in showing the maximum stratospheric subsidence near the tropopause but shows almost level flow and little or no subsidence at 17 km. Reed also assumed that all the subsidence takes place between 18 km and 8 km with maximum total displacement of 2·3 km and, even with these limitations, found that the vertical motion between ridge and trough should cause a 13 per cent increase in ozone. In October (Fig. 8) a 25 per cent variation was recorded and in July (Fig. 7) a 20 per cent. In neither of these have we any reason to expect advection to be a factor in the change. The tropopause is displaced by more than 2·3 km, which is an argument for allowing a greater downward displacement in the air than Reed assumed. Granting this we have a plausible explanation of the ozone changes at Oxford by vertical divergence in summer and autumn and by a combination of vertical divergence and advection in winter; and in all cases these mechanisms are merely redistributing the ozone in space, because all relevant action is assumed to occur within the region in which the ozone is conservative. It is possible of course to picture a different role for the vertical motion – a circulation in which air is carried above the critical height in one area and down below it in another. This must be one of the mechanisms responsible for the seasonal changes of ozone. But we have little knowledge about vertical movements near the 30-km level and no evidence (with the possible exception of the rare iridescent clouds at twilight) to connect them with our changes of weather and with the long-wave oscillations in the upper atmosphere.

8. Ozone surveys

A plausible explanation is not a proof. We cannot close our eyes to the fact that the degrees of freedom for explanation and theory are not at all closely limited by the available data. So, although it does seem probable that vertical convergence and divergence (along with advection during part of the year) will explain the facts, some more direct evidence is required. The ozone survey begun in recent years in W. Europe will furnish evidence on the role of advection, when sufficient individual trajectories touching two or more well-spaced stations can be collected. The Gassiot Committee of the Royal Society, the International Ozone Commission of the U.G.G.I., U.N.E.S.C.O. and several national services, including the London Meteorological Office, have taken part in building up the W. European network of stations. The instruments are Dobson spectrophotometers of the modern type, compared and calibrated at Oxford before issue. The stations are : one each in Spitsbergen, Iceland, the Azores; also, Tromsø, Uppsala, Aarhus, Brussels, Arosa, Lerwick, Aldergrove, Camborne, Yarmouth (Hemsby), Oxford and, soon to start, two in France and one in Spain.

To complement this survey of horizontal distribution, one on the vertical distribution is also most desirable. The best and most direct observations would be by spectroscope carried aloft by large balloon or by rocket. Neither of these can become routine or be brought into frequent use. When comparing their value against results by the less effective ' umkehr ' method, we must remember that they too have their limitations. The experimental points, each of which has its possible error, have to be joined up by a smooth curve, which is then differentiated to determine the variation of ozone with height. Johnson, Purcell and Tousey (1951) who have made a very careful and critical estimate of their rocket data, have stated that :

The shapes of the concentration curves are extremely sensitive to the way in which the smoothed curves . . . are drawn through the experimental points. We have chosen to smooth the data as much as possible, consistent with our allowed probable error . . . If the curves are made to pass closer to or through the most probable positions of the experimental points, it is possible to obtain concentration curves that have other features, such as maxima, minima and shoulders.

It will be recalled that the rough preliminary and much-quoted estimate of the rocket result on 10 Oct. 1946 showed a very marked double maximum. The revised careful re-evaluation of the same data gives but one broad maximum.

Partial information about the height of the ozone layer may also be obtained from the infra-red spectrum of the sun. The principle involved is the change of absorption coefficient of ozone with pressure at these wavelengths. Strong in America suggested some years ago this method of determining the effective air pressure of the ozone layer, and Dr. Goody and Mr. Walshaw have it as one of the items of their current infra-red investigations at Cambridge.

9. The 'umkehr' method

The 'umkehr' method is less reliable and gives less direct information than the balloon method but has the advantage of being simple and cheap in its observational technique and capable of further development, inasmuch as the modern spectrophotometer, with photomultiplier, is very sensitive and has been designed also for observations on several pairs of wavelengths in the ultra-violet. Let me attempt a brief explanation of why a series of so-called 'umkehr' observations does tell us about the height and vertical distribution of the ozone.

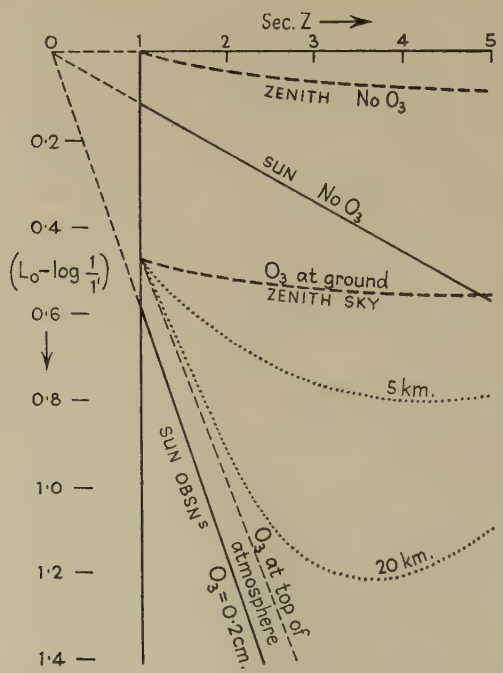

Figure 12. Diagrammatic representation of the effect of the height of an ozone layer on the values of log I/I' in the light from the zenith blue sky.

The ozone spectrophotometer compares the intensities of two neighbouring wavelengths in the ultra-violet, the shorter being in and the longer just on the fringe of the ozone absorption band. A measurement consists of noting the linear distance the optical wedge has to be moved to make the longer wavelength beam of equal intensity with the shorter ozone-absorbing beam. When the instrument is pointed at the sun, the observations lie on a straight line, if distance along optical wedge (or log I/I') is plotted against optical path through the ozone (or secant of sun's zenith angle); see the steeply-sloping line, marked $O_3 = 0{\cdot}2$ cm in Fig. 12. On the other hand, when the instrument is pointed at the blue zenith sky, the observations fall on a curve, not quite equivalent to but rather like the dotted curve marked 20 km in the figure. The turn-up of the curve between sec $Z = 3$ and 4 is the equivalent of what Götz found and explained under the title ' umkehr.' Now, given the sun observations for $O_3 = 0{\cdot}2$ cm, it is not difficult on certain simplifying assumptions to calculate what the zenith blue-sky readings should be if all the 0·2 cm of ozone is collected in a thin horizontal layer at any height. If the ozone is at ground level, the readings should show little change with change of sec Z and be represented by the dashed line, marked ' O_3 at ground.' On the other hand if the ozone molecules are at the top of the atmosphere, the readings on the optical wedge ought to increase rapidly and steadily with sec Z and be represented by the dashed curve marked ' O_3 at top of atmosphere.' These are the two extremes of the possible positions. The readings for intermediate positions of a thin ozone layer must lie somewhere between these two extremes, those for 5 km and 20 km respectively, being entered on the diagram. For ozone distributed over a considerable height instead of in one thin layer the calculation becomes complicated, (see Chapman's (1935) description of cases to which mathematical analysis can be applied). When sec Z is small, between 1·0 and 1·5, the vertical distribution has very little effect and the value of I/I' is determined by the total amount of ozone and the mean air pressure at which it lies. The initial slope of the curve, starting from sec $Z = 1$, is a measure of the mean pressure at which the ozone lies. When sec Z is large, as is evident from the rate at which the 20-km curve diverges from the ' top of atmosphere ' curve, the distribution of ozone in the higher layers is the all-important factor.

The Götz, Meetham and Dobson paper (1934) tells how the vertical distribution of ozone is estimated from the observations and the most significant testimony to the value of the early umkehr estimates has been their reasonably close agreement with the results obtained later on by rocket and by balloon. The chief arguments against the method are : (i) its insensitivity to the finer details in vertical distribution, (ii) the possible variation of ozone amount and distribution during the period covered by a full set of observations, and (iii) the assumption that scattering of the second and higher orders lead to components in I and I' of the same ratio as the components for primary scattering, on which the underlying theory is based. The effect of multiple scattering is now being examined in several quarters. G. F. Walton (1951) working with Prof. S. Chapman at Oxford, has come to the conclusion that corrections have to be applied on account of the secondary scattering and has calculated the corrections suitable for a few given distributions. He has also made use of almost simultaneous observations taken on different pairs of wavelengths, a modified umkehr method, to estimate roughly a vertical distribution.

If we can get the complete answer to our doubts and questions about the effect of multiple scattering on the zenith sky readings of the ozone instrument, there is little doubt that the old and the modified umkehr methods can give us much information of value on vertical distribution and therefore may in turn provide some of the evidence we need to convince us, or otherwise, of the role of vertical motion and vertical convergence and divergence on variations in ozone amounts.

10. Conclusion

To summarize, this rather specialized address has been illustrating present knowledge in but one aspect of the ozone problem, the relation of day-to-day changes in ozone amount with weather. Recent observations at Oxford strongly support the view already deduced by several writers that in our latitudes the ridges and troughs in the zonal flow of the upper air are localities of minima and maxima respectively of ozone. The relations with the smaller systems require and deserve separate examination.

There is good reason to believe that both advection and vertical divergence are responsible in spring for the ozone changes, but in late summer and autumn only the vertical motion can be responsible. The quantitative and convincing answer will require more detailed knowledge of the horizontal and vertical distributions of ozone on individual days than we have had in the past. It is hoped that the network of ozone stations recently established in W. Europe will both add to our knowledge of the relationship between ozone and weather and also furnish evidence about the role of advection in these changes; if several of the stations can collect series of data for umkehr estimates of height and vertical distribution of the ozone, better evidence on the part being played by vertical movements on ozone changes may also emerge.

I take this opportunity to express my indebtedness to the discussions I have fortunately been able to have during the past few years with the man who opened out this whole subject, Dr. G. M. B. Dobson.

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On the theory of large-scale disturbances in a two-dimensional baroclinic equivalent of the atmosphere*

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Summary

The equations of non-viscous, adiabatic flow have been reformulated to apply to a two-dimensional equivalent of a three-dimensional baroclinic fluid. The state of this ' equivalent-baroclinic ' fluid is to be identified with conditions at a definable ' level of equivalence ' in the true atmosphere. The two equations of equivalent-baroclinic flow, in which neither vertical motions nor vertical dependence enter explicitly, involve only two dependent variables – the temperature and height of an isobaric surface. Since those equations form a closed system, they constitute a mathematically complete theory of the development and movement of large-scale disturbances at the level of equivalence. The stability criterion for a special type of flow, previously studied by Fjörtoft and others, has been derived from the theory and compared with Fjörtoft's earlier results. This comparison shows that the stability characteristics of three-dimensional baroclinic flow are reproduced in two-dimensional equivalent-baroclinic flow, and, accordingly, that the present theory allows for the development of disturbances once they are initiated. An iterative scheme, based on successive calculations of the temperature and height tendencies, has been proposed for solving the general non-linear equations of equivalent-baroclinic flow. This method provides the basis for a new technique of numerical weather prediction.

1. Introduction and general remarks

During the past few years, increasing attention and effort have been given to formulating and solving the problem of numerical weather prediction. With the recent development of high-speed automatic computing devices, the theoretical meteorologist has gradually come to realize that the general equations of hydrodynamics can be integrated, in principle and if necessary, by brute machine force. As if by the removal of a bloc, this realization alone seems to have opened the meteorologist's mind to several other possibilities – some of which, in fact, were already open to him. This turn of events has had not only the direct effect of widening the application of numerical methods, but has had the rather paradoxical effect of encouraging greater use of exact analytic methods. With the assurance that purely numerical methods could always be depended on as a last resort, the meteorologist has begun to explore more thoroughly the limits of difficulty which separate a limited class of problems that are amenable to exact mathematical analysis from a general class that must be treated by the more pedestrian numerical methods.

Since the question of integrating the fundamental equations by numerical methods is no longer one of bare possibility, but of feasibility and economy, the primary concern is to formulate the problem in the simplest and most effective way possible. It is obviously desirable that the problem be recast in a form that (a) is simple enough to permit an easy understanding of basic physical mechanisms, reflected in what is otherwise sheer mathematical formalism, but which (b) still preserves the essential meteorological features of the problem and (c) presents no serious computational or analytical difficulties. The major difficulties inherent in the most general formulation of the problem have been discussed by Charney (1949) and by Thompson (1952).

* The work reported here is a part of the preliminary research for the author's doctoral thesis.

Aside from matters of convenience, moreover, it has finally been recognized that the problem of numerical weather prediction has not been stated as a real and sensible question, until the general equations have been specialized to refer explicitly to the meteorological behaviour of the atmosphere – rather than to its acoustic and aerodynamical behaviour. The problem of distinguishing and isolating the ' meteorological ' motions of the atmosphere from its many other possible modes of motion has been dealt with extensively by Charney (1948) and Thompson (1952). Using different variants of the general method of scale analysis, both arrive at the rather remarkable result that exactly those features that make the numerical prediction problem difficult are the ones which truly characterize the problem, and which can be turned to advantage in specializing the general equations. It has been shown, in fact, that the introduction of the conditions for geostrophic and hydrostatic equilibrium into a suitable form of the vorticity equation is necessary and sufficient to exclude solutions corresponding to the obviously non-meteorological types of motion, e.g., high-speed sound and gravity waves.

In order to illustrate the manner in which the so-called ' filtering approximations ' (i.e., the geostrophic and hydrostatic approximations) are to be incorporated into the hydrodynamical equations, and to provide the simplest possible formulation of a truly meteorological problem, Charney and Eliassen (1949) have developed a theory of ' equivalent-barotropic ' flow. By requiring that the direction of the horizontal component of velocity be independent of height, they have established a correspondence between the three-dimensional motion of the true atmosphere and the two-dimensional motion of the ' equivalent-barotropic ' atmosphere, whose state is to be identified with actual conditions at the ' equivalent-barotropic ' level. The latter is defined as the level at which the true wind equals the mean wind, integrated vertically with respect to pressure. It is a matter of experience that this level lies at about half the pressure-depth of the atmosphere, roughly coinciding with the so-called level of non-divergence.

The equation of equivalent-barotropic flow is of the same general form as the vorticity equation for the quasi-horizontal motion of a homogeneous non-viscous fluid, a system that was studied earlier by Rossby* (1939). Numerical solutions of the general non-linear equation, valid for actually observed initial conditions, have been exhibited by Charney, Fjörtoft, and von Neumann (1950) and by staff members of the Institute of Meteorology, University of Stockholm (1952). Analytic solutions of a related linear equation, valid for any given set of initial conditions, have been presented by Thompson (1952).

Although the methods by which the equation of equivalent-barotropic flow has been solved vary widely, the conclusions and results are much the same. The general level of accuracy of predictions based on solutions of the equivalent-barotropic equation is comparable with that attained by a trained forecaster, armed only with more conventional (and somewhat less objective) techniques. The few results available at the present time indicate that the numerical methods are quite successful in predicting the movement or propagation of already developed disturbances, but fail to predict the development of new disturbances. Since the vorticity equation can be regarded as a species of extrapolation formula, giving the *instantaneous* speed of a circulation centre, one is tempted to conclude that what the forecaster lacks in efficiency as a mathematical integrator is just about made up by his ability to recognize the early symptoms of development.

With the remark that many developments which the forecaster regards as ' new ' are probably due to dispersion effects in an essentially barotropic medium, it must be admitted

* In passing, it should be noted that the assumptions adopted by Rossby (1939), to demonstrate the mathematical existence of long waves, were precisely those that were required to exclude the irrelevant and troublesome types of motion. Because he dealt with the horizontal motion of a homogeneous (and, consequently, incompressible) fluid, the system was incapable of propagating either sound or gravity waves. By default, therefore, all that remained were the long, slowly-moving Rossby waves. Although Rossby's assumptions were not fully justified, the work of Charney and Eliassen must be regarded as the vindication of his results.

that the equation of equivalent-barotropic flow requires almost complete conservation of absolute vorticity, and accordingly, that it precludes the development of new centres of circulation, i.e., ' new ' in the sense that they were nowhere present before. It is easy to see, in a general way, why the equivalent-barotropic theory fails in this respect. Although it applies to baroclinic flow of a special (and commonly observed) type, it does require that the direction of the wind be the same as that of the vertical shear. This implies that, in quasi-geostrophic flow, the contours of an isobaric surface at the equivalent-barotropic level must coincide with the isotherms. Thus, with reference to the concepts introduced by Fjörtoft (1951), one is forced to conclude that the advection of ' thermal vorticity ' by the thermal wind vanishes at centres of maximum or minimum relative vorticity and, consequently, that the intensity of the centres must remain substantially unchanged.

It is probably safe to say that numerical techniques will lead to no significant improvement over less objective methods, until the underlying theory is capable of dealing with more general types of baroclinic flow – and, in particular, until it contains the mechanism by which new disturbances develop. Many theories, of course, have been proposed to explain the growth of cyclone waves, long waves, primary depressions, secondary depressions, and other special types of disturbances. More recently, considerable advances toward a general theory of development have been made by Sutcliffe (1947), who has at least succeeded in defining a single, universal index of development. None of these, however, can be regarded as a complete theory. Even the most quantitative and most general of them can predict only the instantaneous *tendency* of development, and cannot (given data for only one time) predict the entire evolution of a disturbance, from the early stages of rapid growth to the final stages of occlusion.

The most obvious and direct course – to make certain that the ingredients essential to baroclinic development are not left out of the theory of large-scale disturbances – would be to treat the three-dimensional motions of the atmosphere with complete generality, without any restriction on the dependence of velocity on height. In fact, it has already been suggested by Charney (1948) that the *large-scale* three-dimensional motions are governed by a very simple system of conservation equations, expressing the individual invariance of potential vorticity and potential temperature, taken together with the conditions for quasi-geostrophic and hydrostatic equilibrium. Since the vertical component of velocity enters linearly and undifferentiated in both equations, it can be eliminated to obtain a single equation involving only pressure. For purposes of discussing the resulting pressure equation, it is sufficient to point out that the coefficient of the derivative of highest order would contain the static stability as a factor. Thus, if the static stability were small, calculations of the instantaneous pressure tendency would be extremely sensitive to errors in estimating the stability and, under some conditions, any method of iterated extrapolation would amplify those errors to the point of obscuring the true solution. It is frequently observed, of course, that large regions of the troposphere do approach the state of neutral (zero) static stability. It would be desirable, therefore, to eliminate the vertical component of velocity by some other means, possibly by integrating out the vertical coordinate in much the same way as has been done in deriving the equation of equivalent-barotropic flow. Moreover, aside from its evident computational advantages, a two-dimensional model can be visualized easily, making it possible to interpret the mathematical theory in terms of relatively simple geometric and physical concepts.

The purpose of this paper is to develop a theory of ' equivalent-baroclinic ' flow, i.e., flow which is two-dimensional, in the mathematical sense that the vertical coordinate and vertical component of velocity do not appear explicitly in the governing equations, but whose behaviour is equivalent to that of true baroclinic flow. It will be found possible

to synthesize a two-dimensional model that reproduces not only the propagation characteristics, but also the stability characteristics of large-scale disturbances in three-dimensional baroclinic flow. The motions of this equivalent-baroclinic atmosphere are governed by a simultaneous system of two equations involving only two unknowns, the height and temperature of an isobaric surface at an identifiable ' level of equivalence.'

Because they form a closed system, the two equations of equivalent-baroclinic flow constitute a complete mathematical theory of the development and propagation of large-scale atmospheric disturbances. As it turns out, the equation that expresses the dynamical constraint on the motion is essentially a vorticity equation, of the same general form as Rossby's equation of potential vorticity. The other, which explicitly introduces the thermodynamical element, contains a physical mechanism similar to that expressed in Sutcliffe's (1947) development formulae. Thus the two major viewpoints, as exemplified by the concepts of Rossby (1940) and Sutcliffe, simply represent complementary aspects of the same general theory. Each is substantially correct, but neither is in itself sufficient to deal with the entire problem.

2. The vorticity equation for equivalent-baroclinic flow

In an earlier paper (1952) the author has noted that, because the pressure force is a potential vector, the vorticity equation does not involve the small ageostrophic resultant of the individually large pressure and Coriolis forces. For this reason, it does not invariably give the local time derivative of vorticity as the small difference between individually large terms of the same sign. It is natural, therefore, to centre the development of suitable prediction formulae around some form of the vorticity equation. In this section we shall derive a vorticity equation that holds exactly under general conditions of baroclinity, later applying it to a special (but still non-trivial) type of baroclinic flow.

Throughout this derivation, it will be assumed legitimate to treat the atmosphere as a non-viscous, ideal gas in hydrostatic equilibrium. Before exploiting the consequences of these assumptions, however, something should be said in defence of the most severe restriction, namely, that the flow is non-viscous. In the first place, since it will later be stipulated that no heat energy is added to the system, it is only consistent to require that no energy be transformed irreversibly into the heat energy of random molecular motion, through the action of dissipative forces. It is observed, in fact, that the forces of molecular viscosity are much smaller than the pressure and Coriolis forces – but this is not to say that the Reynolds stresses, due to eddies smaller than the mesh-size of the observation network, are also negligible. In this connection, it is relevant to note that the total kinetic energy of eddies of various size ranges generally decreases with decreasing size, provided the scale is much greater than that of boundary-layer turbulence. Moreover, the energy of small-scale, mechanically induced eddies is apparently concentrated in a rather shallow layer next to the ground surface. For these reasons, and because we shall later confine attention to the vertically-integrated mean wind, the equations of non-viscous flow will be assumed to apply in the large.

To simplify manipulation, it is convenient to introduce a special type of coordinate system, in which two of the coordinate variables are distances measured in ' horizontal ' planes, but in which either the potential temperature or pressure replaces height as the third coordinate variable. The horizontal coordinates, x and y, are distances in the eastward and northward directions, respectively, measured along planes tangent to the geopotential surfaces at their points of intersection with a single orthogonal curve. For the present purposes, the tangent planes will be regarded as mutually parallel and everywhere ' horizontal.' It might be mentioned, by way of orientation, that the potential

temperature θ in adiabatic flow may be regarded as an independent variable identifying a set of material surfaces, in which case the coordinate system reduces to a variant of the 'quasi-Lagrangian' systems proposed by Starr (1945).

By expressing the horizontal pressure force in terms of the (x, y, θ, t) coordinates, and expanding the horizontal acceleration by means of well-known vector identities, we may write the horizontal vector equation of motion in the form

$$\left(\frac{\partial \mathbf{V}}{\partial t}\right)_\theta + \mathbf{k} \times (\zeta_\theta + \lambda)\,\mathbf{V} + \nabla_\theta \left(\frac{\mathbf{V}\cdot\mathbf{V}}{2} + E\right) + \frac{\partial \mathbf{V}}{\partial \theta}\frac{d\theta}{dt} = 0 \quad . \quad (1)$$

where the subscript θ indicates that the potential temperature has been held fixed in the process of partial differentiation with respect to x, y, or t. The vector $\mathbf{V}$ is the *horizontal* wind, ∇ is the *horizontal* vector gradient, $\mathbf{k}$ is a unit vector directed vertically upward, and $\zeta_\theta = \mathbf{k}\cdot\nabla_\theta \times \mathbf{V}$. The variable $E = gz + c_p T$ is the Montgomery stream-function and λ is the Coriolis parameter. In this and all subsequent expressions, unless otherwise indicated, the undefined symbols carry their usual meanings.

Combination of the equation of mass continuity with the condition for hydrostatic equilibrium leads to a form of the continuity equation due to Godart (1942).

$$\frac{\partial}{\partial p}\left(\frac{dp}{dt}\right) + \nabla_p \cdot \mathbf{V} = 0 \quad . \quad . \quad . \quad . \quad (2)$$

in which the subscript p indicates that pressure has been held fixed in differentiating with respect to x, y, or t. The hydrostatic equation may be written as

$$\frac{\partial E}{\partial \theta} = k^{-1} p^k \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where $k = R\, c_p^{-1}$.

The vorticity equation is obtained by applying the curl operator $\mathbf{k}\cdot\nabla_\theta \times (\ )$ to Eq. (1). Bearing in mind that the curl of any potential vector vanishes, and rearranging terms,

$$\frac{d}{dt}(\zeta_\theta + \lambda) + (\zeta_\theta + \lambda)\,\nabla_\theta \cdot \mathbf{V} = \mathbf{k}\cdot\frac{\partial \mathbf{V}}{\partial \theta} \times \nabla_\theta \frac{d\theta}{dt} \quad . \quad . \quad . \quad (4)$$

Now, by combining Eq. (3) with the relationship obtained by omitting the acceleration terms from Eq. (1), it can be shown that the vertical shear of the geostrophic wind is oriented perpendicular to the vector $\nabla_\theta p$. Thus the quantity $(\partial \mathbf{V}/\partial p)\cdot\nabla_\theta p$, which is the difference between $\nabla_\theta \cdot \mathbf{V}$ and $\nabla_p \cdot \mathbf{V}$, vanishes in geostrophic flow. Since we shall later introduce the geostrophic approximation into the vorticity equation – as a device for excluding solutions corresponding to gravity waves – $\nabla_\theta \cdot \mathbf{V}$ will be replaced by $\nabla_p \cdot \mathbf{V}$. Finally, substituting from Eq. (2), we may rewrite Eq. (4) as

$$D_p(\zeta_\theta + \lambda) - (\zeta_\theta + \lambda)^2 \frac{\partial}{\partial p}\left(\frac{1}{\zeta_\theta + \lambda}\frac{dp}{dt}\right) = \mathbf{k}\cdot\frac{\partial \mathbf{V}}{\partial \theta} \times \nabla_\theta \frac{d\theta}{dt} \quad . \quad (5)$$

where $D_p(\ ) = (\partial/\partial t)_p + \mathbf{V}\cdot\nabla_p(\ )$. This equation provides the basis for much of the remaining mathematical development.

Eq. (5) has been put in a form that is especially convenient for estimating the relative importance of the vorticity-generating effects of non-adiabatic heating and the vertical divergence of the upward flux of reciprocal absolute vorticity. These effects are represented by the right-hand side and second term of the left-hand side of Eq. (5), respectively. Since one of our primary concerns is to account for the development of disturbances, we shall consider tropospheric conditions that are typical of a rapidly intensifying low-pressure centre. Under such circumstances, it is not unusual to find that the maximum value

of vorticity (within a fixed isobaric surface) increases at the rate of about λ per day. Taking the vertical component of velocity to be of the order of 10 cm/sec at the 500 mb level, the magnitude of the second term on the left-hand side of Eq. (5) is also of the order of λ per day. The right-hand side, however, is of the order of

$$\frac{\lambda}{\Delta\theta}\frac{d\theta}{dt}$$

where $\Delta\theta$ is the potential temperature difference between the ground and tropopause levels. Except in rather thin cloud layers or in isolated precipitating clouds of great vertical extent, the non-adiabatic temperature changes in the free atmosphere are of the order of 1°C/day. Thus the magnitude of the right-hand side of Eq. (5) is of the order of ·03 λ per day, between one and two orders of magnitude less than the observed changes of vorticity in rapidly developing low centres. It is therefore reasonably safe to conclude that, so far as its vorticity-generating effects are concerned, non-adiabatic heating is of negligible importance in comparison with the vertical divergence of the vertical flux of reciprocal vorticity. Accordingly, although the foregoing conclusion obviously cannot apply to the course of events over long periods of time, the flow may be regarded as adiabatic for purposes of dealing with the rapid development of disturbances over periods of a day or so.

Introducing the condition for adiabatic flow, and integrating vertically with respect to pressure between the limits zero (the pressure at an infinite height) and p_h (the pressure at the ground surface, located at height h), we find that Eq. (5) reduces to

$$\int_0^{p_h} D_p\left(\frac{\lambda}{\zeta_\theta+\lambda}\right)dp - \beta\int_0^{p_h}\frac{v}{\zeta_\theta+\lambda}\,dp + \frac{\lambda}{\zeta_h+\lambda}\frac{dp_h}{dt} = 0 \quad . \quad . \quad (6)$$

where $\beta = \partial\lambda/\partial y$ and v is the northward component of horizontal velocity. It must be emphasized that this equation does not involve the horizontal velocity divergence, and involves the vertical component of velocity only in the individual rate of change of pressure at the ground surface. That is to say,

$$\frac{dp_h}{dt} = \left(\frac{\partial p_0}{dt}\right)_z + \mathbf{V}_h\cdot\nabla_z p_0 - g\rho_h w_h$$

where p_0 is the pressure at a *level* surface next to the ground. The surface value w_h of the vertical component of velocity is related to the surface wind and the slope of the underlying terrain by the lower boundary condition, $w_h = \mathbf{V}_h\cdot\nabla h$. Although the extension of the theory to include the general case of flow over sloping terrain is obvious and straightforward, it will simplify the remainder of this development to regard the lower boundary as a perfectly flat, level surface. This simply has the effect of suppressing the forced oscillations induced by boundary irregularities, leaving the properties of the dominant free oscillations substantially unchanged.

In order to put Eq. (6) into a form which is comparable with that of other vorticity equations, we shall consider a type of flow that might be characterized as 'almost irrotational,' in the sense that the absolute magnitude of the relative vorticity is much less than the Coriolis parameter. With this restriction, Eq. (6) can be rewritten as

$$\overline{D_p(\zeta_\theta)} + \beta\bar{v} - \frac{\lambda}{p_0}\frac{dp_0}{dt} = 0 \quad . \quad . \quad . \quad . \quad (7)$$

where, for brevity,

$$\overline{(\ \ )} = p_0^{-1}\int_0^{p_0}(\ \ )\,dp$$

In this form, the vorticity equation is roughly equivalent to Rossby's (1940) equation of potential vorticity.

The remainder of this section will deal with a special type of baroclinic flow that has been studied previously by Sutcliffe (1947), Charney (1951), and Fjörtoft (1951) – namely, flow in which the direction of the vertical wind shear (though not its magnitude) is independent of height. Thus, the horizontal velocity may be represented as

$$\mathbf{V} = \overline{\mathbf{V}} + A(p)\,\mathbf{t}(x, y, t) \qquad . \qquad . \qquad . \qquad (8)$$

where $\mathbf{t}$ is a unit vector in the direction of the vertical wind shear. At this point, it will simplify matters to note that the vorticity in isentropic surfaces is, to all intents and purposes, equal to the vorticity in isobaric surfaces. With good approximation, the difference between those quantities is

$$\zeta_p - \zeta_\theta = \frac{\lambda}{Ri}$$

where $\zeta_p = \mathbf{k} \cdot \nabla_p \times \mathbf{V}$ and Ri is the Richardson number. Since the Richardson number of large-scale atmospheric flow is characteristically large, frequently being of the order of 100 or greater, the difference $(\zeta_p - \zeta_\theta)$ is generally less than the probable error in computing either ζ_θ or ζ_p from direct measurements of the wind. Replacing ζ_θ by ζ_p, and introducing the constraint expressed in Eq. (8), we may rewrite Eq. (7) in the form

$$\frac{\partial \bar{\zeta}}{\partial t} + \overline{\mathbf{V}} \cdot \Delta\bar{\zeta} + \mathbf{V}^* \cdot \nabla\zeta^* + \beta\bar{v} - \frac{\lambda}{p_0}\frac{dp_0}{dt} = 0 \qquad . \qquad . \qquad (9)$$

in which $\bar{\zeta} = \mathbf{k} \cdot \nabla \times \overline{\mathbf{V}}$, $\mathbf{V}^* = (\overline{A^2})^{\frac{1}{2}}\,\mathbf{t}$, and $\zeta^* = \mathbf{k} \cdot \nabla \times \mathbf{V}^*$. It will be recognised that Eq. (9) is virtually identical to the vorticity equation derived by Fjörtoft (1951) under less general conditions. The significance of the present derivation is that *no restriction has been placed on the horizontal velocity divergence nor on the vertical motions of the free atmosphere.* The fact that Eq. (9) agrees with Fjörtoft's equation is simply evidence that vertical motions have no *direct* effect on the mechanism by which 'mean' vorticity is generated, for Eq. (9) verifies that the intensification of cyclonic circulation centres is brought about primarily by the advection of positive 'thermal vorticity,' ζ^*, by the 'thermal wind,' $\mathbf{V}^*$, blowing into those centres. This notion is also in accord with Sutcliffe's theory of development.

Although it is not strictly permissible to do so, it will be assumed that Eq. (9) applies to the unaveraged horizontal motions at a level where the true wind $\mathbf{V}$ equals the vertically-integrated mean wind $\overline{\mathbf{V}}$, that is to say, at the equivalent-barotropic level. With this interpretation, we shall next introduce the filtering approximations, replacing the mean wind $\overline{\mathbf{V}}$ by the geostrophic wind at the equivalent-barotropic level and replacing the shear vector $\mathbf{V}^*$ by a vector proportional to the thermal wind. Referred to pressure coordinates, the condition for geostrophic equilibrium takes the form

$$\overline{\mathbf{V}} = \mathbf{k} \times g\lambda^{-1}\,\nabla_p z$$

where z is the height of an isobaric surface at the equivalent-barotropic level. Thus, with good approximation,

$$\bar{\zeta} = g\lambda^{-1}\,\nabla_p{}^2 z$$

To simplify the problem of expressing $\mathbf{V}^*$ in terms of the thermal wind, $A(p)$ will be regarded as a linear function of *pressure*, and the pressure-height $\bar{p}$ of the equivalent-barotropic level will be taken as about one half of p_0, the surface pressure. Introducing these conditions into the thermal wind equation, we obtain the expressions

$$\mathbf{V}^* \simeq \mathbf{k} \times \frac{R}{\sqrt{3}\,\lambda} \nabla_p T$$

$$\zeta^* \simeq \frac{R}{\sqrt{3}\,\lambda} \nabla_p^2 T$$

where T is the absolute temperature of an isobaric surface at the equivalent-barotropic level. It remains to express the last term of Eq. (9) in terms of z and/or T. It has been noted elsewhere (Charney and Eliassen 1949 and Thompson 1952) that this term is at least one and generally two orders of magnitude less than the term representing the horizontal advection of vorticity, and might even be omitted altogether. With the assurance that the final result will not be much affected in any case, it is safe to approximate the last term of Eq. (9) by setting the pressure change at the ground proportional to the height change at the equivalent-barotropic level, by a factor depending on the ratio of the wind speed at the ground to the mean wind speed (Thompson 1952).

$$\frac{1}{p_0}\frac{dp_0}{dt} \simeq \frac{g}{RT_0}\frac{|\mathbf{V}_0|}{|\bar{\mathbf{V}}|}\frac{\partial z}{\partial t}$$

Substituting all these results into Eq. (9), we finally obtain an equation that involves only two dependent variables, the height and temperature of an isobaric surface at the equivalent-barotropic level.

$$\nabla^2\left(\frac{\partial z}{\partial t}\right) + g\lambda^{-1} J(z, \nabla^2 z) + \beta\frac{\partial z}{\partial x} + \frac{R^2}{3g\lambda} J(T, \nabla^2 T) - \nu^2\frac{\partial z}{\partial t} = 0 \quad . \quad (10)$$

where $J(\phi, \psi) = \mathbf{k} \cdot \nabla_p \phi \times \nabla_p \psi$ and $\nu^2 = \lambda^2 |\mathbf{V}_0|/RT_0 |\bar{\mathbf{V}}|$. It is understood that the indicated differentiations are to be carried out with p held fixed. Eq. (10) is the vorticity equation for equivalent-baroclinic flow – one of the two equations essential to a complete theory of development of large-scale disturbances in the baroclinic equivalent of the atmosphere. A similar equation has been derived previously by Charney (1951); as pointed out earlier, however, the basic Eq. (9) holds over a wider range of conditions than was assumed in Charney's derivation.

3. The connection between the equivalent-barotropic level and the levels of non-divergence and maximum individual pressure change

It has been pointed out by Charney (1951) that the equivalent-barotropic level generally lies at about the same altitude as the level of non-divergence. Although there is no obvious and direct connection between these two levels, the empirical relationship is not entirely fortuitous, and there must be some deep-seated physical reason for it. Since the close coincidence of the equivalent-barotropic level and the level of non-divergence is a key postulate in subsequent development of the theory, we shall attempt to show that there is at least an approximate connection between these two levels.

In order to define the equivalent-barotropic level, it is first necessary to define the mean wind, which is, in earlier notation,

$$\bar{\mathbf{V}} = p_0^{-1} \int_0^{p_0} \mathbf{V}\, dp$$

Integrating the right-hand side by parts, and expressing the fact that $\mathbf{V} = \bar{\mathbf{V}}$ at the equivalent-barotropic level,

$$\int_0^H (p_0 - p)\frac{\partial \mathbf{V}}{\partial z}\, dz = \int_H^\infty p\frac{\partial \mathbf{V}}{\partial z}\, dz \quad . \quad . \quad . \quad (11)$$

where H is the height of the equivalent-barotropic level. For a given vertical distribution of wind, the equation above may be taken as a definition of H.

In order to make a simple (though rather rough) accounting of the local variations of θ in the p-surfaces, we shall suppose that the entire θ-field is translated horizontally, without change in shape and at a vector phase velocity $\mathbf{C}$ which is constant in direction. To be exact, it is assumed that

$$\left(\frac{\partial}{\partial t} + \mathbf{C} \cdot \nabla\right)_p \theta = 0$$

As a consequence of the thermal wind equation, the special type of baroclinic flow we have chosen to consider is such that the phase of the θ-field is the same at all levels, whence $\mathbf{C}$ is independent of height. We next cross-multiply Eq. (11) by the vector $(\bar{\mathbf{V}} - \mathbf{C})$ and replace the vertical shear by the thermal wind, bearing in mind that the direction of the vertical shear is independent of height. The result may be put in the form

$$\int_{\bar{p}}^{p_0} (p_0 - p)\, T\, (\mathbf{V} - \mathbf{C}) \cdot \nabla_p (\ln \theta)\, dp = \int_0^{\bar{p}} p\, T\, (\mathbf{V} - \mathbf{C}) \cdot \nabla_p (\ln \theta)\, dp$$

Finally, by introducing the definition of $\mathbf{C}$ and the condition for adiabatic flow, the equation above can be rewritten as

$$\int_{\bar{p}}^{p_0} (p_0 - p)\, T \frac{\partial}{\partial p} (\ln \theta) \frac{dp}{dt}\, dp = \int_0^{\bar{p}} p\, T \frac{\partial}{\partial p} (\ln \theta) \frac{dp}{dt}\, dp \quad . \quad . \quad . \quad (12)$$

This equation may be regarded as an alternative definition of $\bar{p}$, the pressure-height of the equivalent-barotropic level.

In order to interpret Eq. (12), we shall consider the behaviour of the product $T\, \partial/\partial p\, (\ln \theta)\, dp/dt$. The first two factors are single-signed and generally do not vary rapidly with height. On the other hand, as pointed out by Sutcliffe (1947), the magnitude of the individual pressure change has a very distinctive variation with height, having a small value at the ground, increasing to a pronounced maximum at about the 500 mb level, and decreasing to zero at very great heights.* (See Fig. 1). Thus the function $T\, \partial/\partial p\, (\ln \theta)\, dp/dt$ displays the same general behaviour as dp/dt, having small values at the ground and at very great heights, and reaching its extreme value at about the level of maximum individual pressure change. Since the material derivative of pressure reaches its extreme value at about half the pressure-depth of the atmosphere and has a roughly symmetrical profile around its extreme value, Eq. (12) implies that the pressure-height $\bar{p}$ of the equivalent-barotropic level must correspond approximately to that of the level of maximum individual pressure change.

Owing to the quasi-geostrophic character of the large-scale motions, the main contribution to the individual pressure change comes from the vertical advection of pressure. That is,

$$\frac{dp}{dt} \simeq -g\rho w$$

Thus the level of maximum individual pressure change is near the level at which the vertical mass transport reaches its maximum (or minimum) and, accordingly, at which the vertical mass divergence vanishes. Moreover, because the local changes of density are very small – requiring almost complete balance between the horizontal and vertical components of mass divergence – the level of maximum individual pressure change must also lie near the level of zero horizontal divergence.

* Sutcliffe has assumed that the material derivative of pressure is to be fixed by setting the vertical integral of divergence equal to zero. The correct condition is that the material derivative of pressure vanishes at an infinite height, because the vertical transport of mass must also vanish at an infinite height.

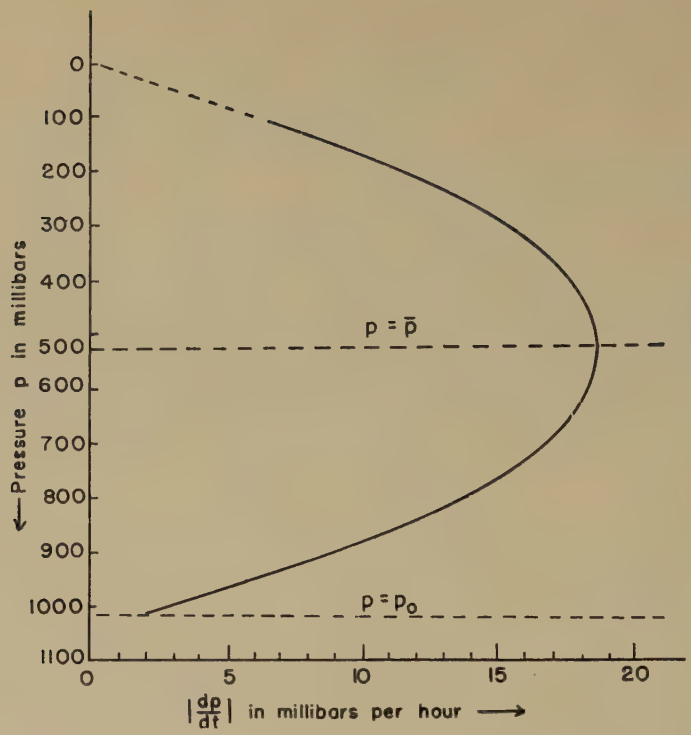

Figure 1. Typical vertical profile of individual rate of pressure change.

Although the foregoing argument is far from rigorous, it is at least sufficient to establish the conclusion that the equivalent-barotropic level, the level of maximum individual pressure change, and the level of non-divergence must all lie at about the same altitude. According to the data presented by Bjerknes and Holmboe (1945), Sutcliffe (1947), and Charney and Eliassen (1949), they are located at around 500 mb pressure-altitude, i.e., at about half the pressure-depth of the atmosphere.

In view of the remarkable properties of this level, and because the present theory has not been restricted to equivalent-barotropic flow, the ' equivalent-barotropic ' level evidently has much broader significance than its name might imply. It is proposed, therefore, that the level at which the true wind equals the mean wind be given the more general designation of ' level of equivalence.'

4. The thermal vorticity equation

It was emphasized earlier that Eq. (10), one of the two fundamental equations of equivalent-baroclinic flow, involves both the height and temperature of an isobaric surface at the level of equivalence. In order to form a closed system of equations, therefore, it is necessary to derive an independent equation that contains no variables other than the two already involved. In this section we shall derive the ' thermal vorticity equation,' which also involves only the temperature and contour height and otherwise has all the desirable characteristics outlined earlier. As it turns out, it expresses in more general and usable form an effect first studied by Sutcliffe (1947).

The starting point of this development is an equation that is obtained by imposing the condition for adiabatic flow on Eq. (5),

$$D_p\left(\frac{1}{\zeta_\theta+\lambda}\right)+\frac{\partial}{\partial p}\left(\frac{1}{\zeta_\theta+\lambda}\frac{dp}{dt}\right)=0$$

We first differentiate this equation vertically with respect to p, *and apply the result to conditions at the level of equivalence*, noting that $\zeta_\theta \ll \lambda$ by hypothesis. With the latter approximation,

$$D_p\left(\frac{\partial\zeta_\theta}{\partial p}\right)+\frac{2}{\lambda}\frac{\partial\zeta_\theta}{\partial p}\mathbf{V}^*\cdot\nabla\zeta^*+\frac{\partial\mathbf{V}}{\partial p}\cdot\nabla_p(\zeta_\theta+\lambda)-\lambda\frac{\partial^2}{\partial p^2}\left(\frac{dp}{dt}\right)=0$$

Now, it has already been shown that the difference between ζ_θ and ζ_p is negligible. Replacing ζ_θ by ζ_p, and making use of the thermal wind equation,

$$\frac{\partial\zeta_\theta}{\partial p}=-\frac{R}{\lambda\bar{p}}\nabla_p{}^2T$$

By introducing this result into the equation above, together with the previously derived expression for ζ^*, and by again noting that $\zeta_\theta \ll \lambda$, it can be shown that

$$\frac{\partial}{\partial t}(\nabla_p{}^2T)+g\lambda^{-1}J_p(z,\nabla_p{}^2T)+J_p(T,\lambda+g\lambda^{-1}\nabla_p{}^2z)+\frac{\lambda^2\bar{p}}{R}\frac{\partial^2}{\partial p^2}\left(\frac{dp}{dt}\right)=0\ . \quad (13)$$

It must be emphasized that this equation applies only at the level of equivalence.

The decisive turn of the argument lies in expressing the last term of Eq. (13) – which, in its present form, involves the vertical motions – in terms of quantities that do not involve the vertical motions explicitly. In the preceding section, it was shown that the individual pressure change reaches its extreme value at about the level of equivalence, decreasing to much smaller values at the ground and to zero at infinitely great heights. For the present purposes, it will be assumed that the individual pressure change varies smoothly, attaining its extreme value at the pressure altitude $\bar{p}$, and decreasing to zero at $p=0$ and $p=p_0$. Thus, approximating by finite differences,

$$\frac{\partial^2}{\partial p^2}\left(\frac{dp}{dt}\right)\simeq-\frac{2}{\bar{p}(p_0-\bar{p})}\frac{dp}{dt}$$

We next introduce the equation of state and the condition for adiabatic flow. Noting at the same time that $\bar{p}$ is approximately $\frac{1}{2}p_0$,

$$\frac{\partial^2}{\partial p^2}\left(\frac{dp}{dt}\right)\simeq-\frac{2g\theta}{R\bar{p}T^2}\frac{\partial z}{\partial\theta}D_p(T)$$

Finally, substituting the expression above into Eq. (13), we obtain an equation that involves the derivatives of only two variables, the height and temperature of an isobaric surface at the level of equivalence.

$$\nabla^2\left(\frac{\partial T}{\partial t}\right)+g\lambda^{-1}J(z,\nabla^2T)+\beta\frac{\partial T}{\partial x}+g\lambda^{-1}J(T,\nabla^2z)-\mu^2g\lambda^{-1}J(z,T)-\mu^2\frac{\partial T}{\partial t}=0\ . \quad (14)$$

where $\mu^2=\dfrac{\lambda^2}{R^2T^2}\left(\dfrac{1}{2g\theta}\dfrac{\partial\theta}{\partial z}\right)^{-1}$. This is the 'thermal vorticity equation.' Taken together with Eq. (10), it completes a closed system of two equations in two unknowns.

Since they do not refer explicitly either to the vertical component of velocity or to dependence on the vertical coordinate, the fundamental Eqs. (10) and (14) apply to a two-dimensional system which, in a mathematical sense, is equivalent to the true three-dimensional atmosphere. The state of this two-dimensional baroclinic equivalent of the atmosphere, as we have gone to some length to show, is to be identified with actual

conditions at the 'level of equivalence.' In principle, therefore, the behaviour of atmospheric flow at that level is completely determined by Eqs. (10) and (14), and the physical concepts expressed in those equations comprise a theory of the development and movement of large-scale transient disturbances.

5. The stability characteristics of disturbances in equivalent-baroclinic flow

In the final analysis, of course, the only completely fair and convincing test of this theory would be to solve Eqs. (10) and (14) exactly, subject to observed initial conditions. As will be shown in the next section, it is possible to do so. Because the equations are non-linear, however, the method of solution requires many iterations of a cycle of laborious and rather involved numerical calculations, a process that can be carried out efficiently only by recourse to semi-automatic computing machines.

On the other hand, it was pointed out earlier that the principal issue is not the possibility of predicting the propagation of already developed disturbances, but of predicting the development of new disturbances. The critical question, therefore, is whether or not the stability characteristics of three-dimensional flow can be reproduced in a two-dimensional equivalent.

The problem of establishing the stability criteria for three-dimensional baroclinic flow has been the subject of several recent papers by Charney (1947), Kuo (1952) and Fjörtoft (1949), the last of which is the most comprehensive. It is natural, therefore, to inquire if Eqs. (10) and (14) contain a mechanism of instability (development) similar to that studied by Fjörtoft. To put the results into a form comparable with his, it will be necessary to specialize the general equations to conform with the conditions Fjörtoft has assumed. Although the restrictions he has imposed are rather severe, they do not significantly alter the three-dimensional character of the flow. Thus, if the stability characteristics of two-dimensional equivalent-baroclinic flow agree with those of three-dimensional flow in this special case, it is fairly safe to conclude that the general equations of equivalent-baroclinic flow will apply under a much wider range of conditions.

To simplify analysis of the three-dimensional equations, Fjörtoft has assumed that the wind velocity and the amplitude of the disturbances are independent of y, and that the vertical wind shear is constant. Under comparable conditions, the perturbation equations corresponding to Eqs. (10) and (14) are, with good approximation,

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\frac{\partial v}{\partial x} + \beta v + \overline{A^2}\frac{\partial^2 \phi}{\partial x^2} = 0$$

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\left(\frac{\partial^2 T}{\partial x^2} - \mu^2 T\right) - \frac{dT_0}{dy}\left(\frac{\partial^2 v}{\partial x^2} + \mu^2 v\right) = 0$$

where U is the speed of the horizontally uniform west-east current in which the disturbances are embedded, ϕ is the angle (counted positive counterclockwise) that the isotherms make with the positive x-axis, dT_0/dy is the temperature gradient required to maintain a constant vertical wind shear consistent with the value of $\overline{A^2}$ and the differentiations are to be carried out at constant pressure. For the present purpose, it is sufficient to consider wave disturbances which may change in amplitude, but do not otherwise alter their shape while travelling at a constant complex speed c toward the east. In this case, the above perturbation equations reduce to

$$(U - c)\frac{\partial^2 v}{\partial x^2} + \beta v + \overline{A^2}\frac{\partial^2 \phi}{\partial x^2} = 0 \quad . \quad . \quad . \quad . \quad (15)$$

$$(U - c)\left(\frac{\partial^2 \phi}{\partial x^2} - \mu^2 \phi\right) + \left(\frac{\partial^2 v}{\partial x^2} + \mu^2 v\right) = 0 \quad . \quad . \quad . \quad (16)$$

Since these equations involve only the variables v and ϕ, they form a complete system.

In order to derive the stability condition, we seek solutions of Eqs. (15) and (16) in the form

$$v \propto e^{i\alpha(x-ct)}$$

$$\phi \propto e^{i\alpha(x-ct)}$$

where α is the wave number of the disturbances. Thus the condition that v and ϕ satisfy Eqs. (15) and (16) becomes

$$c = U - \frac{\beta}{2\alpha^2} \pm \Delta^{\frac{1}{2}}$$

in which

$$\Delta = \left(\frac{\beta}{2\alpha^2}\right)^2 - \overline{A^2}\left(\frac{\mu^2 - \alpha^2}{\mu^2 + \alpha^2}\right) \quad . \quad . \quad . \quad (17)$$

The stability characteristics of equivalent-baroclinic flow are reflected in the behaviour of the discriminant Δ. If Δ is positive, for example, the phase velocity is real and the disturbances neither increase nor decrease in amplitude. In this case, the disturbances are said to be ' neutral.' On the other hand, if Δ is negative, the phase velocity is complex, the imaginary part giving the logarithmic time rate of increase in amplitude. In this case, the disturbances are always unstable.

The quantitative results of this analysis are summarized in Fig. 2, which shows the dependence of Δ on the wave length and vertical wind shear, as given by Eq. (17). As it turns out, Δ is relatively insensitive to small variations of the static stability (except when the static stability is very small). Accordingly, the values of Δ shown here were calculated for a single value of $\partial\theta/\partial z = 3{\cdot}5°\text{A km}^{-1}$ as used by Fjörtoft to represent conditions throughout the troposphere. The solid curves are lines along which Δ is constant, and the dashed curve gives the wavelength of maximum instability for given

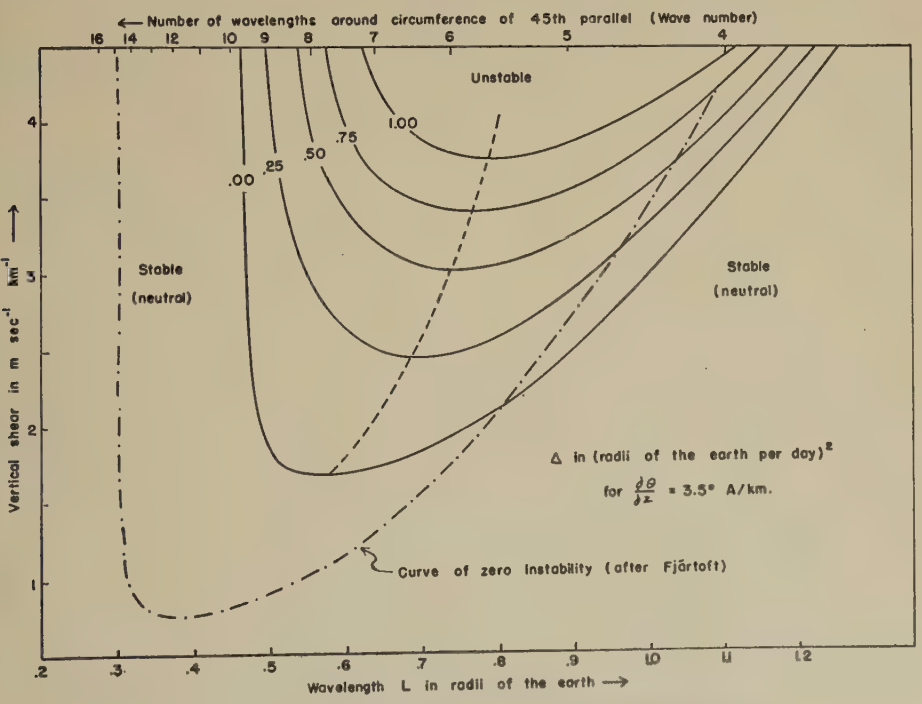

Figure 2. Stability characteristics of equivalent-baroclinic flow.

vertical wind shear. The dash-dot curve is Fjörtoft's curve of ' zero instability,' which separates the neutral waves from the unstable waves.

This diagram makes it clear that the stability characteristics of equivalent-baroclinic flow are very similar to those of the three-dimensional baroclinic flow studied by Fjörtoft (1949). As he has also found, there is a critical value of the vertical wind shear, below which all disturbances are neutral, and above which there is an increasingly broad band of wavelengths for which the disturbances are unstable. For any given value of the wind shear, however, the very short and very long waves are neutral. Another striking feature is the sharp ' cut-off ' between wave numbers 9 and 10. For any reasonable value of the vertical shear, disturbances with wave number equal to or greater than 10 are invariably neutral, while those with wave number 9 may be unstable.

On general physical grounds, one might suspect that the mechanism by which disturbances develop would also tend to destroy the conditions that were initially favourable to such development, e.g., vertical wind shear in excess of the critical value. Tentatively accepting this point of view, it follows that the critical shear must be approached from below and is never far exceeded. To round out this hypothesis, we shall also suppose that the atmosphere is continually subjected to more or less random impulses, in the sense that the energy spectrum of the impulses is roughly uniform and with no pronounced and persistent maxima. It is a consequence of these suppositions that the predominant wavelength of the transient disturbances does not correspond to a pronounced peak in the impulse spectrum, but is the result of a selection process whereby only certain bands in the impulse spectrum are picked out and amplified.

The significant fact is that the mechanism of baroclinic instability is precisely the kind of mechanism required to select a limited class of wavelengths from a uniform spectrum of random impulses. When the vertical shear approaches the critical value from below, the first – and, therefore, the most frequent – wavelength to be excited would be approximately the wavelength of maximum instability for the critical shear. This conclusion must be modified slightly, because the critical shear is undoubtedly exceeded by some finite amount, and because the wavelength of maximum instability increases with increasing vertical shear. One might expect, therefore, that the predominant wave number of the transient disturbances would be slightly less than the wave number of maximum instability for the critical shear, i.e., around 7 or 8. As a further consequence of this argument, the mean spectrum of ' eddy ' kinetic energy should have a pronounced maximum at about wave number 7 or 8. It has already been noted that the energy spectrum must show an abrupt cut-off between wave numbers 9 and 10, with a somewhat smoother decrease toward the long wavelength end of the spectrum.

The task of calculating the mean spectrum of eddy kinetic energy over a long period of time is a very formidable one, and no extensive data of this type are available at present. Fortunately, Charney has computed the spectrum of the kinetic energy of the north-south motions for a three-day period. His results, as taken from his (1951) *Compendium* article, are reproduced in Fig. 3. Allowing for the fact that the maximum between wave numbers 2 and 3 is a manifestation of the semi-permanent forced oscillations, it appears that the band in which the greatest eddy kinetic energy is concentrated lies between wave numbers 7 and 8. Moreover, although Charney's method of computation probably suppresses components with wave numbers greater than 9, his curve shows a sharp drop between wave numbers 9 and 8, at about the cut-off wavelength. Thus Charney's data provide striking confirmation of the stability characteristics of equivalent-baroclinic flow. It must be borne in mind, however, that this evidence does not carry the weight of years, but of days. At least in part, the almost complete agreement between theory and fact must be put down as coincidence.

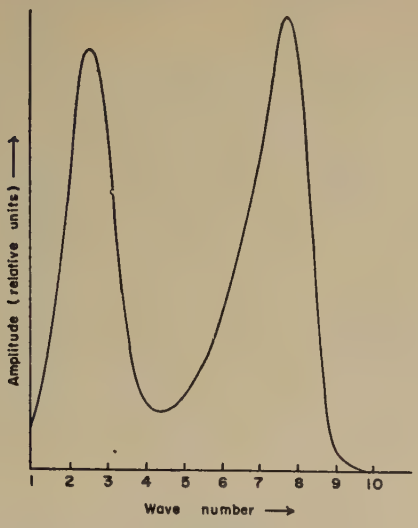

Figure 3. Mean spectrum of kinetic energy of north-south motions (after Charney). 500-mb level at Lat. 45°N.

For purposes of comparing Eq. (17) with Fjörtoft's (1949) results, it is convenient to identify RT with gH, where H is a measure of the effective *half* depth of the atmosphere. With this definition, the discriminant Δ becomes

$$\Delta = \left(\frac{\beta L^2}{8\pi^2}\right)^2 - \frac{1}{3}H^2\left(\frac{dU}{dz}\right)^2\left[\frac{1 - \dfrac{2\pi^2 g}{\lambda^2}\left(\dfrac{H}{L}\right)^2 \dfrac{1}{\bar{\theta}}\dfrac{\partial\bar{\theta}}{\partial z}}{1 + \dfrac{2\pi^2 g}{\lambda^2}\left(\dfrac{H}{L}\right)^2 \dfrac{1}{\bar{\theta}}\dfrac{\partial\bar{\theta}}{\partial z}}\right] \quad . \quad . \quad (18)$$

in which L is the wavelength. Except for a constant factor, this expression is almost identical with Fjörtoft's stability discriminant. The latter, in fact, may be regarded as a special form of Eq. (18), valid for very long wavelengths. It may be concluded, therefore, that the mechanism of instability contained in the equations of equivalent-baroclinic flow is essentially the one studied by Fjörtoft in his treatment of three-dimensional baroclinic flow. It might be noted, in passing, that Eqs. (10) and (14) should form a satisfactory basis for a general theory of atmospheric instability, for they include not only the effects of baroclinic instability, but also the effects of the barotropic instability discussed by Kuo (1949).

As might be expected, the principal differences between Eq. (18) and Fjörtoft's stability condition arise at the short wavelength end of the spectrum. Fjörtoft's curve of zero instability places the predominant wave number of the transient disturbances at 12, with the cut-off at about wave number 15. It is also relevant to point out that, because the real part of Fjörtoft's stability discriminant becomes infinite as the wavelength approaches zero, his result implies that the phase speed of the very short waves becomes indefinitely large. According to Eq. (17), however, the phase speed of very short waves approaches a finite limit which is independent of wavelength. The non-dispersive character of the short waves has been discussed by Eady (1951). In short, where the present theory does disagree with Fjörtoft's results, independent evidence tends to support the stability condition for equivalent-baroclinic flow.

In view of the general agreement of these results with Fjörtoft's theory of instability and because the stability condition is strongly confirmed by Charney's data, it appears that the stability characteristics of three-dimensional flow have been successfully incorporated into a two-dimensional baroclinic equivalent of the atmosphere.

6. A METHOD OF NUMERICAL PREDICTION

Eqs. (10) and (14), the fundamental equations of equivalent-baroclinic flow, provide the basis for a new method of numerical prediction. That is to say, since these equations contain no time derivatives of order higher than the first, the future distributions of the temperature and height of an isobaric surface at the level of equivalence are completely determined, in principle, by their initial distributions. The purpose of this section is to indicate how the fundamental equations are actually to be solved, subject to observed initial conditions.

The key to the method for solving the general non-linear equations lies in regarding the instantaneous local-time derivatives (or tendencies) of temperature and contour height as the dependant variables, and in regarding all terms in Eqs. (10) and (14) *that do not involve time derivatives* as known functions of the horizontal space coordinates. With this interpretation, Eqs. (10) and (14) take the form

$$(\nabla^2 - \nu^2)\left(\frac{\partial z}{\partial t}\right)_i = f_i(x, y) \quad . \quad . \quad . \quad . \quad (19)$$

$$(\nabla^2 - \mu^2)\left(\frac{\partial T}{\partial t}\right)_i = g_i(x, y) \quad . \quad . \quad . \quad (20)$$

The functions $f_i(x, y)$ and $g_i(x, y)$ are the sums of all terms in Eqs. (10) and (14), respectively, that *do not* contain time derivatives. As emphasized earlier, f_i and g_i involve the horizontal space derivatives of z and T only. The subscript i refers to the value of a quantity at the ith stage of an iteration process, the subscript ' zero ' referring to conditions at the initial moment.

Both f_0 and g_0 can be calculated from the initial values of z and T. Thus, the instantaneous initial tendencies of z and T can be computed by solving Eqs. (19) and (20). The distributions of z and T at a very short time Δt later than the initial instant are then obtainable by extrapolation from the initial values, according to the computed initial tendencies of z and T. Having regenerated the initial conditions, it is then possible to calculate f_1 and g_1, the values of f and g at a time Δt later than the initial moment. This step completes the first cycle of an iteration process, which can obviously be repeated until the aggregate of short time intervals adds up to the required forecast period.

It remains to show how Eqs. (19) and (20) are to be solved for all functions f and g. For the purpose of this discussion, it is sufficient to point out that both are non-homogeneous equations of the Helmholtz type, a species which appears frequently in mathematical physics and whose properties are known. Equations of this type may be solved equally well by a combination of analytic and numerical methods or by purely numerical techniques, such as the relaxation method. In order to study the general behaviour of solutions, however, it is preferable to display them in closed form. By applying Green's theorem, the solutions of Eqs. (19) and (20) may be given explicitly, in terms of the known functions f and g and certain analytic functions that are independent of the initial data. Simultaneously replacing the local time derivatives by the corresponding ratios of finite differences,

$$z(x, y, t + \Delta t) = z(x, y, t) - \frac{\Delta t}{2\pi}\int\int_S K_0\left(\nu\sqrt{(x-\xi)^2 + (y-\eta)^2}\right) f(\xi, \eta, t)\, d\xi\, dn \quad . \quad (21)$$

$$T(x, y, t + \Delta t) = T(x, y, t) - \frac{\Delta t}{2\pi}\int\int_S K_0\left(\mu\sqrt{(x-\xi)^2 + (y-\eta)^2}\right) g(\xi, \eta, t)\, d\xi\, d\eta \quad . \quad (22)$$

where the Green's function K_0 is the zero-order Bessel function of the second kind with imaginary argument, and ξ and η are dummy variables of integration corresponding

to x and y respectively. The region of integration S covers the entire (x, y) plane. According to its asymptotic expression, $K_0(kr)$ behaves like $r^{-\frac{1}{2}} e^{-kr}$ for large r, so that the integrals above must converge to some finite limit. Moreover, as the distance from the point (x, y) increases, the Green's function decreases so rapidly that contributions from beyond a sufficiently large finite distance are negligible, in the sense that they can be made less than any pre-assigned value. As it turns out, in fact, the radius of the 'effective domain of dependence' is small in comparison with the circumference of the earth, thus justifying the use of plane coordinates.

Eqs. (21) and (22) are the prediction formulae, from which the values of z and T at time $(t + \Delta t)$ can be calculated from their values at time t. Since f and g at time $(t + \Delta t)$ are determined by the values of z and T at that same time, Eqs. (21) and (22) enable us to calculate z and T at time $(t + 2\Delta t)$, and so to repeat this process until the period of the prediction reaches the desired length. Eqs. (21) and (22) demonstrate that it is possible, in principle, to predict the temperature and height of an isobaric surface at the level of equivalence, given only the temperature and height at some arbitrarily chosen initial moment.

With the predicted values of temperature and contour height at the level of equivalence, one can infer something about other meteorological properties of the atmosphere, some of which are directly connected with the more tangible aspects of weather. For example, by making use of the hydrostatic equation, the contours of pressure surfaces above and below the level of equivalence can be extrapolated from the contours at that one level. A more significant fact, perhaps, is that the vertical component of velocity at the level of equivalence is determined by the non-advective changes of temperature. That is, according to the adiabatic hypothesis,

$$w(x, y) = -\frac{\theta}{T}\left(\frac{\partial \theta}{\partial z}\right)^{-1}\left[\overline{\mathbf{V}} \cdot \nabla T - \frac{1}{2\pi}\int\int_S K_0\left(\mu\sqrt{(x-\xi)^2 + (y-\eta)^2}\right) g(\xi, \eta)\, d\xi\, d\eta\right]$$

Thus it is possible to reconstruct the pattern of instantaneous vertical motion from the temperature and contour distributions at the level of equivalence, where the vertical mass transport reaches its maximum. Needless to say, the possibility of forecasting the large-scale vertical motions would lead us closer to quantitative methods for predicting cloud formation and precipitation, or, in other words, to predicting weather.

7. Summary and conclusions

In the first four sections of this paper, the fundamental equations of non-viscous flow have been reformulated in such a way that they apply to a two-dimensional baroclinic equivalent of the true atmosphere, i.e., two-dimensional in the sense that the vertical coordinate and the vertical component of velocity do not appear explicitly in the governing equations. The state of this 'equivalent-baroclinic' atmosphere is to be identified with actual conditions at a definable 'level of equivalence,' and is representative of conditions through a very deep layer of the atmosphere. It has been shown that the level of equivalence, which is the level at which the true wind equals the vertically integrated mean wind, also lies near the levels of maximum vertical mass transport and maximum individual pressure change.

Of the several possible types of solutions of the fundamental equations, those corresponding to sound and gravity waves have been excluded, by introducing the conditions for geostrophic and hydrostatic equilibrium into the vorticity equation. The resulting equation involves only two dependent variables, namely, the temperature and

height of an isobaric surface at the level of equivalence. An independent equation in those same variables has been derived by considering the individual changes in the 'vorticity' of the thermal wind, and by introducing explicitly the condition for adiabatic flow. It is worth emphasizing that neither of those equations contains the vertical component of velocity or the horizontal mass divergence.

The two equations referred to above, Eqs. (10) and (14), form a closed system, and therefore constitute a mathematically complete theory of the development and propagation of large-scale disturbances in equivalent-baroclinic flow. Eq. (10) has the same general form as Rossby's equation of potential vorticity, and is virtually identical with an equation derived by Fjörtoft (1951) under less general conditions. Eq. (14) contains a mechanism of temperature change that has been studied extensively by Sutcliffe (1947). Thus, by dealing with both equations simultaneously, we have reconciled the two major views of the prediction problem to form a single complete theory.

As a simple and rather critical test of the theory, we have derived from Eqs. (10) and (14) the stability criterion for horizontally uniform flow with constant vertical wind shear. This special case is comparable with the three-dimensional baroclinic flow studied by Fjörtoft. The results of this analysis, as summarized in Fig. 2, are in good qualitative agreement with Fjörtoft's theory. It turns out, in fact, that his stability discriminant is a special case (valid for very long wavelengths) of the more general form given in Eq. (17). Where the two theories do disagree, independent evidence appears to support the stability criterion for equivalent-baroclinic flow. It is concluded, therefore, that the stability characteristics of three-dimensional baroclinic flow have been successfully reproduced in the two-dimensional baroclinic equivalent in this special case, and that the general equations of equivalent-baroclinic flow contain a mechanism of development operating under a much wider range of conditions.

Finally, we have proposed an iterative scheme for solving Eqs. (10) and (14), the general non-linear equations of equivalent-baroclinic flow. It is found that the instantaneous tendencies of both temperature and contour height can be expressed as infinite surface integrals, taken over the entire (x, y) plane. In either case, the integrand consists of two distinctly different factors : (a) Green's function, which is independent of the initial data and can be tabulated once and for all, and (b) a quantity that involves only the initial values of temperature and contour height. The analytic expressions for the temperature and height tendencies, as presented in Eqs. (21) and (22), provide the basis for a method of numerical prediction. As a by-product of that method, it is possible to infer the contour patterns above and below the level of equivalence, as well as the pattern of large-scale vertical motion at that level.

Needless to say, the computations required to carry out such an iterative scheme are very expensive in terms of time, money, effort and patience. For this reason, and because the field of numerical prediction is still in the early stages of rapid development, it appears justifiable to report on the theoretical aspects at the present time, even though the evidence for or against the theory is not complete. It is hoped that the theory will be of some interest in itself, and that the reader will reserve final judgment until more direct evidence is available. Numerical solutions of the general non-linear equations will be included in a forthcoming paper.

Acknowledgment

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Theory of airflow over mountains: II – The flow over a ridge

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SUMMARY

Three kinds of flow over unlevel ground are defined according to their scale – aerodynamic, barostromatic, and geostrophic flow – and it is seen that the practical difficulties in the way of obtaining the correct value of the relevant non-dimensional numbers in model experiments are almost prohibitive.

The flow over a ridge with varying degrees of static stability is calculated, and the vertical displacement is seen to depend very much on the details of the airstream. When the flow is oblique to the ridge the air is permanently displaced sideways from its original path; and if the original airstream contains shear, shear in almost any direction may appear over a ridge. This may be displayed in billow-cloud formations.

Diurnal variations in the length of lee waves are due to the development and disappearance of an adiabatic layer at the ground. The flow is calculated in a particular case and the temperatures recorded by radiosonde balloons are shown to contain spurious inversions as the balloon passes through the waves.

The disappearance of the adiabatic layer at the ground makes orographic effects a maximum around sunset on sunny days; and the appearance of a statically neutral layer in middle levels, for instance by condensation, renders steady laminar flow impossible.

1. INTRODUCTION

This paper describes some useful conclusions that can be drawn from perturbation theory about the motion of air over hills. The deduction of the fundamental equations will not be repeated since it has already been given in Scorer (1949); that paper will be referred to as I, and this paper is therefore numbered II.

First of all it is necessary to describe the kind of flow to which the discussion applies. We shall then discuss the main obstacle to laboratory experiments on airflow over hills, namely the difficulty of scaling correctly. Since perturbation theory provides the only mathematical treatment of airflow over hills, we have exploited it as far as seems justified.

List of symbols used

a, b	lengths determining the dimensions of the ridge.
A, B, C, D	arbitrary constants.
f	twice the vertical component of the earth's rotation.
g	gravity.
h, H	depths of layers of air.
k	wave number.
l	defined by $l^2 = g\beta/U^2 - U''/U$, where a prime denotes $\partial/\partial z$.
p	disturbance of pressure.
q	a function approximating to μ.
Q	non-dimensional number defined by Eq. (3).
S	non-dimensional number defined by Eq. (1).
T_0, T	absolute temperature and disturbance thereof.
U, u	horizontal velocity of undisturbed airstream and disturbance thereof.
W	vertical velocity of radiosonde balloon.
w	vertical velocity of the air.
X	horizontal coordinate defined by Eq. (17).
x, z	horizontal and vertical coordinates.
β	stability; βT_0 = adiabatic lapse rate − actual lapse rate.

ζ vertical displacement of the air.
η, ξ horizontal displacements of the air.
ϕ angle between ridge and normal to airstream.
r, θ polar coordinates.
μ defined by Eq. (7), and equal to ik.
τ_0, τ reciprocal of potential temperature, and disturbance thereof.

Suffixes

z denotes value at height z.
a denotes value in lowest (adiabatic) layers.
i denotes value in middle (inferior) layers.
s denotes value in upper (superior) layers.
0 denotes value in undisturbed stream.
1 denotes value at ground.

Second suffix denotes value for a particular wave number.

2. Types of flow and the relevant non-dimensional numbers

(a) *Aerodynamic flow*

Though most of the atmosphere is stably stratified there is generally a statically neutral layer at the ground. If the disturbance to horizontal flow due to a hill or other obstacle does not extend beyond the limits of this adiabatic layer the flow is termed *aerodynamic* because the pressure field is determined in the same way as in ordinary aerodynamics.

If the viscosity is unimportant then the flow may be potential flow, and is analogous to the flow of an incompressible fluid of uniform density. By obtaining the correct Reynolds number when the viscosity is important model experiments may be used to deduce properties of the flow. Examples of the possibilities here are finding the best location for a windmill on a small hill, or a good position for an anemometer over a building.

Irreversibility in the flow is usually due to the production of a wake, which appears always on the lee side of an obstacle. Two kinds of wave can be propagated – sound waves and shock waves, the latter being irreversible.

(b) *Barostromatic flow*

When the hill influences the flow at levels where the air is stably stratified the flow becomes quite different from aerodynamic flow. Because the main reason for this is the stratification of density the flow is called *barostromatic**, though it is possible that in some layers the *dinostromatic** force may be as important. This latter force is produced by a vertical gradient of shear, that is a stratification of the horizontal component of vorticity. In the linearized equations these two forces are represented by the terms $g\,\beta/U^2$ (barostromatic) and U''/U (dinostromatic), and they occur together with k^2 (see Eq. (2) below) where $2\pi/k$ is the wavelength or some other representative length in the problem. Therefore, in order to perform model experiments, the relationship between these terms must be made the same in the model as in the full-scale case. Viscosity has

* These words are derived from βάρος = weight, στρῶμα = layer, δῖνος = rotation, and refer to the influence of the stratification of potential density and of vorticity. For advice on this matter I am indebted to Mr. M. M. McCrum, of Corpus Christi College, Cambridge.

been omitted from the equations so far, so that the relevant non-dimensional number is

$$S = (g\beta - UU'')/k^2 U^2 \quad . \quad . \quad . \quad . \quad (1)$$

In model experiments the adiabatic lapse-rate can be taken as zero and so if the model is 3×10^{-5} times the size of an actual hill and the vertical temperature gradient in the model airstream is as large as 1°C/cm, if the full scale values are $U = 10^3$, $\beta = 2 \times 10^{-7}$, the airspeed in the model is given by (all units CGS), $U = 67$ cm sec^{-1} (ignoring U''). With smaller temperature gradients in the model airstream the airspeed would be correspondingly less, and such slow speeds in an ordinary wind tunnel are difficult to achieve, if the temperature gradient is to be controlled, without serious trouble from boundary layers on the walls of the tunnel. At higher speeds the effect of the static stability is inadequately represented. The significance of S is that, when U'' is ignored, it is the ratio of the potential energy required to lift unit mass to a representative height (determined by k) against the static stability to the kinetic energy of unit mass.

It is scarcely a practical possibility to control the quantity U'' in model experiments in such a way that the number $U''/k^2 U$ takes the same value in the model as in the full scale, and it deserves little attention until methods of controlling β have been evolved.

If in model experiments the boundary layer at the walls occupies a substantial part of the wind tunnel an attempt must be made to obtain a reasonable Reynolds number, and it is likely that if the molecular viscosity is used for the model the Reynolds number obtained will make the experiment correspond to a full-scale case with a Reynolds number derived from an eddy viscosity in the range 10^3 to 10^5, which is reasonable.

In view of these considerations it is plain why model experiments so far carried out on flow over mountains have met with little success. Also it is extremely difficult to obtain anything like a uniform vertical temperature gradient in a wind tunnel.

Irreversibility in barostromatic flow is due to the formation of a train of lee waves.

Shock waves and sonic waves can, of course, occur in a barostromatic airstream but ordinary gravity waves are the important mechanism whereby the effects of non-horizontal motion are transmitted from one part of the fluid to another.

(c) *Geostrophic flow*

When the time taken for air to cross the mountain system is of the order of a day the earth's rotation is important, and because it produces the dominant force the flow is appropriately termed *geostrophic*. Irreversibility of the flow is manifested by the appearance of lee depressions, and in these the motion associated with passage over the mountain is horizontal. Barostromatic and aerodynamic forces are involved but any wave motion is primarily an oscillation to either side of the geostrophic wind. Besides these waves, appropriately called Queney waves, there are also Rossby waves depending on the horizontal variation of the geostrophic force. Fultz (1951) has discussed some of the non-dimensional numbers relevant to geostrophic flow, but since he did not discuss Queney waves, which demand a relationship between the geostrophic and barostromatic forces, it is appropriate to mention them here.

The equation for wave motion on a rotating flat earth is (I, p. 44) for a uniform airstream

$$(1 - f^2/k^2 U^2)\,\zeta'' + (l^2 - k^2)\,\zeta = 0 \quad . \quad . \quad . \quad (2)$$

The term in ζ' has been omitted since it can be shown to be negligible or irrelevant. Its neglect is equivalent to assuming that sonic disturbances, are irrelevant in the meteorological problem.

The non-dimensional number, $Q = f^2/k^2 U^2$ . . . . (3)

must take the same value in model and full scale if the effect of the earth's rotation is to be properly accounted for in relation to the mountains whose size is represented by $2\pi/k$. Furthermore the two terms of Eq. (2) must bear the same relationship in model and full scale, thus obtaining the correct ratio between the static stability and the earth's rotation. This will in fact be ensured if the correct value of S has been obtained.

In aerodynamic flow the vertical and lateral velocities are of equal importance, and gravity plays no part : in geostrophic flow the disturbance from rectilinear flow is mainly horizontal, but in barostromatic flow it is the vertical displacements that are of fundamental significance.

3. The flow over a ridge

(*a*) *The validity of the approximation to be used*

We have previously (Scorer 1951) obtained the equation

$$w'' + (l^2 - k^2)\, w = 0 \quad . \quad . \quad . \quad . \quad (4)$$

which determines the vertical velocity at any level if two boundary conditions are given. The vertical displacement is

$$\zeta = \int \frac{w}{U}\, dx \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and we consider flow over the ridge whose height is given by

$$\zeta_1 = \frac{a^2 b}{b^2 + x^2} = a^2 \int_0^\infty \mathrm{e}^{-kb + ikx}\, dk \quad . \quad . \quad . \quad (6)$$

Assuming that there exists a layer at the top of the atmosphere where l^2 is independent of z then the solution of Eq. (4) corresponding to wave fronts progressing upwards, or to waves of finite kinetic energy if they progress horizontally is*, using Eq. (5)

$$\zeta \propto \mathrm{e}^{-\mu z}/ikU$$

where

$$\mu = +(k^2 - l^2)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

and so since Eq. (6) is the second boundary condition the complete solution is

$$\zeta = \frac{U_1}{U} a^2 \int_0^\infty \mathrm{e}^{-kb + ikx - \mu z}\, dk \quad . \quad . \quad . \quad (8)$$

and our problem is to evaluate this integral.

Lyra, using the Green's function method, found it more convenient to take a rectangular mountain profile but his problem was fundamentally the same. He found that each streamline executes a series of undulations on the lee side of the ridge, the undulations decreasing in intensity downstream, but that the number of observable undulations was greater the greater the altitude of the streamline. Also the observable undulations extended further downstream at greater heights. In this way a wave motion on the lee side was predicted but it was unlike the waves most often observed in that they did not achieve a maximum amplitude at some level and fade away at higher levels. Queney (1948) using the same mountain profile as is taken here, showed that the importance of these Lyra waves depended upon the steepness of the mountain slopes, and that they could in many cases be ignored. We now discuss how this happens.

* Lyra and Queney chose a negative sign for μ when μ is imaginary, but as the controversy is not resolved and does not affect the conclusions of this paper it will not be entered into here.

The function $+(k^2 - l^2)^{\frac{1}{2}}$ may be approximated to by a series of straight lines. For instance we may take in its place the function

$$\left.\begin{aligned} q &= i\,l, && 0 \leqslant k \leqslant l/2 \\ &= i\,(2l - 2k), && l/2 \leqslant k \leqslant l \\ &= (k - l)/(2^{\frac{1}{2}} - 1), && l \leqslant k \leqslant l/(2 - 2^{\frac{1}{2}}) \\ &= k, && l/(2 - 2^{\frac{1}{2}}) \leqslant k \end{aligned}\right\} \quad . \quad . \quad (9)$$

The exponent in the integral in Eq. (8) is now a linear function of k and we readily obtain

$$U\zeta/U_1 = a^2\,(1 - e^{-\frac{1}{2}lb + \frac{1}{2}ilx})\,e^{-ilx}/(b - ix) + \text{terms in } z^{-1}, e^{-lz}, e^{ilz} \quad . \quad (10)$$

For large values of z the first term dominates, but at lower levels the later terms also contribute substantially and a wavy streamline is obtained, while at the ground they interfere to give no waves. A closer approximation to the correct answer could be obtained by taking a larger number of straight line sections for q but still it would be the early terms that would dominate at higher levels. The later terms interfere at low levels and are small at high levels. All terms decrease as x increases. The main, though not the only, contribution to Lyra's waves comes from the first term where we have approximated to $+(k^2 - l^2)^{\frac{1}{2}}$ near $k = 0$ by putting $k = 0$. The approximation used in I involves putting $k = 0$ in this part of the exponent for the whole range of integration; and assumes that, when this approximation is bad, that is for large values of k, the integrand is small on account of the factor e^{-kb}. Thus for instance, if $b = 3 \times 10^5$ and $l^2 = 2 \times 10^{-10}$, when $k^2 = \frac{1}{4}\,l^2$ then $e^{-kb} \simeq \frac{1}{7}$ and since the integrand is highly oscillatory the approximation is good. In the case of a ridge of rectangular profile the solution is obtained from the solution for $b = 0$ and it is therefore seen that Lyra could not make the approximation (or its equivalent in the Green's function method). Queney (1948) shows diagrams in which the approximation is effectively made for cases where b is large enough, and we shall make it here, and obtain

$$U\zeta/U_1 = a^2\,e^{-ilz}/(b - ix) = a^2\,(b\cos lz + x\sin lz)/(b^2 + x^2) \quad . \quad . \quad (11)$$

Since b is the horizontal distance from the summit of the point where the ridge height is half the summit height it is a measure of the steepness of the slope. When the slope is steep the approximations involved in linearizing the equations are invalid and it will appear later that the flow is likely to separate from the surface and invalidate the ground boundary condition.

(b) *The effect of varying degrees of stability in a uniform stream*

First we note that the approximation just made is invalid if $l^2 = 0$, i.e., the aerodynamic case. The flow pattern in this case is therefore not the limit of the barostromatic case as $l^2 \to 0$. In the atmosphere the lapse rate is either adiabatic or quite substantially different from it and so we are not concerned with the cases where l^2 is an order of magnitude smaller than the average.

When $l^2 = 0$ we find from Eq. (8) that, with $U = U_1$

$$\zeta = a^2 \int_0^\infty e^{-k(b+z) + ikx}\,dk = a^2\,\frac{b + z}{(b + z)^2 + x^2} \quad . \quad . \quad . \quad (12)$$

This is illustrated in Fig. 1.

If we now take $l^2 = 2{\cdot}25 \times 10^{-10}$ and 10^{-10} we obtain the cases illustrated in Figs. 2, 3 respectively. The significant fact is that stable stratification causes the effect of the hill to extend through the whole depth of the atmosphere; and, more important at present,

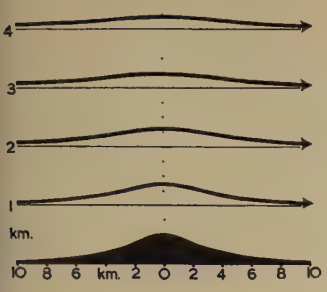

Figure 1. Aerodynamic flow of a uniform airstream over a ridge (no static stability). The amplitude of the disturbance decreases with height.

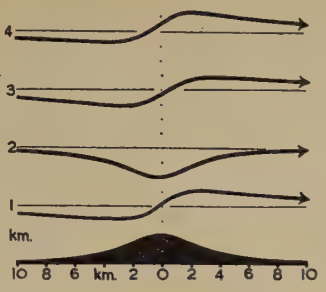

Figure 2. Barostromatic flow of a uniform air stream over a ridge $l = 1{\cdot}5\ \text{km}^{-1}$, e.g., with a wet adiabatic lapse rate and a speed of 13 m sec^{-1} or an isothermal lapse-rate and a speed of 8 m sec^{-1}.

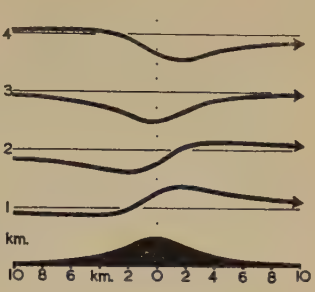

Figure 3. As Fig. 2 but with $l = 1\ \text{km}^{-1}$. With the same lapse rates as in Fig. 2 the wind is 1½ times as strong. In Figs. 2, 3 the amplitude of the disturbance does not decrease with height.

the form of the disturbance at any level depends completely on the value of l^2. In particular the assumption that there is a level, at and above which there is no disturbance and below which the disturbance varies linearly, is seen to be quite without justification. The fact that this solution asserts that the whole depth of the atmosphere is disturbed will be commented upon later, and is no objection to the conclusions arrived at here.

(c) *Oblique flow over a ridge*

The equation of continuity is (effectively, with an infinite velocity of sound)

$$\partial u/\partial x = -\partial w/\partial z \quad . \quad . \quad . \quad . \quad (13)$$

and so if ξ is the displacement of a particle in the direction of the undisturbed motion from the position it would have in undisturbed motion, then by Eqs. (5) and (13)

$$\xi = \int \frac{u}{U}\,dx = -\int \frac{\partial \zeta}{\partial z}\,dx \quad . \quad . \quad . \quad . \quad (14)$$

and for the case under discussion

$$\xi = ia^2 l\, e^{-ilz} \int dx \int_0^\infty dk\, e^{-kb+ikx}$$

$$= \tfrac{1}{2} a^2 l\, e^{-ilz} \left\{ \log \frac{b^2}{b^2+x^2} + 2i \tan^{-1} \frac{x}{b} \right\} \quad . \quad . \quad (15)$$

if we suppose that $\xi = 0$ over the crest of the ridge, for reference.

For a ridge inclined at angle ϕ to the wind direction, taking (r, θ) as coordinates, and (ξ, η) as the displacements in the direction of the wind and perpendicular to it to the left, we find that, since l^{-1} is proportional to the component of the wind across the ridge, and the component along the ridge behaves independently,

$$\xi, \eta = \tfrac{1}{2} a^2 l \sec\phi\, e^{-ilz\sec\phi} \left\{ \log \frac{b^2}{b^2+X^2} + 2i \tan^{-1} \frac{X}{b} \right\} (\cos\phi, -\sin\phi) \,. \quad (16)$$

Where

$$X = r\cos(\theta+\phi) \quad . \quad . \quad . \quad . \quad . \quad (17)$$

We can now illustrate the flow of the airstream in Fig. 2 over the ridge if it is inclined at angle ϕ to the stream. Figs. 4, 5, 6 are for $\phi = 45°$, and it is seen that besides vertical undulations considerable yaw is produced.

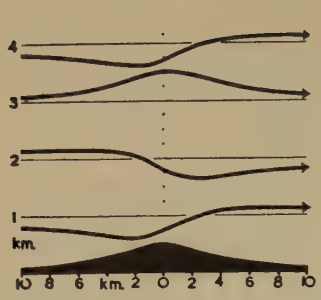

Figure 4. Barostromatic flow ($l = 1{\cdot}5\ \text{km}^{-1}$) inclined at 45° to the ridge. The section is in the direction of the air stream, which is the same as in Fig. 2.

Figure 5. The projection of the streamlines of Fig. 4 on to a horizontal plane. The numbers against the streamlines indicate their height above the ground.

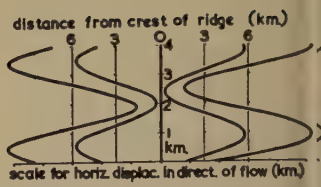

Figure 6. The projection of a 'marked' column of air (assumed vertical over the crest of the ridge) on to a vertical plane lying across the ridge, for the case of Fig. 4.

Billow clouds are thought (Scorer 1951a) always to lie across the shear vector, and if they lie in several different directions we do not conclude that some are longitudinal and some transverse, nor that at their level there is general shear in the airstream, nor that their various directions indicate that they lie at different levels : for the component of shear in the direction of the undisturbed stream, when flow is perpendicularly across the ridge is,

$$\frac{\partial u}{\partial z} = \frac{\partial}{\partial z}\left(-\int \frac{\partial w}{\partial z}\,dx\right) \qquad \text{by Eq. (13)}$$

$$= -\frac{\partial}{\partial z}\int \frac{\partial}{\partial z}\left(U\frac{\partial \zeta}{\partial x}\right)dx$$

$$= -U\frac{\partial^2 \zeta}{\partial z^2} = U l^2 \zeta$$

$$= U a^2 l\,(b\cos lz + x \sin lz)/(b^2 + x^2) \qquad . \qquad . \qquad (18)$$

If the wind is oblique to the ridge crest the component parallel to the ridge remains undisturbed and so the shear is always across it and equal to

$$U a^2 l^2 \sec^2\phi\,\{b\cos(lz\sec\phi) + X\sin(lz\sec\phi)\}/(b^2 + X^2) \qquad . \qquad (19)$$

where X is the horizontal distance from the ridge crest. If there already exists a component of shear along the ridge in the undisturbed wind field the total shear can assume almost any direction as the air proceeds across the ridge. If the ridge is not two-dimensional then evidently the shear, in an airstream that has no shear in the undisturbed state, will vary according to the direction of the slope of the ground.

(*d*) *The validity of the perturbation technique*

In Fig. 5 we see that at $z = 2$ km the component of the wind across the ridge over the crest of the ridge vanishes. Thus the disturbance of the horizontal wind component is as large as the component itself, which is in violation of the assumption on the basis of which the equations leading to Eq. (4) were linearized. Thus second-order terms in the equations of motion have been neglected unjustifiably. However, we can first of all regard the solution as valid for a lower ridge (smaller value of a^2), all the disturbances being exaggerated in the diagrams. Alternatively we can say that the answer is qualitatively correct and the extra terms will only have a modifying influence, and in this case we must

judge the results by their agreement with observation : in any case, since there is at present no better general mathematical treatment we must exploit those solutions which can be obtained. For instance we may say that the boundary conditions are correct, so that if a method can be found for solving the full equations numerically over limited regions and the linearized equations are valid elsewhere then a complete solution can be obtained. A method to do this under certain additional restrictions has been evolved by Davies (1952). We shall recognize these difficulties in a later discussion of the other forms which the flow over a ridge can take (or see Scorer 1952).

4. Lee waves

(a) *Adiabatic layers at the ground*

It has been shown (Scorer 1951b) that in order that a train of lee waves shall occur over the level ground behind mountains the quantity l^2 must decrease upwards. This is simply because while Eq. (4) is satisfied w vanishes at great height and at the ground and so w'' must change sign : therefore since w is continuous and cannot change sign at a level above the highest zero of w'' l^2 exceeds k^2 at some levels and is exceeded by k^2 in the highest levels. Nevertheless it is possible for lee waves to occur when l^2 is zero near the ground. For if we suppose that there are three layers denoted in order downwards by suffixes s, i, a, since in the top layer where w must tend to zero upwards

$$w_s \propto \mathrm{e}^{-\mu_s z} \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

In the middle layer

$$w_i \propto \cosh \mu_i z + \frac{\mu_s}{\mu_i} \sinh \mu_i z \quad . \quad . \quad . \quad . \quad (21)$$

for then the conditions that w and w' are continuous at the interface, at $z = 0$, are satisfied. If we suppose the adiabatic layer to lie between the ground, at $z = -H$, and the level $z = -h$, and impose the boundary conditions

$$w_a(-H) = 0,\ w_a(-h) = w_i(-h),\ w_a'(-h) = w_i'(-h) \quad . \quad (22)$$

since w_a is of the form

$$w_a = A\,\mathrm{e}^{kz} + B\,\mathrm{e}^{-kz} \quad . \quad . \quad . \quad . \quad . \quad (23)$$

we obtain an equation determining k which is

$$\mu_i \tanh k(h - H) = k \frac{\mu_i + \mu_s \tanh \mu_i h}{\mu_i \tanh \mu_i h + \mu_s} \quad . \quad . \quad . \quad (24)$$

In order that $w_s \to 0$ as $z \to \infty$, μ_s is real and $l_s^2 < k^2$. If we suppose μ_i to be real then the second member of Eq. (24) is positive while the first member is negative because $H > h$. μ_i is therefore imaginary and $l_i^2 > k^2 > l_s^2$ and the condition is imposed that if a real value of k satisfies Eq. (24) l^2 must decrease upwards. The presence of a statically neutral layer at the ground does not alter the essential physics of the problem.

If μ_i is imaginary and equal to $i\nu_i$ then

$$\mu_s = \frac{k + \nu_i \tan \nu_i h \tanh k(h - H)}{\tanh k(h - H) - \dfrac{1}{\nu_i} \tan \nu_i h} \quad . \quad . \quad . \quad (25)$$

Now $\mu_s > 0$, which is impossible if $\tan \nu_i h > 0$ and Eq. (25) holds, and so an absolute lower limit of $\pi/2\nu_i$ is placed upon h since $\nu_i > 0$. In fact, for a given value of k, h must exceed this value considerably. The extreme case is $\mu_s = 0$, i.e., $k = l_s$ and substitution

of this value of k in Eq. (25) gives a value for h, assuming H and l_i to be given. If the depth of the lower stable layer does not exceed this value lee waves cannot possibly occur. If they exist and the convective layer gradually thickens during the day then k will gradually decrease (the wavelength will increase) and the waves will disappear when $k = l_s$. Roper (1952) has observed the reverse of this process in the evening, with the waves appearing and shortening in length, and Manley (1945) describes how in the evening ' helm bars ' move towards the hill; while on one occasion he observed the helm bar to move away from the Crossfell ridge (the wavelength increased) after sunrise and finally diappear.

Many attempts have been made to predict the wavelengths from ordinary routine wind and temperature observations but this is not a practical possibility because the upper boundary condition cannot conveniently be expressed if the wind and temperature profiles are only known as tables of values. It is more sensible to regard the wavelength as about the most easily observable of all quantities and make deductions about wind and temperature. For instance Manley quotes the proverb ' the bar never crosses Eden,' that is to say the wavelength never exceeds about 16 km. Since $k^2 > l_s^2$ the proverb asserts, in principle, that $l_s > 2\pi/16$ km^{-1}. If we write $l^2 = g\,\beta/U^2$ then with a wind of 15 m sec^{-1} the stability is always in excess of the value $\beta = 6 \times 10^{-8}$, i.e., the lapse rate in the higher layers round about 3 km does not exceed 8°C/km, which is a reasonable conclusion.

(*b*) *Calculation of streamlines in a particular case*

We now describe a possible procedure for calculating the streamlines in a case designed to illustrate these properties. We wish to retain both realism and mathematical simplicity, and take an airstream in which there are three layers denoted by suffixes s, i, a, the lowest being an adiabatic layer. If the suffix is z it refers to height z, without specifying in which layer this is; if the suffix is 1 it refers to the ground. A second suffix is sometimes used to denote values for a particular wave number, and is either zero or k (the lee-wave number denoted by k_0 in I).

As shown in I (where $f_z(k)$ was used to denote $\zeta_{z,k}$) the vertical displacement is

$$\left.\begin{aligned} \zeta &= a^2 \frac{\zeta_{z,0}}{\zeta_{1,0}} \frac{b + ix}{b^2 + x^2} + 2\,i\,a^2\,\pi \frac{\zeta_{z,k}}{\delta\zeta_{1,k}/\delta k} e^{-kb + ikx}, && x > 0 \\ &= a^2 \frac{\zeta_{z,0}}{\zeta_{1,0}} \frac{b + ix}{b^2 + x^2}, && x < 0 \end{aligned}\right\} \quad . \quad (26)$$

where $\zeta_{z,0}$ satisfies Eqs. (4), (6), with $k = 0$.

In the numerical work it is convenient to use the following units : length = 1 km, time = 100 sec, velocity = 10 m sec^{-1}; then in the case shown for illustration, in Fig. 7, we put

$$k = 1, \quad l_s^2 = 0{\cdot}75, \quad l_a^2 = 0. \quad . \quad . \quad . \quad . \quad . \quad (27)$$

$$\left.\begin{aligned} w_{a,k} &= \sinh z, && 0 \leqslant z \leqslant 0{\cdot}5 \\ w_{i,k} &= -0{\cdot}4193\,z^2 + 1{\cdot}5469\,z - 0{\cdot}1475, && 0{\cdot}5 \leqslant z \leqslant 2{\cdot}5 \\ w_{s,k} &= 3{\cdot}836\,e^{-3/2}, && 2{\cdot}5 \leqslant z \end{aligned}\right\} \quad . \quad (28)$$

$$\left.\begin{aligned} U_a &= 1 + \tfrac{1}{4}z, && 0 \leqslant z \leqslant 0{\cdot}5 \\ U_i &= -{\cdot}09375\,z^2 + {\cdot}71875\,z + {\cdot}7890625, && 0{\cdot}5 \leqslant z \leqslant 2{\cdot}5 \\ U_s &= 1{\cdot}375 + \tfrac{1}{4}z, && 2{\cdot}5 \leqslant z \end{aligned}\right\} \quad . \quad (29)$$

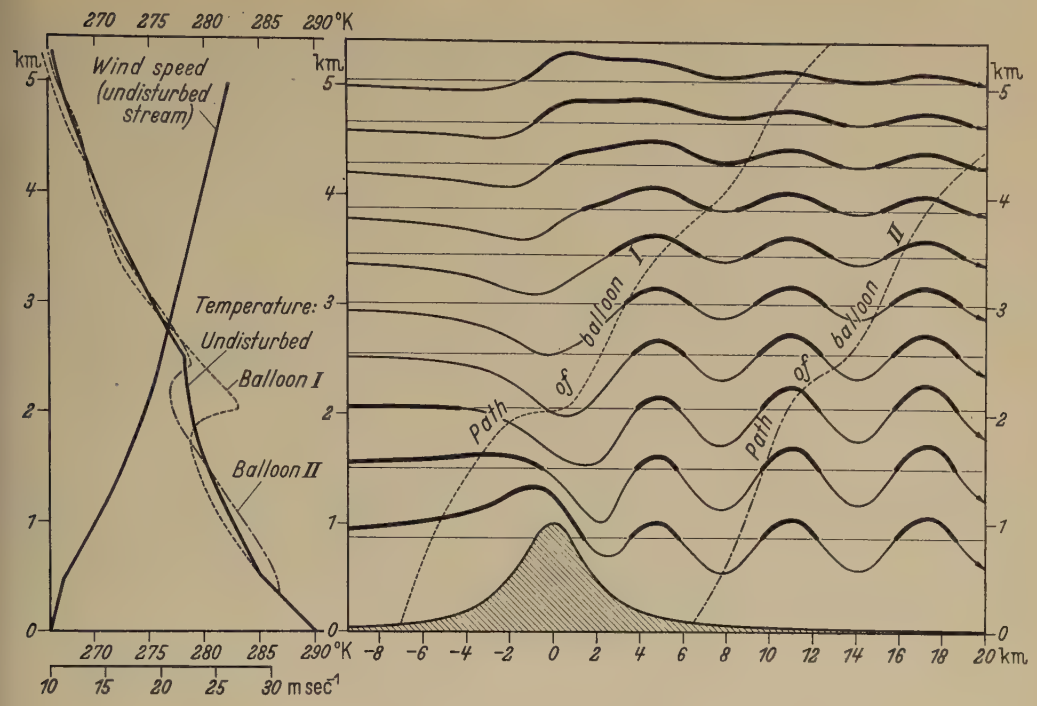
270 275 280 285 290°K
km
Wind speed (undisturbed stream)
Temperature: Undisturbed
Balloon I
Balloon II
Balloon I
Balloon II
Path of balloon
Path of balloon
-8 -6 -4 -2 0 2 4 6 8 10 12 14 16 18 20 km
10 15 20 25 30 m sec⁻¹

Figure 7. Airflow over a ridge under conditions favourable for the occurrence of a train of lee waves. Velocity and temperature profiles of the undisturbed air stream are shown on the left. The lapse rate is adiabatic in the lowest 500 m. Possible tracks of radiosonde balloons with a rate of ascent of 5 m sec^{-1} are shown together with the temperature profiles they would measure.

These values have been chosen in such a way that U is a continuous function of z, so that at the interfaces $w_{z,k}$ and $w_{z,k}'$ are continuous. l_i is now defined as a function of z by

$$w_{i,k}'' + (l_i^2 - k^2)\, w_{i,k} = 0 \qquad . \qquad . \qquad . \qquad . \qquad (30)$$

and then $w_{i,0}$ is defined by

$$w_{i,0}'' + l_i^2\, w_{i,0} = 0 \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (31)$$

which must be integrated numerically using the conditions that $w_{i,0}$ and $w_{i,0}'$ are continuous at the interfaces, and in the upper layer

$$w_{s,0} \propto \exp(-i\, l_s\, z) \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (32)$$

In the adiabatic layer $w_{a,0}'' = 0$.

To evaluate Eq. (26) we note that

$$\frac{\zeta_{z,0}}{\zeta_{1,0}} = \frac{U_1}{U_z}\frac{w_{z,0}}{w_{1,0}}, \quad \frac{\zeta_{z,k}}{\partial\zeta_{1,k}/\partial k} = \frac{U_1\, w_{z,k}}{U_z\, \partial w_{1,k}/\partial k} \qquad . \qquad . \qquad . \qquad (33)$$

To calculate $w_{z,k}/(\partial w_{1,k}/\partial k)$ we denote an infinitesimal increment in k by κ, then $w_{z,k}$ is known, and $w_{z,k+\kappa}$ satisfies

$$w_{z,k+\kappa}'' + (l^2 - (k+\kappa)^2)\, w_{z,k+\kappa} = 0 \qquad . \qquad . \qquad (34)$$

so that if we put $w_{z,k+\kappa} = w_{z,k} + \kappa\,\psi$ (to first order in κ) . . . (35)

we find $\psi_i'' = (k^2 - l_i^2)\,\psi_i + 2\,k\, w_{i,k}$ . . . (36)

which must therefore be integrated from the upper boundary condition down to the ground. The value of ψ at the starting point of the integration is irrelevant because the complementary function of the equation being integrated vanishes at the ground, k being the lee-wave number. In the adiabatic layer, with $k = 1$,

$$\psi_a'' = \psi_a + 2 \sinh z$$

and so

$$\psi_a = A \cosh z + B \sinh z + z \cosh z$$

and

$$\partial w_{1,k}/\partial k = (w_{1,k+\kappa} - w_{1,k})/\kappa = \psi_1 = A \quad . \quad . \quad (37)$$

and A is found by making ψ and ψ' continuous at $z = 0{\cdot}5$. It is found that $A = 4{\cdot}58$.

The temperature profile of the undisturbed stream is defined by the definition of β,

$$\beta = (10 - \partial T_0/\partial z)/T_0 \quad . \quad . \quad . \quad . \quad . \quad (38)$$

and $\beta_a = 0$, $\beta_i = (l_i^2 U_i^2 - U_i'' U_i)/g$, with a similar formula in the top layer. In this example the formula for U_s in Eq. (29) is not important; it is simply chosen to be realistic and simple. Any other formula could be chosen, and the corresponding temperature profile determined by the condition $l_s^2 = 0{\cdot}75$. In the case chosen U' is a continuous function of z at $z = 2{\cdot}5$.

Now

$$w = U \frac{\partial \zeta}{\partial x}$$

$$u = -\int w' \, dx = (U\zeta)' \quad . \quad . \quad . \quad . \quad (39)$$

Discontinuities of horizontal velocity will therefore arise at discontinuities of U'. If there are any turbulent stresses they are likely to be discontinuous at discontinuities of lapse rate, at the top of the convection layer, for instance, where there is therefore likely to be a discontinuity of U'. Discontinuities of U' are not readily detectable but discontinuities of lapse rate of temperature are, and when they are observed it is reasonable to deduce that they will also be levels of large shear over small regions in hilly districts.

Discontinuities of temperature will also produce local shear layers but these are to be considered less important than discontinuities of the shear U' or of lapse rate. It has been assumed throughout this work that the temperature is a continuous function of z and that inversions are merely layers of great static stability.

The discontinuity of horizontal velocity is

$$\Delta u = U' \Delta \zeta + \zeta \Delta U'$$

$$= \zeta \Delta U' \quad \text{in our case.} \quad . \quad . \quad . \quad . \quad (40)$$

U' is discontinuous at $z = 0{\cdot}5$ in the example to illustrate the point. At other levels U' is continuous and is supposed to be the thermal wind while in the adiabatic layer U' is supposed to be reduced by the convection. If convection ceases it will be some time before the original (thermal wind) velocity profile is restored and so in the evening these shear layers will be observed in hilly country (e.g., Roper 1952, Scorer and Ludlam 1952).

Undulating motion at the top or bottom of cloud layers will produce discontinuities of temperature, and as a second-order effect, discontinuities of horizontal velocity because these are then discontinuities in w', but we will not discuss this further here.

(c) *The effect of waves on radiosonde balloons*

If the rate of ascent of a balloon is W, then the direction of its path makes an angle with the horizontal whose tangent is $(W + w)/(U + u)$ and so the path may be drawn. The temperature is determined by drawing the dry adiabatic through the point on the undisturbed temperature curve at the level from which the air came to the new level. Wave troughs are warmer than the wave crests. In Fig. 7 the tracks of balloons with rates of ascent of 5 m sec^{-1} from two release points are shown, and the temperatures which they would record are shown in dotted lines against the temperature of the undisturbed airstream. One hesitates to contemplate the meaning of soundings in hilly country in strong winds!

The disturbance of pressure is

$$p = \rho_0 (uU + U' \zeta) \quad . \quad . \quad . \quad . \quad (41)$$

the greatest value of which is of the order of a millibar or two and so if the temperatures are plotted as functions of pressure (e.g., on a $T - \phi$ gram instead of a $T - z$ gram) no substantial difference from the result shown in Fig. 7 will arise.

5. Effect of vertical variations of l^2 on the amplitude of the disturbance at high levels

(a) *Gradual variations*

We are interested in the amplitude of the first term on the right-hand side of Eq. (26) and assuming the velocity profile to be given we examine the effect of variations of lapse rate. It is necessary therefore, because of Eq. (33), to estimate $w_{z,0}$. Assuming that $l_s^2 > 0$ we have in the uppermost layer

$$w_{s,0} = (A_s + i\,B_s) \cos l_s z + (C_s + i\,D_s) \sin l_s z \quad . \quad . \quad (42)$$

while in the layer below

$$w_{i,0} = (A_i + i\,B_i) \cos l_i z + (C_i + i\,D_i) \sin l_i z \quad . \quad . \quad (43)$$

w and w' are continuous, so putting the interface at $z = 0$ we have

$$A_s + i\,B_s = A_i + i\,B_i\,; \quad l_s (C_s + i\,D_s) = l_i (C_i + i\,D_i) \quad . \quad . \quad (44)$$

The part represented by the cosine is therefore unaltered at the interface but that represented by the sine is less in the upper layer by the factor l_i/l_s. The vertical velocity increases or decreases upwards according as l^2 decreases or increases upwards. When conditions are most favourable for lee waves (l^2 decreases upwards) the part of the disturbance other than the lee waves takes its greatest value.

This conclusion is arrived at on the basis of Eq. (4), by generalizing from a discontinuity to a series of discontinuities, and then to a continuous variation of l^2. It is a legitimate procedure in this case though not (Scorer 1951c) in all cases.

(b) *The effect of an adiabatic layer near the ground*

If we ignore the dinostromatic forces in the adiabatic layer then $w_{a,0}'' = 0$ and we can put, with the ground at $z = 0$

$$w_{a,0} = 1 + (C_a + i\,D_a) z \quad . \quad . \quad . \quad . \quad . \quad (45)$$

and in the layer above

$$w_{i,0} = (A_i + i\,B_i)(\cos l_i z - i \sin l_i z) + (C_i + i\,D_i)(\cos l_i z + i \sin l_i z). \quad (46)$$

Since solutions can be superposed we can treat the two parts of $w_{i,0}$ separately. If the interface is at $z = h$ we apply the condition that w and w' are continuous and arrive at the following, when $C_i = D_i = 0$,

$$A_i^2 + B_i^2 = 1 - h^2/(h^2 + 1/l_i^2) < 1 \quad . \quad . \quad . \quad (47)$$

$C_i^2 + D_i^2$ takes this same value when $A_i = B_i = 0$.
Also

$$(1 + C_a z)^2 + D_a^2 z^2 = 1 - \frac{2hz(h^2 + 1/l_i^2) - z^2(h^2 + z^2)}{(h^2 + 1/l_i^2)^2} < 1 \text{ when } z < h.$$

Thus the greatest amplitude of the disturbance is less than at the ground, both in the adiabatic layer and in the layers above. It is also less than if the lowest layer were stably stratified. We would thus expect the vertical displacement to be less over hills by day, and to increase at night, for this cause alone; but if simultaneously there are changes in the value of U_1, the surface wind, Eq. (33) shows that the effect might be offset by the nocturnal decrease in surface wind. If however, the decrease in surface wind occurred at a later hour than the stabilisation of the lowest layers after a day of convection there would be a short period of maximum orographic influence around sunset. This has been observed by Ludlam (1952).

(c) *An adiabatic layer in middle levels*

Finally we come to a most remarkable result. We suppose that a steady laminar wavy motion of the kind so far discussed occurs when there is an adiabatic layer in middle levels. If again the dinostromatic forces are negligible then

$$w_{a,0} = A + i\,B + (C + i\,D)z \quad . \quad . \quad . \quad (48)$$

At the upper and lower boundaries of this adiabatic layer the ratio $w_{a,0}/w'_{a,0}$ is determined by conditions in the layers above and below, i.e., by some boundary conditions determining the motion. Thus if the boundaries of the layer are at $z = 0$, h we have

$(A + i\,B)/(C + i\,D) = L$ determined by the lower boundary condition

$(A + i\,B + Ch + i\,Dh)/(C + i\,D) = M$ determined by the upper boundary condition

or

$$L = M - h \quad . \quad . \quad . \quad . \quad . \quad (49)$$

which gives us a relationship between the two necessarily independent boundary conditions: this kind of motion is therefore impossible. Thus an airstream with an adiabatic layer in middle levels cannot pass over any hill without a disruption of steady laminar flow. It is beyond the scope of this paper to discuss what the flow is like: it will suffice to remark that the adiabatic layer can easily be realized if a cloud layer forms in air whose lapse rate is the wet adiabatic.

This last result does not apply to the lee wave term in Eq. (26) because Eq. (4) only reduces to $w'' = 0$ in a statically neutral layer when $k = 0$. When $l \neq 0$ in the middle layer the boundary conditions determine a relationship between the coefficients in the expression for $w_{z,0}$. Only when this expression is a linear function of z does this property arise. Strictly, the layer need not be adiabatic, it is only required that $g\beta/U^2 - U''/U = 0$ but an adiabatic layer with a linear velocity profile is the likeliest case.

6. Conclusion

The results obtained here for a ridge may mostly be applied to isolated (three-dimensional) hills which can be synthesized by integrating the effect of ridges all lying through the hill top. The discussion has been restricted to steady two-dimensional laminar flow, but the extension to three dimensions and to unsteady and non-laminar flows must eventually be discussed. In these fields the mathematical theory is likely to be largely intractable and observational work to be of greater relative importance, and here the work of Förchtgott is pre-eminent. Some of his early results have been described by Scorer (1952). They have been concerned mainly with the airflow over the long ridges in Bohemia and Silesia.

Further understanding of how air flows over mountains is required before such effects as orographic rainfall can be properly explained.

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On the velocity, temperature, and humidity profiles in the lower atmosphere†

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Summary

On the basis of the assumption that the eddy viscosity, eddy conductivity, and eddy diffusivity are determined by the first and second differential coefficients of the mean velocity U, mean potential temperature Θ, and mean humidity χ (specific humidity, mixing ratio, or water-vapour pressure) the following formulae are obtained for the lower atmosphere over level ground (valid up to a height of about 25 m),

$$\frac{U}{U_*} = \frac{1}{k}\log\frac{z-d}{a}, \quad \frac{\Theta-\Theta_0}{\Theta_*} = \frac{1}{l}\log\frac{z-d}{b}, \quad \frac{\chi-\chi_0}{\chi_*} = \frac{1}{m}\log\frac{z-d}{c}$$

Here Θ_0 and χ_0 denote potential temperature and humidity at the ground. U_* is the friction velocity and Θ_* and χ_* denote corresponding quantities for temperature and humidity.

The above logarithmic formulae are in good agreement with Best's measurements when the parameters d, a, b, k, l are chosen to give the best fit with his observed velocity and temperature profiles which are grouped according to stratification and mean velocity.

The zero-plane displacement d is inferred to be the same for all profiles. Measurements by Sverdrup indicate that this is true for low temperatures whereas measurements by Rider and Robinson indicate that it is not always true for higher temperatures.

Theoretical reasoning leads to the conclusion that d should increase with thermal instability and this is verified by Best's data. It is also inferred that k, l, and m should increase with instability, whereas a is found to decrease with instability.

The constant-transfer layer of the lower atmosphere

It has been shown by Ertel (1933) and Calder (1939) that under steady conditions the shearing stress can be considered constant up to a height of about 25 m over level ground. Further, if radiation and condensation processes can be neglected the vertical transfer of heat and water vapour must also be constant under steady conditions. In the following we shall refer to the layer throughout which these conditions are satisfied as the constant-transfer layer.

1. The velocity profile in the case of neutral equilibrium

In a note published in this *Journal* (Björgum 1952) I have shown that, in the case of neutral equilibrium and smooth ground (e.g., ice), the logarithmic velocity profile represented by

$$\frac{U}{U_*} = \frac{1}{k}\log\frac{z}{a_s} = \frac{1}{k}\log\frac{U_* z/\nu}{U_* a_s/\nu} \quad . \quad . \quad . \quad (1.1)$$

is uniquely determined in the fully turbulent region of the constant-transfer layer. Here, U = mean velocity, U_* = friction velocity, k = Kármán's constant, z = height above ground, ν = kinematic viscosity. Further $U_* a_s/\nu$ is a universal constant approximately equal to 1/9.

† Most of the theoretical part of this paper was presented to the I.A.M. of the U.G.G.I. General Assembly at Brussels Aug.-Sept. 1951.

It is well known that the logarithmic velocity profile can also be derived in the following way :

Let us put

$$\sigma = K\frac{dU}{dz} \quad . \quad . \quad . \quad . \quad . \quad (1.2)$$

where K denotes eddy viscosity and σ is the shearing stress referred to unit mass. We have here neglected molecular viscosity as we shall only be concerned with the fully turbulent region.

If we now assume that K is determined by dU/dz and d^2U/dz^2, dimensional analysis leads to von Kármán's formula†

$$K = k^2\frac{\left(\frac{dU}{dz}\right)^2\left|\frac{dU}{dz}\right|}{\left(\frac{d^2U}{dz^2}\right)^2} \quad . \quad . \quad . \quad (1.3)$$

where k again denotes Kármán's constant. Introducing this expression for K into Eq. (1.2) we obtain ($dU/dz > 0$),

$$\sigma = k^2\frac{\left(\frac{dU}{dz}\right)^4}{\left(\frac{d^2U}{dz^2}\right)^2} \quad . \quad . \quad . \quad . \quad (1.4)$$

and it is easily verified that this equation is satisfied by Eq. (1.1) with $\sigma = U_*^2$.

However, Eq. (1.4) has also the more general solution

$$\frac{U}{U_*} = \frac{1}{k}\log\frac{z-d}{a} \quad . \quad . \quad . \quad . \quad (1.5)$$

where d and a denote the empirical roughness parameters introduced by A. C. Best (1935, p. 37) and by W. Paeschke (1937, p. 177). In order that Eq. (1.5) shall reduce to Eq. (1.1) in the case of a smooth surface we may put

$$a = a_s + a_r \quad . \quad . \quad . \quad . \quad (1.6)$$

where a_r depends on the roughness of the ground.

Best and Paeschke found that under neutral conditions d approximately equals the average height h of the roughness elements of the ground. Further Paeschke found that $a \simeq h/7{\cdot}35$, a result which is not in contradiction to Eq. (1.6) as a_s could be neglected in the cases considered by Paeschke (cf., also the table on p. 103 of Sutton, 1949).

According to Sutton (1949, p. 57), Deacon has also found Eq. (1.5) to be in good agreement with measurements, provided that both a and d were chosen independently to give the best fit. Further Roll (1948), Neumann (1949) and Pasquill (1950a and b) have arrived at the same result, such that Eq. (1.5) can be said to be well established under neutral conditions.

† J. van Isacker (1951, p. 4) has criticized the use of formulae of this type as they are not invariant. However, the generalized formula

$$K = k^2\frac{|\nabla\times\mathbf{V}|^3}{(\nabla^2\mathbf{V})^2}$$

where $\mathbf{V}$ denotes mean velocity is seen to be invariant as $\nabla\times\mathbf{V}$ and $\nabla^2\mathbf{V}$ both are so.

Let us next consider the effect of the roughness parameters. For this purpose write Eq. (1.5) in the form

$$U + \frac{U_*}{k} \log a = \frac{U_*}{k} \log (z - d) \quad . \quad . \quad . \quad (1.7)$$

If we here introduce new coordinates z' and U' defined by

$$z - d = z', \quad U + \frac{U_*}{k} \log a = U' + \frac{U_*}{k} \log a_s \quad . \quad . \quad (1.8)$$

Eq. (1.7) reduces to

$$U' = \frac{U_*}{k} \log \frac{z'}{a_s} \quad . \quad . \quad . \quad . \quad . \quad (1.9)$$

It is seen that this equation is of the same form as (1.1). Thus in a U, z diagram the velocity profile for rough ground can be obtained from that for smooth ground by a translation of the latter a distance d in the z direction and a distance ΔU in the negative U direction where

$$\Delta U = U' - U = \frac{U_*}{k} \log \frac{a}{a_s} = \frac{U_*}{k} \log \left(1 + \frac{a_r}{a_s}\right) \geqslant 0 \quad . \quad (1.10)$$

This process is illustrated in Fig. 1.

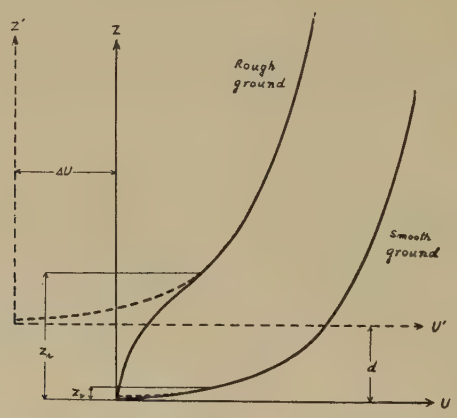

Figure 1. Illustration of the effect of roughness of the ground.
d = zero-plane displacement, ΔU = reduction of mean velocity, z_v = depth of viscous layer, z_r = depth of roughness layer.

If (1.5) is introduced into (1.3) we find the eddy viscosity to be

$$K = k\, \sigma^{\frac{1}{2}} (z - d) \quad . \quad . \quad . \quad . \quad . \quad (1.11)$$

It is seen that for a constant σ the roughness implies a reduction in the eddy viscosity at a fixed height. Thus, above a certain layer, which may be referred to as the *roughness layer*, the intensity of the *Austausch* process is reduced.

The reduction of the mean velocity due to roughness is given by

$$\frac{U_*}{k} \log \left\{ \frac{z}{z - d} \left(1 + \frac{a_r}{a_s}\right) \right\} \geqslant 0 \quad . \quad . \quad . \quad (1.12)$$

It is seen that for a constant σ this reduction decreases with height, and it may therefore be said that the influence of the roughness decreases with height.

Finally, it should be noted that the zero-plane displacement d is a rough measure of the depth of the roughness layer because of Eq. (1.11). ΔU of Eq. (1.10), on the other hand, is not a direct measure of the total mean-velocity reduction, which is given by (1.12), since the latter also depends on d.

2. On the turbulent transfer of heat and water vapour in the lower atmosphere

For the vertical transport of any quantity x, the upward flux is given by

$$\overline{w\,x} \quad . \quad . \quad . \quad . \quad . \quad (2.1)$$

where w denotes the vertical velocity component.

In the case of heat transfer, x can be interpreted as the heat which must be supplied to unit volume of air in order that it shall achieve the average state of the air at the height considered. According to the first principle of thermodynamics this heat is approximately given by

$$x = \rho\left(c_p\, t - \frac{p}{\rho}\right) = c_p\, \rho\, T\, \frac{\theta}{\Theta} \quad . \quad . \quad . \quad (2.2)$$

where ρ = density, p = pressure fluctuation, T = mean temperature, t = temperature fluctuation, Θ = mean potential temperature, θ = potential-temperature fluctuation, c_p = specific heat at constant pressure. Introducing (2.2) into (2.1) we obtain the following expression for the *downward* flux of heat

$$Q = -\,c_p\, \rho\, \frac{T}{\Theta}\, \overline{w\theta} \quad . \quad . \quad . \quad . \quad (2.3)$$

which is in accordance with Priestley and Swinbank's (1947) theory.

Let us put

$$-\,\overline{w\theta} = L\,\frac{d\Theta}{dz} \quad . \quad . \quad . \quad . \quad (2.4)$$

where L denotes eddy conductivity. Eq. (2.3) then reads

$$Q = c_p\, \rho\, \frac{T}{\Theta}\, L\, \frac{d\Theta}{dz} \quad . \quad . \quad . \quad . \quad (2.5)$$

Now, when radiative heat transfer is negligible, Q can be taken constant under steady conditions. Further, as we shall only consider the lower atmosphere up to a height of 25 m we may treat ρ, and T/Θ as constants. Consequently we may write

$$q \equiv \frac{Q\Theta}{c_p\, \rho\, T} = L\, \frac{d\Theta}{dz} \quad . \quad . \quad . \quad . \quad (2.6)$$

where q can be considered constant.

The *downward* flux of water vapour due to turbulence is given by

$$F = -\,\rho\, \overline{wr} \quad . \quad . \quad . \quad . \quad . \quad (2.7)$$

where r denotes the specific humidity (or mixing ratio) which is approximately given by

$$r = \frac{5}{8}\,\frac{e}{p} \quad . \quad . \quad . \quad . \quad . \quad (2.8)$$

where e denotes water-vapour pressure and p air pressure. As p varies little within a height of 25 m we may take r proportional to e for a single profile.

For the fully turbulent region we may now put

$$f \equiv \frac{F}{\rho} = M \frac{d\chi}{dz} \quad . \quad . \quad . \quad . \quad . \quad (2.9)$$

where χ denotes average water-vapour pressure and M is the eddy diffusivity. Here again we may consider f constant under steady conditions.

3. The profiles under non-neutral conditions

In accordance with the assumption that led to Kármán's formula in the case of neutral equilibrium, let us now assume that, in the case of stratification, the eddy viscosity, here denoted by K', the eddy conductivity L, and the eddy diffusivity M are all determined by the first and second differential coefficients of the mean velocity U, mean potential temperature Θ, and mean water-vapour pressure χ. Dimensional reasoning then leads to the following relations

$$\frac{K'}{K} = F_1(\phi, \psi) \quad . \quad . \quad . \quad . \quad . \quad (3.1)$$

$$\frac{L}{K} = F_2(\phi, \psi) \quad . \quad . \quad . \quad . \quad . \quad (3.2)$$

$$\frac{M}{K} = F_3(\phi, \psi) \quad . \quad . \quad . \quad . \quad . \quad (3.3)$$

where ϕ and ψ are given by

$$\phi = \frac{\dfrac{d^2\Theta}{dz^2}\dfrac{dU}{dz}}{\dfrac{d\Theta}{dz}\dfrac{d^2U}{dz^2}} \quad . \quad . \quad . \quad . \quad . \quad (3.4)$$

$$\psi = \frac{\dfrac{d^2\chi}{dz^2}\dfrac{dU}{dz}}{\dfrac{d\chi}{dz}\dfrac{d^2U}{dz^2}} \quad . \quad . \quad . \quad . \quad . \quad (3.5)$$

Introducing (3.1) to (3.3) into Eqs. (1.2), (2.6) and (2.9) respectively, we obtain

$$\sigma = k^2 \frac{\left(\dfrac{dU}{dz}\right)^4}{\left(\dfrac{d^2U}{dz^2}\right)^2} F_1(\phi, \psi) \quad . \quad . \quad . \quad (3.6)$$

$$q = k^2 \frac{\left(\dfrac{dU}{dz}\right)^3 \dfrac{d\Theta}{dz}}{\left(\dfrac{d^2U}{dz^2}\right)^2} F_2(\phi, \psi) \quad . \quad . \quad . \quad (3.7)$$

$$f = k^2 \frac{\left(\dfrac{dU}{dz}\right)^3 \dfrac{d\chi}{dz}}{\left(\dfrac{d^2U}{dz^2}\right)^2} F_3(\phi, \psi) \quad . \quad . \quad . \quad (3.8)$$

It is easily verified that a solution of these equations is represented by

$$\frac{U}{\sigma^{\frac{1}{2}}} = \frac{1}{k} \log \frac{z-d}{a} \quad . \quad . \quad . \quad (3.9)$$

$$\frac{\Theta - \Theta_0}{q} \sigma^{\frac{1}{2}} = \frac{1}{l} \log \frac{z-d}{b} \quad . \quad . \quad . \quad (3.10)$$

$$\frac{\chi - \chi_0}{f} \sigma^{\frac{1}{2}} = \frac{1}{m} \log \frac{z-d}{c} \quad . \quad . \quad . \quad (3.11)$$

where b, c, l and m denote new parameters.

It is thus seen that the above assumption implies logarithmic profiles as a possible solution in the case of a stratified lower atmosphere. In the subsequent section we shall see that this result seems to be in fair agreement with measurements (cf. Figs. 2-4).

Eqs. (3.9) to (3.11) show that the same zero-plane displacement d should occur in all simultaneous profiles. This result means that the thickness of the roughness layer should be approximately the same for all profiles. This seems to be a reasonable conclusion and we shall also find some experimental evidence for it.

As to the variation of d with stratification, the presence of convectional eddies (cf., M. A. Giblett *et al.* 1932, pp. 62-63) in the case of instability should tend to propagate the influence of the roughness higher than in the case of neutral equilibrium. Conversely, in the case of stable stratification the stability should depress this tendency. Accordingly, we arrive at the conclusion that d should increase with instability and decrease with stability. We shall see that the profiles examined below reveal such a tendency in the variation of d.

As to the variation of the parameters a, b and c, one would expect instability to diminish the effect of the roughness of the ground because the air particles should then ascend and descend more easily so that the roughness elements should be more easily passed. Consequently we should expect that the expression

$$\frac{U_*}{k} \log \left\{ \frac{z}{z-d} \frac{a}{a_s} \right\}$$

from (1.12) should decrease with instability. Now since d is expected to increase with instability, a would be expected to decrease with instability, but this conclusion is not unambiguous, as we shall see k will increase with instability.

The effect of stratification on the parameters k, l and m is easily derived from the following formulae for the eddy viscosity, eddy conductivity, and eddy diffusivity obtained by the use of Eqs. (3.9) to (3.11).

$$K' = k \, \sigma^{\frac{1}{2}} (z-d) \quad . \quad . \quad . \quad (3.12)$$

$$L = l \, \sigma^{\frac{1}{2}} (z-d) \quad . \quad . \quad . \quad (3.13)$$

$$M = m \, \sigma^{\frac{1}{2}} (z-d) \quad . \quad . \quad . \quad (3.14)$$

As d increases with instability, the variation of this quantity causes a diminution of K', L and M. Since experience shows that K', L and M increase with instability k, l and m must also increase with instability and even more rapidly than do the former quantities. Sheppard (1947) has also concluded that k should increase with instability.

In summing up the above reasoning we may say that in the case of unstable stratification the effect of the roughness decreases on the whole because the air passes the roughness elements more easily, but the influence of the roughness will penetrate higher because of the presence of convectional eddies. This is also verified by the

empirical result that frictional eddies are generally less developed in the case of instability (cf., Giblett *et al.* 1932). In addition to the alteration of the influence of roughness, the stratification also affects the state of turbulence throughout the constant-transfer layer through the variation of the eddy viscosity, eddy conductivity and eddy diffusivity. It also seems highly probable that the depth of the constant-transfer layer varies with stratification. Thus in the case of instability one would expect a deeper constant-transfer layer as an effect of the convectional eddies.

Finally, let us introduce the friction velocity U_* and the corresponding quantities for temperature and humidity defined by

$$\Theta_* = q\,\sigma^{-\frac{1}{2}}, \quad \chi_* = f\,\sigma^{-\frac{1}{2}} \qquad . \quad . \quad . \quad (3.15)$$

into Eqs. (3.9) to (3.11) whereby

$$\frac{U}{U_*} = \frac{1}{k}\log\frac{z-d}{a} \qquad . \quad . \quad . \quad (3.16)$$

$$\frac{\Theta-\Theta_0}{\Theta_*} = \frac{1}{l}\log\frac{z-d}{b} \qquad . \quad . \quad . \quad (3.17)$$

$$\frac{\chi-\chi_0}{\chi_*} = \frac{1}{m}\log\frac{z-d}{c} \qquad . \quad . \quad . \quad (3.18)$$

Because of the analogy with friction velocity, Θ_* and χ_* may be called *friction temperature* and *friction humidity*, respectively.

4. Provisional comparison with measurements

(*a*) *Best's measurements*

Best (1935) measured the mean velocities (3-min periods) at heights 2·5, 5, 10, 25, 50, 100, 200 and 506 cm over short grass 1-2 cm high and expressed them as percentages of the simultaneous velocity at 1 m. They were then grouped and meaned for each height according to temperature gradient and velocity at 1 m. The result is given in the table on p. 34 in Best's paper.

In Figs. 2 and 3 the corresponding ' mean ' profiles are represented. The values of d have been determined by plotting the observations with a logarithmic height scale and by trying different values of d and choosing the one which gives the best fit with a linear law (cf. Pasquill 1950b). The values of d and a are attached to each profile in the diagrams. The parameter k, on the other hand, could not be determined as there was no independent determination of the shearing stress.

For the greatest instability the best fit was obtained by $d = 5{\cdot}5$ cm, but the observations at 2·5 and 5 cm had then to be discarded as they were situated in the roughness layer. The same applies to the observations at 2·5 cm for the profile for which d was found equal to 3·3 cm.

The diagrams show that all velocity profiles are well described by logarithmic laws for all stratifications and both groups of mean velocities at 1 m.

Further, it is seen that d increases and a decreases steadily with instability in the case of low velocities (cf., Fig. 2). In the case of higher velocities (cf., Fig. 3) these variations are not as regular.

Best (1935, p. 18) has also given two temperature profiles. In Fig. 4 these profiles are represented with appropriate values of d obtained by the method described above. Again it is seen that the logarithmic law is obeyed with great accuracy, and the values of d are as might be expected.

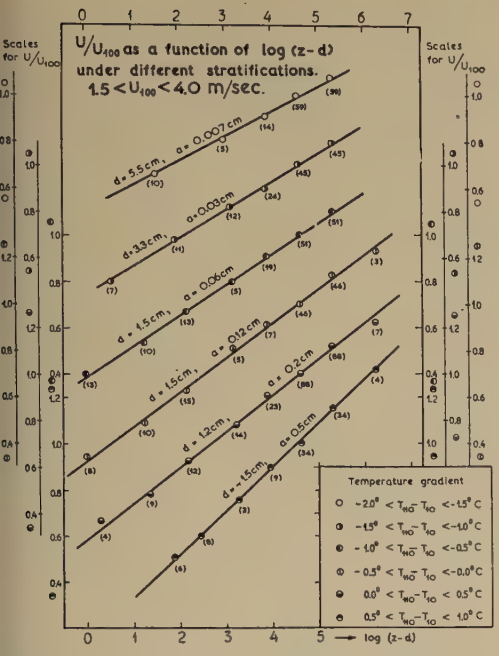

Figure 2. Comparison of theory with Best's wind data for $1{\cdot}5 < U_{100} < 4{\cdot}0$ m/sec. Figures in parenthesis give number of observations.

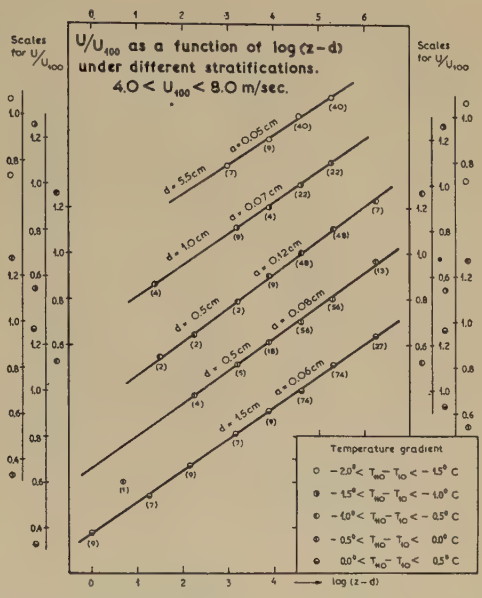

Figure 3. Comparison of theory with Best's wind data for $4{\cdot}0 < U_{100} < 8{\cdot}0$ m/sec. Figures in parenthesis give number of observations.

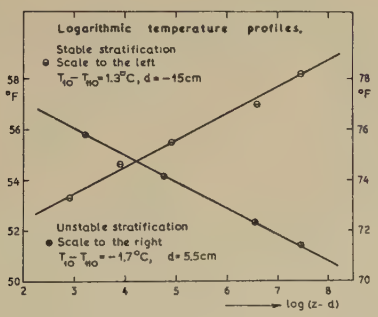

Figure 4. Comparison of theory with Best's temperature data.

To the author's knowledge there are no published humidity profiles grouped according to temperature gradient and mean velocity other than those given by Sverdrup, which will now be considered.

(*b*) *Sverdrup's measurements*

Sverdrup (1936) measured simultaneously the mean velocity, mean temperature and mean humidity at 3 or 4 different heights over a large snowfield. He also grouped the profiles according to temperature gradient and mean velocity, but his process of averaging differs from Best's as he summed the observed values directly for each height. The higher values therefore play a more dominant role than do the lower values in his 'mean' profiles.

Figs. 5-7 show velocity, temperature and humidity profiles for four groups of stable stratification and different mean velocities. Here d has been determined to give the best fit for the velocity and temperature profiles by the method described above.

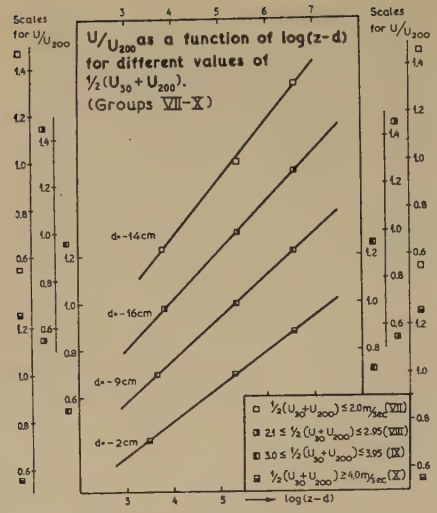

Figure 5. Comparison of theory with Sverdrup's data. Velocity profiles.

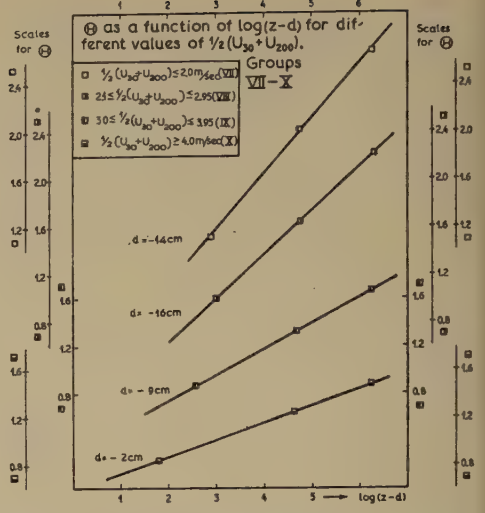

Figure 6. Comparison of theory with Sverdrup's data. Temperature profiles.

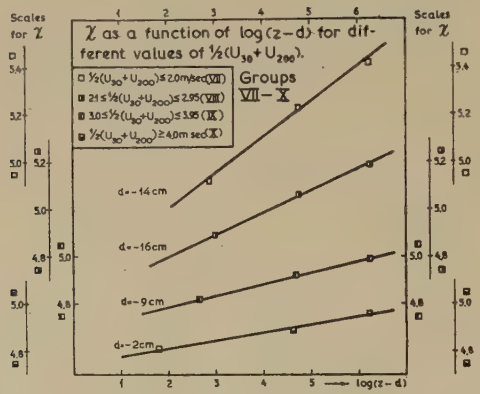

Figure 7. Comparison of theory with Sverdrup's data. Humidity profiles.

It is seen that not only the velocity and temperature profiles, but also the humidity profiles, are well described by logarithmic laws with the same value of d. However, there are only two degrees of freedom for three simultaneous profiles so that it might be premature to conclude that d is the same for all simultaneous mean profiles. But Rider and Robinson have found that in many cases simultaneous velocity, temperature and humidity profiles coincide if the scales are adjusted for this purpose and this result is an independent verification of the constancy of d since,

$$\frac{dU}{dz} \cdot \frac{d\Theta}{dz} \cdot \frac{d\chi}{dz} = \frac{\sigma}{k} \cdot \frac{q}{l} \cdot \frac{f}{m} \qquad . \quad . \quad . \quad (4.1)$$

for the logarithmic laws described above.

Rider and Robinson (1951) assumed on this basis that the eddy convectivity and eddy diffusivity are equal. In the present case, on the other hand,

$$K : L : M = k : l : m \qquad . \quad . \quad . \quad . \quad (4.2)$$

so that these quantities may well be different.

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Bubble theory of penetrative convection

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Summary

The process of convection is described in terms of a unit or proton of convection – the bubble. A rising bubble of warm air sheds its outer skin steadily into a disturbed wake until it becomes exhausted completely or spreads out at a stable layer. The wake is a region where the ascent of further bubbles is favoured. The wakes of small bubbles close to the ground are aggregated into larger bubbles, which are more dilute, up to a level where they begin to penetrate hitherto undisturbed air, and then they waste away as they ascend further.

The air above a bubble is lifted (and cooled) as the bubble approaches and then drains down the outside, the air close to the bubble being mixed into the wake. The wake of a clear bubble is buoyant but that of a cloudy bubble may sink if it is sufficiently chilled by dilution with surrounding clear air.

The drag on a bubble is estimated from observations on rising cumulus towers, and a linear relation between buoyancy and limiting velocity is proposed. From this the horizontal velocity of the bubble relative to the surrounding air is deduced to be about the same as the vertical velocity when it ascends through strong shear.

A bubble is a compact stable configuration for a rising element. Near the ground the heat is transported by small bubbles, which are less efficient, and therefore require a greater lapse rate, than the larger ones which can operate higher up. The process of aggregation is again renewed within large clouds so that the bubbles found at the top may be more dilute than if they had ascended directly from the base. Clouds growing in a shearing current will grow into the shear.

1. Introduction

The ideas of the early glider pilots, described in the writings of Hirth (1933), and later Albrecht (1942) for instance, suggest that the thermals in which they soared were rising bubbles of warm air. Hirth, and those who have copied his ideas, imagined the bubbles to break away from the super-adiabatic layer at the ground rather as water falls in drops from a ceiling on which steam is condensing, but the ultimate fate of the bubbles was not considered except that they were imagined to reach a level of no buoyancy somewhere, and spread out and lose their identity there. In this paper the approach is different. We develop a theory from aerodynamical reasoning as an aid to the interpretation of observations.

It is plain that convection is one of the most complicated of all meteorological processes, particularly over land. The condition is one of persistent static instability to which many physical processes contribute and in which units of motion of a wide range of sizes are found superposed and intermingled. At first sight this is clearly a field for the application of statistical methods, but we have found that our main interest lies in the behaviour of the individual protons of the motion – the bubbles. Indeed our main object is to understand the process in terms of these fundamental units of motion and mathematical precision is not to be expected of the theory at present. Once the bubble theory has been put forward, we choose to wait for the accumulation of evidence which can be interpreted by the theory before attempting mathematical formulation. No attempt is made here to discuss precipitating clouds or the effect of freezing.

The term ' penetrative ' is used, after Sir Napier Shaw, to denote that we are studying the behaviour of the masses of air which rise through an environment much bigger than themselves. It is used in contrast, for example, to ' cellular ' convection, which is a motion in which the whole fluid circulates.

Before introducing the model bubble which is the ' proton ' from which convection is imagined to be built up we refer to a kind of penetrative convection that can be readily created artificially, but which will be seen to be of a very special kind, namely steady jets.

2. Steady jets

Jets are of two kinds. The first is the diverging jet which is treated by means of the mixing-length theory of turbulence (e.g., Schmidt 1941). The chief characteristic is that it mixes with its surroundings. The heat emitted from a point source is confined within a cone with its apex at the source and widening upwards. The air outside the cone is moving horizontally in towards the cone. On entering the cone it is mixed into the air of the jet by turbulence which derives its energy from the buoyancy of the air. The mixing length of the turbulence is proportional to the width of the jet and the appearance of the jet is similar at all heights. The mean velocity within the cone is almost vertical and the temperature anomaly and vertical velocity increase towards the centre of the jet but decrease upwards. The buoyancy is exactly spent on the momentum added as the air enters the cone horizontally and begins to move vertically. This kind of jet requires a continuous concentrated source of heat, and though we do not assert that it never occurs in the atmosphere it can be fairly described as uncommon.

The second kind of penetrative jet converges upwards. It is bounded by a free stream surface across which there is a discontinuity of velocity and density. The walls are nearly vertical and the velocity and pressure almost uniform across the jet. The flow is steady so that if suffixes 0 and 1 denote values outside and inside the jet and the air outside is motionless

$$p_0 = p + \rho_0 gz$$

$$p_1 = p + \rho_1 gz - \tfrac{1}{2}\rho_1 (w^2 - W^2) .$$

where W is the vertical velocity at $z = 0$ where the pressure is p. Thus since the pressure is the same inside and outside the jet we obtain

$$w^2 = W^2 + 2gz(\rho_0 - \rho_1)/\rho_1 \quad . \quad . \quad . \quad . \quad (1)$$

Since the flux up the jet is the same at all heights, if it is circular with radius R at $z = 0$ and r elsewhere

$$R^2 W = r^2 w$$

$$r^4 = \rho_1 R^4 W^2/\{W^2 \rho_1 + 2gz(\rho_0 - \rho_1)\} \quad . \quad . \quad (2)$$

The air accelerates upwards through the jet, which slowly converges. At some height the velocity is so large that laminar flow cannot continue and the jet disintegrates into meandering lumps which mix into the environment. This kind of jet can be demonstrated by holding a smoking cigarette motionless in calm air or by obtaining large rate of emission of hot gases from a bonfire. When the rate of emission is small the jet soon becomes narrow and disintegrates ; as the rate is increased the height of disintegration is raised.

In order to maintain either kind of jet a continuous localized source is required. For the diverging jet sufficient turbulence must be present from the bottom to diffuse the buoyancy and momentum, while the converging jet is without turbulence.

3. The properties of a rising bubble

A localized source may be unable to maintain a steady jet and the buoyant gas will then rise from it in lumps whose properties we now study.

(i) *Bubbles of air rising through a liquid.*

These were studied by Davies and Taylor (1950) who showed that their tops are spherical. This is because the pressure within the bubble is uniform, the air having a negligible density. The pressure field over the cap of a sphere in potential flow is equal

and opposite, to the first order in the distance from the top, to the hydrostatic pressure field ; so that once the shape is spherical it will remain so, for the two fields cancel out to produce uniform pressure over the surface. The liquid below the bubble is turbulent and the bottom of the bubble is horizontal but very disturbed. The accelerations in the wake of the bubble and the stresses produced there by the degeneration of kinetic energy keep the pressure at the base of the bubble the same as at the top and less, therefore, than the pressure outside the wake at the same level. Experiments made at Imperial College show that if the bubble is allowed to rise through a layer of dyed fluid, the wake is seen to be tinted, showing that the wake is composed of liquid from all the levels through which the bubble has risen, mixed thoroughly together. The buoyant gas is gathered together into the umbrella-shaped cap whose perimeter oscillates violently as it throws off the turbulent wake. Sometimes small bubbles break off from the perimeter, but this is rare. The general coherence of the bubble is thought to be uninfluenced by surface tension because of its flattened appearance and disturbed base : the spherical cap appears to be a stable configuration.

Figure 1. Schematic representation of path of small bubble relative to that of a large bubble of air rising through water (1/3 natural size).

If a bubble (Fig. 1) is adjacent to the wake of a larger one it may be ' sucked ' in and then rise rapidly up through the wake and amalgamate with the larger one. If the larger one, which rises more rapidly, is below the smaller, though not vertically beneath, it appears to ' push ' the smaller one aside and overtake it.

(ii) *Instability at a surface*

Instability at a surface of discontinuity of density in the atmosphere is displayed when mammatus cloud forms. When a layer of cloud is formed by the spreading out of cumulus it often assumes a mammatus form as it thickens. The characteristic of this form of cloud is its sharply outlined base ; since the air of the cloud is well above its condensation level and the air below is unsaturated, up and down movements do not make the base diffuse like the base of a cumulus, where the drops are smaller than in the body of the cloud. If the layer as a whole descends the cloudy air will warm more slowly than the dry air below, and if it becomes colder it will sink into it. The base while remaining sharply outlined takes on a disturbed appearance. The phenomenon is displayed *in excelsis* on the edges of large cumulonimbus when cloudy air of low-level origin has

spread out over clear air and is subsequently subjected to widespread sinking. The upper mass penetrates into the lower one, the clear outlines of the penetrating masses being visible because of the different origin of the masses. Soon the bubbles become invisible when the cloud in them evaporates. It is reasonable to suppose that the lower mass penetrates into the upper one in a similar manner.

A bubble of warm air rising through cool air will cohere under a spherical dome as in water, but we do not know what shape its base will take, if indeed it can be defined. The appearance of the upper surface of a bubble can be seen on the rising towers of cumulus congestus. It is covered by a multitude of small nodules caused by the penetration of small bubbles of warm air into the air above, and it looks like a cauliflower. The small bubbles do not continue to rise with the large bubble but flow down its cap and soon mix with its surrounding air and generally sink, fresh air coming to the surface and forming new small bubbles. Thus the wake does not originate at the perimeter of the bubble but on its upper surface. Surface tension is thought to prevent this process occuring on the upper surface of a bubble of air in water.

(iii) *The wake*

The only guidance from aerodynamic theory about the shape of the wake is from Oseen's solution for the flow past a sphere. Since the solution is in any case not accurate close to the surface the shape of the wake, in this case paraboloidal, is not likely to depend critically upon the shape of the bubble. When the bubble reaches its limiting velocity the buoyancy is equal and opposite to the upward momentum added to the wake, but because the bubble continuously sheds its outer skin the wake consists of a mixture of the surrounding air with the air of the bubble itself. In the absence of condensation the wake is therefore buoyant and rises some distance further, though more slowly than the bubble. The bubble is eventually completely wasted away and the top of what is is now only wake ceases to rise as rapidly.

When the bubble is above the condensation level the mixture of bubble air with drier surroundings may, and generally will, produce air that will sink because it is colder than the surroundings. The outside of a cumulus congestus generally does move downwards even when the top is growing upwards. The wake will, even so, consist of air that is damper than the surroundings.

The difference between the wake and the surroundings decreases with distance from the bubble which produced it. The surfaces of constant dilution will be oval even though the boundary of the wake may be more nearly paraboloidal. If another bubble rises into the wake its velocity relative to the surroundings and the instability at its surface are less than if it were in fresh air : it will therefore waste away more slowly and will rise to a greater height than its predecessor before exhaustion. If it merely impinges on the wake it will tend to be drawn wholly within it (as in Fig. 1).

(iv) *The shape of a bubble*

The top of a buoyant bubble is more or less spherical for reasons given in (i) ; but when density difference is small we can only guess at its vertical extent. The undiluted part of the bubble cannot be very deep when the terminal velocity is reached because, like the air in a jet, the lower part would either be rapidly diluted or accelerated upwards. The bubble will soon become compact with a rounded upper boundary whose precise shape may also be affected by circulations within it, which have arisen while it takes shape, and by the static stability of the surrounding air.

(v) *The velocity field around a bubble*

Above the bubble the velocity is similar to that ahead of a moving sphere. The air is thus displaced upwards (and cooled at each level if the air is stable) above it, as illustrated by the occasional appearance of pileus over rising towers of cumulus, Fig. 2 (Plate I). The bubble penetrates each layer, which then drains down its sides. The upward displacement of the surrounding air is permanent if it is not stable or if further bubbles follow the first, but if stable it sinks back ultimately to its original level. In the edges of the wake the velocity is upwards if the air is clear (Fig. 3) but generally downwards if evaporation causes chilling when the bubble is cloudy. The whole wake is disturbed, the vertical velocity being greatest immediately below the undiluted bubble. The disturbance in this region may be small compared with the violent eddies on the edges of the wake.

(vi) *An illustration of the bubble-structure of cumulus.*

The photographs of Fig. 4 (Plate I) illustrate a structure typical of large cumulus grown in a small wind shear (amounting on this occasion to about $1\frac{1}{2}$ m sec^{-1} km^{-1}), showing the changes in the cloud shape during a period of about twenty minutes. Although the cloud mass as a whole is recognizable throughout this period, it is seen to be composed of a number of bubbles which emerge in succession on its up-shear (right-hand) side, grow to become its principal tower, and then flatten, subside and evaporate on its down-shear side. Fig. 5 shows the rate of rise and subsidence of the bubble indicated by arrows on Fig. 4, and of its successor, estimated from theodolite observations. The rather sudden deceleration in the last few hundred metres of the ascent and the complete evaporation of the bubble after a subsequent descent of a further few hundred metres are features typical of the erosion of a bubble before it reaches an equilibrium level.

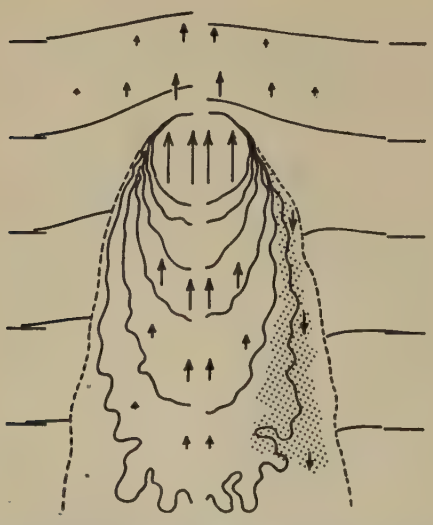


Figure 3. The velocity field around a rising bubble. The left-half of the diagram refers to dry ascent ; in the other half condensation has occured in the bubble and the shaded area represents the region of the wake in which there is sinking motion. The displacement of air particles in the environment from their original level is also indicated.

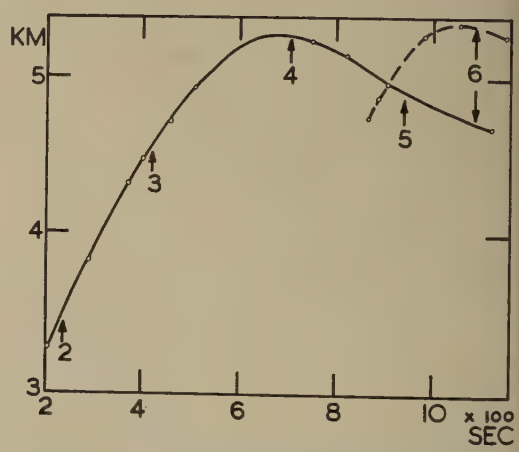
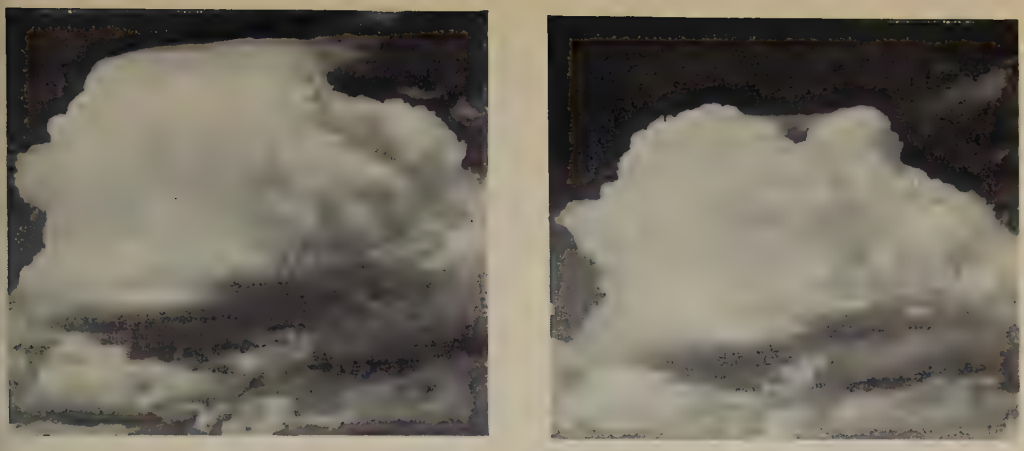


Figure 5. The observed rise and subsidence of the top of the bubble marked by arrows on Fig. 4 (solid line), and of its successor (pecked line). The numbers indicate when the corresponding photographs of Fig. 4 were taken, the time scale commencing with the first exposure.



(a) (b)

Figure 2. Typical pileus cap to rising cumulus bubbles. Within three minutes the bubbles pierce the pileus, which subsides and evaporates (right hand picture).

Figure 4. Cumulus to east of Dunstable, 18 hr, 31 July 1950, showing bubble structure. Cloud base is at 1,300 m; the bubble diameter is about 1·5 km. The clouds move away from and slightly to the left of the observer; the wind shear is directed farther towards the left. The bubbles (one of which is indicated by the white arrows) emerge from the up-wind side of the cloud mass, rise to about 5,300 m (see also Fig. 3), and subside and evaporate on the down-wind side.

4. The drag on a bubble

(i) *The limiting velocity*

In the previous section a mathematical description of the shape and behaviour of a bubble was avoided because not enough is known about the process of wasting away or the motion in the wake to act as a foundation for it. But we may treat the bubble as a whole and estimate the drag on it. There is no aerodynamic analogy and no guidance about the probable dependence of the drag upon the vertical velocity, and the shearing off of the outer skin on the top would suggest that the drag would be less than for a solid body.

An attempt was made to obtain some information by observing with a theodolite the rate of rise of cumulus tops. The height of the tops was found by assuming them to lie vertically above the cloud base, whose height was occasionally observed by aircraft but was otherwise estimated. By following the changes in angular elevation of both the top and the base of a cloud, and by carefully selecting the part of the sky in which the observations were made, the position and speed of ascent of a prominent cloud bubble could in this way be deduced with an error not exceeding 15 per cent and probably considerably less. It was found that on any one occasion there appeared to be a representative maximum upward speed achieved by the most prominent cloud bubbles. The observations were made at Dunstable, and the radiosonde ascents from Larkhill and Downham Market were used to estimate the excess temperature associated with the observed rates of ascent. It was assumed that at the level of the cloud base there already existed an excess temperature of 1°C, and at higher levels the excess temperature was found by assuming the bubble air rose adiabatically from the cloud base : the virtual temperatures of the bubble and its environment were compared and a correction made to take into account the weight of suspended water. The results from the occasion shown in Fig. 4 and seven days in August 1950 are plotted in Fig. 6. In view of the possible sources of error they show a surprising consistency, and in a first treatment suggest that a linear relationship can be assumed between the buoyancy of a bubble and its limiting velocity. It is not likely to be greatly in error over the range of velocities encountered so it is used here because there is no theoretical reason for taking a more complicated relationship.

If w and z are the vertical velocity and height of the bubble, and ΔT its excess temperature, the equation for vertical motion is

$$\dot{w} + Kw - g\Delta T/T = 0 \qquad . \quad . \quad . \quad . \quad (3)$$

The observations indicate a value for K of about $\frac{1}{50}$ sec^{-1}. If ΔT is constant then

$$w = w_0 - Ce^{-Kt} \qquad . \quad . \quad . \quad . \quad . \quad (4)$$

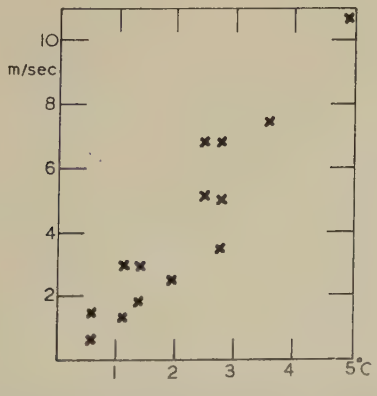


Figure 6. Observed rate of rise of prominent cumulus towers compared with the estimated temperature-excess of the cloud air (corrected for weight of suspended water).

where w_0 is the limiting velocity. If released from rest so that $C = w_0$, the bubble acquires 2/3 of its ultimate velocity in 1 min and 9/10 after 2 min. If it entered a layer in which the buoyancy suddenly increased by half then it would after 1 min. again acquire 9/10 of its limiting velocity.

The life of a bubble which rises to cloud heights is many minutes, so that over nearly all its life its vertical velocity will be indistinguishable from the limiting value.

If the bubble enters a stable layer in which $\Delta T/T = -\beta z$ the solution of Eq. (3) is

$$\left.\begin{aligned} z &= Ae^{\lambda t} + Be^{\mu t} \\ \lambda, \mu &= -\tfrac{1}{2}K \pm (\tfrac{1}{4}K^2 - g\beta)^{\frac{1}{2}} \end{aligned}\right\} \quad . \quad . \quad . \quad (5)$$

If oscillations about equilibrium could occur the period would be $2\pi/(g\beta - \frac{1}{4}K^2)^{\frac{1}{2}}$, and with $K = \frac{1}{50}$, $\beta = 10^{-7}$, the period is infinite. If β takes twice this value the period is about 10 min and after half a complete oscillation the vertical speed would be reduced to $w_0/20$. Oscillations about equilibrium therefore seem improbable, and certainly not observable. If the stability were so great as to reduce the period to a minute or two the amplitude would be less than the normal bubble diameter, and the bubble would flatten out. Indeed this seems to be a characteristic of cumulus, that it does not overshoot a stable layer (unless it completely penetrates it) but flattens out. Towers do appear to sink soon after reaching their maximum height, but this is because they consist only of dilute wake whose buoyancy is negative. They sink in exactly the same way as the walls of a rising tower.

(ii) *Ascent through shear*

If u is the horizontal velocity of the bubble relative to the surroundings whose velocity is U, a function of z, then if the limiting velocity has been achieved

$$w\partial(U + u)/\partial z = Ku$$

$$w/u = K/U', \text{ if } U' \gg u' \text{ (a prime denotes } \partial/\partial z).$$

In uniform shear and constant w, $u' = 0$. In a typical strong shear of $\frac{1}{50}$ sec^{-1} we find that w and u are equal. Thus the bubble travels through the air a distance considerably greater than the height reached, and it therefore wastes away more rapidly in ascending through shear than through a uniform current ; consequently it becomes exhausted at a smaller height than in a uniform current.

This result is in near qualitative agreement with that of Malkus (1952) ; but since the derivation is simpler, and is based on the simplest possible observations that could be contemplated (of w and ΔT), it is preferred to hers.

5. The transport of heat upwards

(i) *The aggregation of small bubbles*

The lapse rate in strong sunshine decreases steadily from the ground upwards until at the greatest heights reached by convection, in the absence of condensation which causes complications not relevant here, it is close to the adiabatic lapse rate. The oscillation of temperature along a horizontal line is so large that the lapse rate only has a meaning as an average. One metre above the ground the oscillation could be 2°C, while the average lapse rate from 50 cm to 150 cm was ten times the adiabatic and implied a temperature difference between these two levels of only 0·1°C. At higher levels the lapse rate has more meaning and we can begin to talk about the surroundings and expect them to show only small horizontal variations.

For a bubble to be worthy of the name it must be fairly homogeneous and in order to create a bubble, say 300 m in diameter, a mechanism for carrying the heat from the

ground up to a height of 100 m (i.e., to near the centre of the mass that will compose the bubble) must be operating. We postulate that smaller bubbles provide this mechanism, and that they coagulate into larger ones through the power of wakes to protect and engulf succeeding bubbles. If there is no wind this favoured site remains over the same piece of ground and must draw fresh bubbles from the same place, thereby exhausting its heat and inhibiting fresh growth. A general wind drift therefore assists the growth of large bubbles. When several smaller bubbles have exhausted themselves into a region of air that air is buoyant and their aggregated wakes may be regarded as a bubble on a larger scale. Since it is composed only of wake, its temperature anomaly is necessarily less than that of the original bubbles.

A bubble cannot contribute to the transport of heat through a layer at the ground whose depth is small compared with its diameter ; but the agregation of smaller bubbles serves to build up irregularities in the layer.

As the larger bubble begins to move away from the ground the motion, on that scale, is rather like cellular convection, but soon a part of the layer breaks away and forms a compact bubble, and that piece of ground is temporarily exhausted as a source of large bubbles. The smaller bubbles, whose temperature anomaly is large, do not ascend far before exhaustion, so that at greater heights above the surface the oscillation of temperature along a horizontal line is less, and the size of bubble effecting the transport of heat greater.

It must now be supposed that large bubbles provide a more efficient mechanism for heat transport than smaller ones. This follows from the decrease of the lapse rate upwards, for at the levels where the larger bubbles operate a smaller lapse rate is required to effect the heat transfer. The heat transfer decreases upwards because the whole convection layer is being heated, but the lapse rate approaches the adiabatic much more rapidly. Also in cellular convection the largest possible cells appear most readily (at a smaller lapse rate) and do not disintegrate. Large bubbles cohere so that there is a dynamical force preventing their disintegration, consistent with the idea that if they remain integral the forces of the static instability which cause the convection are thereby most rapidly relieved.

The rate at which heat is being transported up by a typical thermal may be many (perhaps 100) times the heat input by insolation into the same horizontal area. Some ground can store the heat of insolation and releases it when air motion brings cooler air over it. Also time is required for the small bubbles to aggregate and spread the heat over a depth of many times their diameter. Thus the heat carried away by a large bubble represents the input into the ground over a long period.

The variable nature of the ground also contributes to the process of aggregation of small bubbles into large ones by supplying varying heat sources and anabatic and katabatic winds. If the surface is uniform and flat and has negligible thermal capacity (and/or conductivity), and there is no wind, then the aggregation is inhibited because the release of a bubble automatically decreases the likelihood of another one following in its wake, for the loss of heat from that part of the surface must be made up by insolation before it can become again a bubble source. In the Egyptian desert, for instance, where the thermal capacity of the ground heated is small the lapse rate continues to build up throughout the day to a value greatly in excess of that normally required to support convection if bubbles of sufficient size can be built up. The lapse rate is thus a measure of the largest bubble size that is effecting upward heat transfer at that level.

(ii) *The influence of radiation*

Robinson (1950) regards convection, in which he includes also the eddying motion whose energy is not derived from buoyancy, as a process which destroys the statically unstable temperature gradient set up by radiation, and he estimates the heat transported

away by convection by calculating that put in by radiation. In the lowest few metres this is considerable, so that the process of bubble convection is not required to transport all the heat from the surface to a height of 2 m to 3 m, say. The dryness of the air over the desert may contribute to the great depth of the substantially superadiabatic layer found there in the afternoon, for it is more transparent to radiation. In this view the superadiabatic layer is regarded as maintained by radiation in spite of convection, but we have just suggested that it is necessary to have a steeper lapse rate close to the ground because the larger and consequently more efficient bubbles cannot operate in a layer shallower than their diameter. The actual lapse rate is thus achieved by a balance of these two effects.

In discussing superadiabatic lapse rates, Brunt (1939) points out that convection, as imagined by Robinson, plays the role of viscosity and thermal conductivity and thereby prevents the initiation of ordered convective circulations. He showed that the Rayleigh-Jeffreys criterion for the onset of cellular convection roughly explains how lapse rates of the magnitude of those described by Robinson can occur without ordered convection if disordered convection, such as that produced by a wind over rough ground, is present. The ordered, or cellular, convection imagined in the treatment of Jeffreys as the most efficient corresponds roughly to the beginning of the break-away of an accumulation of wakes of small bubbles as a larger bubble. The small eddies diffuse the bubble and prevent the growth of the large gradient of temperature at the upper surface of the bubble which is conceived as necessary for its coherence.

It is, however, in calm conditions that the greatest lapse rates are observed, which supports the viewpoint of Robinson that all convection (in this case the kind just referred to as disordered) tends to reduce super-adiabatic lapse rates. The decisive contribution of the wind on the present theory is that it encourages the growth of large bubbles by displacing them over new ground.

In the absence of much eddying Brunt pointed out that the loss or gain of heat from a rising (or sinking) element would be mainly by radiation. Robinson's discussion is one-dimensional and it will be necessary to investigate the loss of heat by radiation from a rising bubble below the condensation level before the theory is complete. Frequently the bases of cumulus clouds into which air is rising are concave (Scorer and Ludlam 1952) and a suggested explanation is that the exterior of the bubble is cooled by radiation during ascent to cloud base, while the interior is protected by a thicker coat of water vapour. Since the rising air is usually damper than the surroundings any mechanical dilution of the outside of rising elements could raise the condensation level and so make the base of cumulus convex ; but if the outside air were sufficiently damp the base could become concave.

(iii) *The tectonic limit*

Up to some surface, which we call the tectonic limit, the bubbles aggregate into bigger ones, and the surroundings through which the smaller ones rise are composed of old wake ; but as heating proceeds the larger bubbles continually penetrate into new air, above the tectonic limit. Individual bubbles therefore waste away when they emerge into the air above this surface but gradually as more of them penetrate and scatter their wakes the surface is raised. The lapse rate above this surface is generally stable and the bubbles only penetrate it on account of their excess temperature. Below this surface the down motion can also aggregate into larger bubbles and large cold bubbles can therefore descend to the ground and spread out when the large warm ones leave it. As this overturning begins in each case the motion is very similar to cellular convection in the layer below the tectonic limit, and so we can expect the Rayleigh-Jeffreys criterion as modified by Brunt to have some relevance to it.

6. The growth of cumulus clouds

When the tectonic limit is above the condensation level we can imagine all the processes that go to form the larger bubbles out of smaller ones will contribute to the formation of large clouds from, initially, the wake of one cloudy bubble. On the other hand as soon as a bubble reaches the condensation level a new source of energy is released and the bubble can penetrate stably stratified air. The down currents in the surroundings that are not affected by wakes of what were once cloud bubbles will not become organized into large cold bubbles which descend to the ground as in the air below tectonic limit, but the air will undergo widespread slow sink.

When a large cloud builds up the tectonic limit reaches upwards to include approximately the cloud outline. Within the cloud small bubbles become exhausted before reaching the top, and their wakes amalgamate into large bubbles some of which appear at the top surface of the cloud. External air is mixed into the cloud region when bubbles ascend on the edge, and just as the temperature anomaly of bubbles decreases upwards from the ground so the bubbles found near the cloud top are more dilute than if they had ascended from the base. Thus many of the bubbles found near the top are created within the cloud.

Cloud bases become more or less uniform and flat when the tectonic limit reaches the condensation level. Before this happens cloud bases are expected to be less uniform and more ragged (e.g., Scorer 1952).

If clouds grow in a shearing current the top of the wake of cloud bubbles is displaced down the shear and so apart from other influences, fresh bubbles from below will be guided into the up-shear side of the clouds where the lower part of the wake is found. Bubbles ascending on the down-shear side must ascend through a region where descent of old chilled wakes is occuring and are therefore more rapidly wasted. The new towers therefore tend to grow on the up-shear side. The process is well illustrated by the pictures of Plate I and those in the plates of the paper by Malkus.

The bubble theory has also been applied to explain the onion-like structure of hailstones (Ludlam 1952).

7. Acknowledgments

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The growth of ice crystals in a supercooled water cloud

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SUMMARY

Equations are derived for the growth rate of ice crystals in the form of thin plates and prismatic columns suspended in a supercooled water cloud of given temperature. The effect of the fall motion of the crystal on its development is discussed – in particular a criterion is derived for the transition from polyhedral to dendritic growth. Ice crystals grown in supercooled water clouds produced in large cold chambers reveal a double change of habit in the temperature range 0°C to − 40°C. Experimental determinations of the growth rates of plate crystals growing at − 2·5°C and prisms at − 5°C are in fairly good agreement with the theory.

1. INTRODUCTION

In this paper theoretical expressions are derived for the growth rate of the simple ice-crystals forms in a supersaturated atmosphere and are compared with measured values. After the early stages of growth the crystals generally lose their simple geometrical shapes and develop into dendritic structures. Accurate calculations of the growth rates then become impossible, but a criterion is derived for the transition from polyhedral to dendritic growth, after which the theory is probably not valid.

2. CALCULATION OF THE GROWTH RATE OF A STATIONARY CRYSTAL

It is assumed that the crystal is at rest relative to the air and that the diffusion field around the crystal has achieved a steady state. The rate of increase of crystal mass m is then (Jeffreys 1918),

$$\frac{dm}{dt} = 4\pi CD\,(\rho - \rho_s) \qquad (1)$$

where D is the coefficient of diffusion of water vapour in air, C the electrostatic capacity of the crystal, ρ_s the vapour density at the surface of the crystal and ρ its value at a distance remote from the crystal in the undisturbed field. This formula has been used by Houghton (1950) to calculate the growth rate of crystals by a trial-and-error method based on a table of values of ρ_s against crystal surface temperature. It is possible, however, to derive an analytical expression for dm/dt in terms of parameters which, for a water-saturated environment, are determined only by the temperature and by the geometry of the crystal.

If it is assumed that the vapour pressure p_s in the immediate vicinity of the crystal surface is the equilibrium vapour pressure with respect to ice at the surface temperature T_s of the crystal, i.e., $p_s = p_e\,(T_s)$, Eq. (1) can be written

$$\frac{p - p_e\,(T_s)}{p_e\,(T)} \simeq \frac{RT}{4\pi DC\,p_e\,(T)\,M}\frac{dm}{dt} \qquad (2)$$

where $p_e\,(T)$ is the saturation vapour pressure over a plane ice surface at the ambient temperature T, p is the ambient vapour pressure and M the molecular weight of ice.

While the crystal is growing the surface temperature T_s will be higher than that of the surrounding air. If it is assumed that the latent heat of sublimation L is dispersed solely by conduction through the air, we have

$$L\frac{dm}{dt} = 4\pi CK(T_s - T) \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where K is the thermal conductivity of air.

Now

$$\frac{1}{p_e}\frac{dp_e}{dT} = \frac{JLM}{RT^2} \quad \text{(Clausius-Clapeyron)}$$

and integrating this equation from the crystal surface to a boundary in the environment remote from the crystal gives

$$\ln\frac{p_e(T_s)}{p_e(T)} = \frac{JLM}{R}\frac{(T_s - T)}{TT_s} \simeq \frac{JLM(T_s - T)}{RT^2} \quad . \quad . \quad . \quad (4)$$

From Eq. (3),

$$\frac{p_e(T_s) - p_e(T)}{p_e(T)} = \exp\left(\frac{JL^2M}{4\pi CKRT^2}\frac{dm}{dt}\right) - 1 \quad . \quad . \quad . \quad (5)$$

Adding Eqs. (2) and (5):

$$\frac{p}{p_e(T)} = \exp\left(\frac{JL^2M}{4\pi CKRT^2}\frac{dm}{dt}\right) + \frac{RT}{4\pi CD\,p_e(T)\,M}\frac{dm}{dt} \quad . \quad . \quad (6)$$

When the exponent in Eq. (6) is small compared with unity (this may be proved by solving the equation accurately by successive approximation), we may write

$$\frac{dm}{dt} = 4\pi C\sigma \Big/ \left[\frac{JL^2M}{KRT^2} + \frac{RT}{DMp_e(T)}\right] = \frac{4\pi C\sigma}{f(T)} \quad . \quad . \quad (7)$$

where $\sigma = p/p_e(T) - 1$ is the supersaturation of the environment relative to ice and the term in the square brackets is a function of temperature only at constant air pressure.

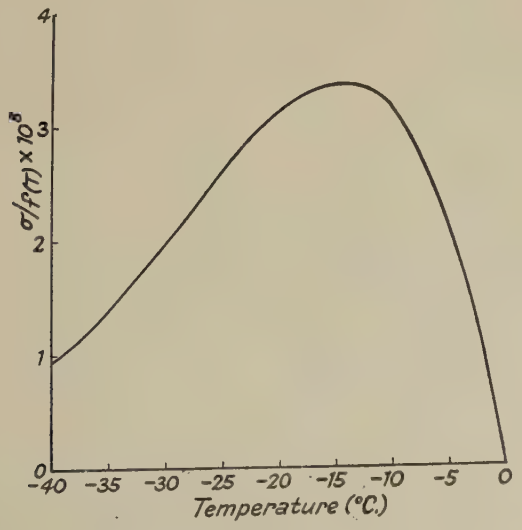

Figure 1. The function $\sigma/f(T)$ plotted against temperature.

Values of $\sigma/f(T)$ are plotted as a function of temperature in Fig. 1, assuming the environment to be saturated with respect to liquid water. There is very good numerical agreement between this curve and Houghton's Fig. 1 when adjustment is made for the fact that the latter was calculated for a constant total pressure of 800mb instead of for standard atmospheric pressure as in the present work.

Eq. (7) can now be applied to calculate the growth rate of ice crystals whose shapes approximate to those of conductors of known capacity. To a first approximation we may follow Houghton and treat the thin hexagonal ice plate as a thin circular disc and the hexagonal prism as a prolate spheroid of revolution of large eccentricity. The following solutions are then relevant :-

(a) *Sphere* $C = r$, $m = 4\pi/3\, dr^3$, where d is the crystal density.

Thus,

$$r\frac{dr}{dt} = \frac{\sigma}{d} f(T) \qquad . \quad . \quad . \quad . \quad (8)$$

(b) *Circular disc* $C = 2r/\pi$.

$$\frac{dm}{dt} = \frac{8\,\sigma\, r}{f(T)} \qquad . \quad . \quad . \quad . \quad (9)$$

(c) *Prolate spheroid* $C = 2ae/\ln\, 1 + e/1 - e$ where the eccentricity $e = (1 - b^2/a^2)^{\frac{1}{2}}$ and a and b are the major and minor semi-axes respectively.

Writing $b/a = \xi$ we have

$$\frac{dm}{dt} = \frac{8\pi\,(1-\xi^2)^{\frac{1}{2}}\, a\,\sigma}{\ln\left[\dfrac{1+(1-\xi^2)^{\frac{1}{2}}}{1-(1-\xi^2)^{\frac{1}{2}}}\right] f(T)} = 8\pi\phi\,(\xi)\,\frac{a\,\sigma}{f(T)} \qquad . \quad . \quad (10)$$

Eqs. (9) and (10) may predict quite accurately the rate of mass increase of small hexagonal ice plates and prisms suspended in a supersaturated atmosphere, but since the exact geometry and the surface structure of the crystals are not taken into account the equations cannot predict how this mass will be distributed, and hence the detailed shape of the crystal. By analogy with the charged conductor we should expect, under steady-state conditions, steeper gradients of vapour concentration and of temperature to be directed towards the corners of a hexagonal plate. Whether or not these preferred sites will be propagated faster than the rest of the crystal surface will depend on the relative rates at which molecules condense preferentially in these regions and diffuse from them to the remainder of the crystal surface. Fig. 1 shows that for a crystal of given geometry the rate of arrival of material from a water-saturated environment is a maximum at about -14°C, so we may expect sprouting of the crystal corners to be most pronounced at this temperature. This rather slight protrusion of corners which may occur under steady-state conditions is to be distinguished from the more pronounced dendritic development which results from the fall motion of the crystal and which is discussed in Section 3. In the case of an electrically charged ellipsoid of revolution the maximum density of surface charge occurs at the ends of the major axis, and the minimum density at the ends of the minor axis, the ratio of the two densities being a/b. Once prismatic growth has started, we should expect it to continue, more vapour arriving per unit surface area on the basal faces than on the prism faces.

3. Transition from polyhedral to dendritic growth

So far we have considered only the case where the crystals possess such a small fall velocity that they may be considered stationary with respect to the air and the diffusion field. The field was also considered to be in a steady state and to extend to distances

far remote from the crystal (theoretically to infinity). However, when a crystal grows to such a size that it attains an appreciable fall speed it will disturb the diffusion field from its equilibrium pattern. The crystal will always be moving into new surroundings and the diffusion field will be limited to a boundary layer around the crystal in which the concentration gradients will be larger than those directed towards a stationary crystal. Hence the rate of mass increase of the crystal will be enhanced by its fall motion.

We may consider the crystal to be introduced into the initially undisturbed medium at time zero and calculate the concentration gradient which will exist at the surface of the crystal at some subsequent time t. For the simplest case of a spherical crystal of radius r the vapour density ρ at distance R is determined by

$$\frac{\partial \rho}{\partial t} = D\left(\frac{\partial^2 \rho}{\partial R^2} + \frac{2}{R}\frac{\partial \rho}{\partial R}\right) \quad . \quad . \quad . \quad . \quad (11)$$

with the boundary conditions $\rho = \rho_0$ everywhere when $t = 0$ and the concentration $\rho\,(r)$ at the crystal surface is constant for all values of t. The solution of Eq. (11) is then

$$\frac{\rho_0 - \rho\,(R)}{\rho_0 - \rho\,(r)} = \frac{r}{R}\left[1 - \operatorname{erf}\left\{\frac{R - r}{2\,(Dt)^{\frac{1}{2}}}\right\}\right] \quad . \quad . \quad . \quad (12)$$

whence

$$\left(\frac{\partial \rho}{\partial R}\right)_{R=r} = (\rho_0 - \rho\,(r))\left[\frac{1}{r} + \frac{1}{(\pi D t)^{\frac{1}{2}}}\right] \quad . \quad . \quad (13)$$

In the steady state when the crystal is in equilibrium with the field, $(t \to \infty)$, Eq. (13) reduces to $(\partial\rho/\partial R)_{R=r} = (\rho_0 - \rho\,(r))/r$. The second term in the square brackets is therefore a measure of the enhancement of the concentration gradient near the crystal surface relative to the equilibrium value and this will be very marked if

$$(\pi D t)^{\frac{1}{2}} \ll r \quad . \quad . \quad . \quad . \quad (14)$$

Now $(\pi Dt)^{\frac{1}{2}}$ has the dimensions of length and will be intimately related to the thickness of the diffusion boundary layer surrounding the crystal.

If we now consider a spherical crystal of radius r falling with a velocity v relative to the air, the time t for which any given parcel of air will remain in contact with it before being replaced by fresh air will be given approximately by $t = 2r/v$; hence the diffusion field will have approximately this time in which to spread out from the crystal surface. The condition for the concentration gradient near the crystal to be considerably augmented because of the fall motion is therefore

$$2\pi D r/v \ll r^2$$

or
$$vr \gg 2\pi D \quad . \quad . \quad . \quad . \quad (15)$$

From Eq. (15) we see that the crystal can be regarded as growing at rest in a steady-state field if $vr/D < 2\pi$, when Eqs. (8), (9) and (10) may be applied to the growth rate without appreciable error.

Let us now consider a falling hexagonal plate, its maximum cross-section being, on average, normal to the direction of fall. The diffusion field will be limited to a boundary layer and the larger concentration gradients will be directed towards the corners which protrude farther into the field. Vapour will therefore arrive preferentially at the corners, the effect being more marked when the boundary layer is narrow. Furthermore, the largest temperature gradients will be directed away from the corners, enabling the latent heat of crystallization to be dispersed more rapidly at these points than from the rest of the surface – an additional factor in their preferential development.

Derivation of a complete mathematical criterion for the transition from polyhedral to dendritic growth would require a solution of the diffusion equation analogous to Eq. (13) for a hexagonal plate crystal but attempts to obtain a solution for even a circular plate have so far proved unsuccessful. The thickness of a boundary layer at a time t after introducing the crystal into the field will, however, be proportional to $(Dt)^{\frac{1}{2}}$ and from analogy with the sphere we may conclude that enhancement of the concentration gradients towards the corners of the crystal will be much more marked than in the steady-state if the diffusion boundary layer is thin compared with the crystal radius (r). The criterion for the onset of dendritic growth will therefore be of the form

$$\beta Dt \ll r^2 \quad . \quad . \quad . \quad . \quad . \quad (16)$$

where β is an undetermined numerical factor. Using a similar argument to that given above for a falling sphere we may write $t \simeq x/v$ where x is the crystal thickness when the criterion becomes

$$vr^2 \gg \beta Dx \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Measurements on crystals in the early stages of growth (see Section 6) give values of $x \simeq 20\,\mu$; putting $\beta = \pi$ (as for the sphere), one deduces that pronounced dendritic growth leading to well-developed stellar crystals will not set in until the crystal radius exceeds $100\,\mu$, which is in good agreement with observation.

Once the crystal corners start to sprout, the diffusion field will organize itself to become convergent towards the growing tips and dendritic growth will tend to continue while the supply of vapour lasts.

4. Experiments on the growth of crystals in a supercooled cloud

A number of experiments in the temperature range 0°C to − 20°C were conducted in a series of cold chambers (about 12 × 8 × 8 ft) at the Low Temperature Research Station, Cambridge, each chamber being thermostatically controlled at a fixed temperature. A supercooled cloud was formed by introducing steam from an electric boiler. Within a few seconds the fog cooled to the temperature of the chamber and was then seeded with dry ice. To prevent overseeding, and hence rapid transformation of the water cloud into a pure ice cloud, the dry ice was packed into a small cardboard tube closed except for a small hole at one end. A single rapid passage of the tube through the cloud was found to produce sufficient crystals, which were collected on glass slides coated with a thin film of Formvar solution. Plastic replicas of the crystals were made as described by Schaefer (1946) for subsequent photography and measurement. In order to obtain growth rates, slides were exposed for 5 sec at intervals of 15 sec until the water cloud disappeared, i.e., after some three minutes. To study the effect of growing crystals at lower supersaturations experiments were made in which seeding was delayed until the water cloud had almost disappeared, so that for the most part the crystals grew at sub-saturation with respect to liquid water.

Similar experiments have been continued down to − 40°C in a thermostatically-controlled cold box of about 2 ft^3 capacity, the temperature being measured with a thermocouple. In order to avoid a non-uniform temperature distribution in the experimental space an inner box of aluminium was placed inside the chamber leaving a 1 cm gap on all sides. The inner walls of the box bounding the experimental space were blackened and coated with glycerol to prevent frost formation. A supercooled cloud was produced by evaporating water into the chamber and minute quantities of dry ice were again used as the seeding agent. The crystals were caught at 20-sec intervals on glass slides coated with Formvar solution, the replicas being photographed and their dimensions measured as described in Section 6.

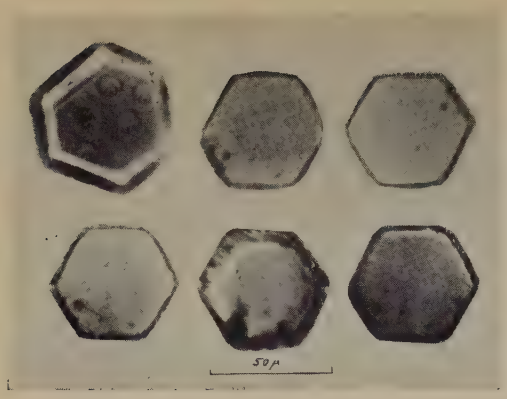

Figure 2. Simple hexagonal plates (— 2·5°C).

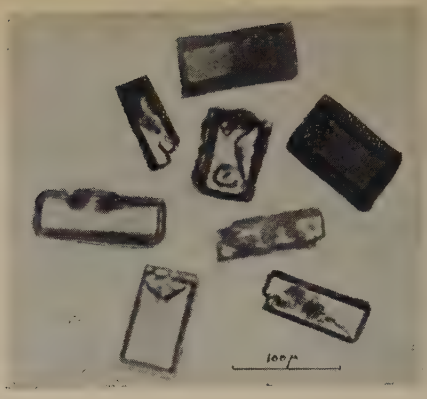

Figure 3. Hexagonal prisms (— 8°C).

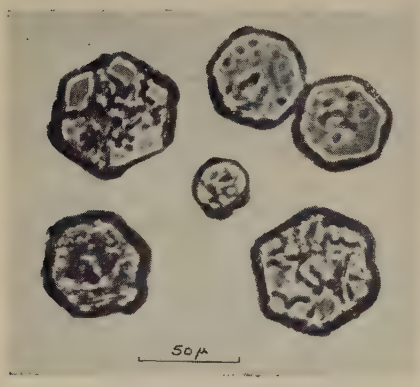

Figure 4. Hexagonal plates with surface markings (— 13°C).

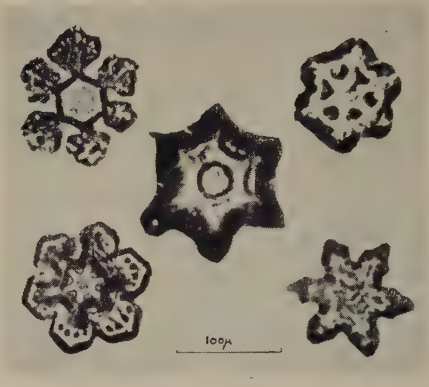

Figure 5. Plates showing sprouting of corners (— 16°C)

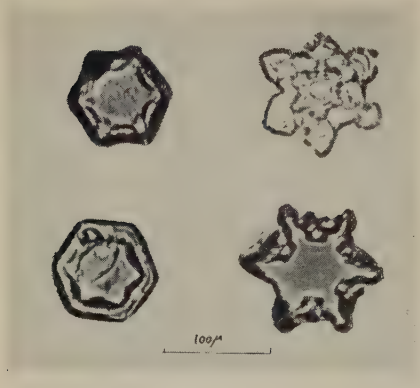

Figure 6. Plate crystals (— 20°C).

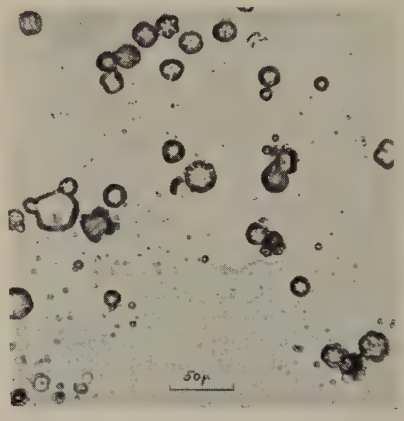

Figure 7. Small hexagonal prisms (— 39°C).

5. The effect of temperature and supersaturation on crystal habit

The predominant habit of crystals grown in a water-saturated environment shows a marked dependence on the temperature and hence on the supersaturation prevailing during the growth period. As shown in Table 1 and Figs. 2-7 (Plate II) a double transition between the hexagonal plates, (preferential growth along the *a* axes), and hexagonal prismatic columns (preferential growth along the *c* axis), occurs in the temperature range 0°C to − 40°C. This double transition has also been found by aufm Kampe, Weickmann and Kelly (1951) and also appears to be a general feature of natural atmospheric ice crystals.

TABLE 1. Change of crystal habit with temperature between 0°C and − 40°C

Temperature	Crystal habit	Remarks	Figure
0°C to − 5°C	Simple hexagonal plates	Plates show no ribs or surface markings. Some have 3 shorter and 3 longer sides.	2
− 5°C to − 10°C	Complete and hollow hexagonal prisms	Some crystals show marked cavities at both ends.	3
− 10°C to − 25°C	Hexagonal plates, stellar and dendritic crystals	Plates show ribs and other surface markings with tendency to sprout at corners. Stars most prominent at about − 15°C.	5 and 6
− 25°C to − 40°C	Hexagonal prisms	Short prisms in which cavities are absent or only very small.	7

The thin, clear plates formed between 0°C and − 5°C showed no tendency to sprout at the corners, indicating that the excess of molecules arriving there was removed rapidly by diffusion to other parts of the crystal surface. The absence of ridges and other surface markings was also evidence of the efficiency of surface diffusion at these temperatures in smoothing out local concentrations of the condensate. The development of the 'triangular' crystal (Fig. 2) may be brought about by three faces of an originally regular hexagonal plate growing faster than the three opposite faces; this may occur as a result of a net transport of molecules over the surface across a diagonal.

There is a rather sharp transition from plate crystals to hexagonal prisms at about − 15°C and an equally sharp reversion to plates at about − 10°C. The second transition to prisms below about − 25°C is much less sharp. The factors which are responsible for this double change of crystal habit in such a narrow range of temperature are not known but these observed transitions suggest that the difference between the activation energies for evaporation and surface diffusion of molecules for the basal and prism planes may be strongly dependent on temperature.

The hexagonal plates occurring at temperatures below − 10°C are quite distinct in appearance from those occurring above − 5°C. The development of ribs and other surface markings, and sprouting of the corners to give rise to stars, indicate that non-uniformities in the arrival of vapour over the crystal surface cannot be entirely smoothed out by surface diffusion. Dendritic growth is most marked at about − 15°C when the arrival of material is a maximum.

The prisms grown at temperature below − 25°C in the small cold box were nearly isometric and often fell end-on in the plastic to give the appearance of small hexagonal plates in the photographs (Fig. 7). They did not exhibit the well-marked cavities often found in atmospheric crystals at the same temperature but in view of their very restricted growth in the small cold box their features are probably not typical of crystals grown under more realistic conditions.

Crystals grown at sub-water saturation for the greater part of their lives were of the same general habit as their counterparts grown in a water cloud but their structure was generally less complex. Development of ribs and corners was much less pronounced than for crystals grown at the same temperatures at water saturation. Similar observations are reported by aufm Kampe *et al.*

6. Measurements on the growths of crystals

One of the objects of the experiments described in Section 4 was to measure the rate of growth of crystals in supercooled-water cloud and to compare the results with the values predicted by Eqs. (9) and (10). Some measurements have been made on the simple hexagonal plates occurring in the temperature range 0° to − 5°C and on prisms found between − 5°C and − 10°C. On a slide exposed for say, 10 sec, there was an appreciable spread of crystal sizes, probably because of small-scale inhomogeneities in the parent cloud, interference between neighbouring crystals, different growth periods for different crystals and variation in the size of the drops from which the crystals originated. It was therefore decided to select only the largest crystals (about 10 per cent of the total) and assume these to be the last to hit the slide.

(*a*) *Measurements on plates*

The simple plates observed above − 5° were chosen because they showed no signs of sprouting at the corners, their conditions of growth then corresponding fairly closely to those assumed in the theory. Also, their surfaces were flat, making it easier to determine their mass from their linear dimensions than would have been the case for the more complex structures occurring between − 10°C and − 25°C. The diameter of the inscribed circle of the hexagon was measured accurately using a 2 mm objective and a 10 × micrometer eyepiece. The thickness of the crystals was determined by focusing on the top and the bottom of a replica and reading the displacement on the calibrated fine-adjustment of the microscope. The mass of about 50 of the largest crystals on each slide was calculated from their linear dimensions and a mean value computed for each slide, i.e., for each growth period. It was therefore possible to determine the increase in mass of a crystal in successive growth periods; these experimental values are compared with those computed from the integrated form of Eq. (9) in Table 2.

TABLE 2. Experimental and theoretical growth rates of hexagonal plate crystals grown at − 2·5°C in a water cloud

Time t (sec)	r (μ)	Thickness x (μ)	Observed mass m (10^{-8} g)	Δm (10^{-8} g)	Δm from Eq. (9)* (10^{-8} g)
20	15	13	0·9		
				0·5	0·34
40	18·5	15	1·4		
				1·2	0·82
80	22	18	2·6		
				1·5	0·95
120	25	20	4·1		

* *Note added in proof.* Eq. (9) is valid only for very thin plates whereas the measured crystals have thicknesses comparable to their diameters. If these are treated theoretically as *oblate* spheroids of capacity $(r^2 - x^2/4)^{1/2}/\cot^{-1}\{x/(4r^2 - x^2)^{1/2}\}$ the computed values of Δm corresponding to those in column 6 of Table 2 become 0·42, 1·0 and 1·2 × 10^{-8} g respectively. These are in closer agreement with the measured values.

(b) *Measurements on prisms*

The masses of the crystals were determined from their linear dimensions as measured with the micrometer eyepiece; the values obtained are compared with those computed from Eq. (10) in Table 3.

TABLE 3. EXPERIMENTAL AND THEORETICAL GROWTH RATES OF PRISMATIC CRYSTALS GROWN AT — 5°C IN A WATER CLOUD

Time t (sec)	a (μ)	b (μ)	ξ	Observed mass m (10^{-8} g)	Δm (10^{-8} g)	Δm from Eq. 10 (10^{-8} g)
40	22·5	12·5	·56	1·7		
					2·5	2·1
80	36·5	15·5	·42	4·2		
					2·8	2·7
120	51	17	·33	7·0		
					2·8	3·1
160	62	18	·29	9·8		

Because measured values of the parameters r, a and $\phi(\xi)$ have to be substituted in Eqs. (9) and (10) in order to calculate values of dm/dt, the measurements of the growth rate do not provide an entirely independent check of the theory. However, the values in Table 3 show that for the prisms where both a and b could be measured accurately, theory and measurement are mutually consistent to a fair degree of accuracy. In the case of the plates the agreement between the computed and measured values of mass increase is not so good; this may be attributed to considerable uncertainty in measuring the crystal thickness by the focusing technique. New methods are now being developed to measure this parameter more accurately and to make measurements on much greater numbers of crystals.

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Condensation nuclei and the development of radiation fog

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Summary

The growth of liquid-water droplets upon condensation nuclei as the relative humidity rises from 80 per cent to 100 per cent is examined. It is assumed that the condensation nuclei consist of sea-salt particles and that the droplets are always in instantaneous equilibrium with the ambient relative humidity. An arbitrary nucleus-size distribution formula, which is easily adjusted to fit an intuitively reasonable size distribution, is also assumed. It is found that the shape of the nucleus-size distribution curve has very little effect upon the computed value of the liquid-water content W of the fog though at saturation W varies as the square root of the mean mass of the nuclei. In near-saturation conditions a small variation in relative humidity has a big effect upon W.

A formula for drop-size distribution in fog at saturation is developed and found to agree with measurements.

Visibility in the developing fog is briefly considered and it is shown that Koschmieder's formula, applied to the growing water drops, leads to results consistent with experience. Neither the mean nucleus mass nor the shape of the nucleus-size distribution curve affect the visibility at saturation.

1. Introduction

The investigation described in this paper was undertaken following some discussion as to whether synoptic processes are predominant in the formation of radiation fog or whether the size and size distribution of the nuclei upon which the fog droplets form may not exert a significant influence.

Fog results when a sufficient number of particles, liquid or solid, are suspended in the air. The solid particles are likely to be of local origin and to have local significance only and so will not be further considered here. The liquid particles result from the condensation of water vapour upon hygroscopic nuclei. These hygroscopic nuclei may have an industrial origin or may derive from the oceans. The balance of evidence seems to favour the view that the nuclei of fog and cloud droplets consist of sea salt and in the following paragraphs we shall accept this view. It is unlikely that the neglect of industrial nuclei will have any serious effect since, as Wright (1939) has pointed out, in the presence of salt nuclei combustion nuclei will not grow to significant sizes.

Condensation on hygroscopic nuclei occurs at relative humidities very substantially below 100 per cent. As the relative humidity rises the droplets increase in size, the rate of growth following a well known pattern (see, e.g., Simpson 1941). While the relative humidity is below a critical value (which exceeds 100 per cent by a very small amount dependent upon the mass of the nucleus) the droplets can attain a size such that they are in equilibrium with the ambient vapour pressure and growth then ceases until the relative humidity increases again. After the relative humidity exceeds the critical value the growth of the drops continues indefinitely even though the relative humidity does not continue to increase, but we shall not here consider that stage in the development of the fog. We shall show that while the drops are small and the relative humidity is not too near the critical value the drops grow very rapidly to the size which is in equilibrium with the ambient vapour pressure and so, for a specified

relative humidity below the critical value, we may always consider the droplets to be in equilibrium with the ambient conditions. Then we shall compute the equilibrium drop-size as a function of the mass of the nucleus and the ambient relative humidity. If the number and size distribution of nuclei are defined we can then compute the liquid-water concentration in the air as a function of the relative humidity.

2. The symbols used

The following symbols have been used :

p' = vapour pressure over a drop of salt solution (dynes cm^{-2}).
p_θ = saturation vapour pressure at temperature θ (dynes cm^{-2}).
θ = temperature (°A).
r = drop radius (cm).
ρ_1 = drop density (g cm^{-3}).
T = surface tension (74 dynes cm^{-1} for water).
M = molecular weight of water (18).
R = gas constant per mol ($8{\cdot}314 \times 10^7$ erg deg^{-1} mol^{-1}).
m = mass of nucleus in one drop (g).
$\Delta\theta$ = excess of droplet temperature over ambient temperature (°A).
λ = latent heat of evaporation (cal mol^{-1}).
D = coefficient of diffusion of water vapour in air (cm^2 sec^{-1}).
k = thermal conductivity of air (cal cm^{-1} sec^{-1} deg^{-1}).
$100H$ = relative humidity.
ψ = ratio of vapour pressure over drop to saturation-vapour pressure at drop temperature.
$P = 4\,TM/\rho_1 R\theta$.
a, b = constants in relation between drop size and nucleus mass for a specified relative humidity (Eq. 8).
r_c, H_c = critical values of drop radius and relative humidity for a specified nucleus mass.
S = total mass of salt per unit volume of air (g cm^{-3}).
F = fraction of S comprised by nuclei smaller than m.
m_0, q = constants in assumed nucleus-size distribution formula (Eq. 9).
$j\,dm$ = number of nuclei per unit volume with mass between m and $m + dm$.
$f\,dr$ = number of drops per unit volume with radius between r and $r + dr$.
W = liquid-water concentration in fog (g cm^{-3}).
V = visibility (cm).
σ = extinction coefficient of atmosphere (cm^{-1}).
K = scattering coefficient of a water drop.

3. Rate of growth of drops at relative humidities below 100 per cent

From Best (1951a) we see that the equations governing the growth of a single drop can be written

$$\frac{p'}{p_{\theta+\Delta\theta}} = \exp\,(P/2r) - \frac{m}{6r^3} = \psi \quad \text{(say)} \quad . \quad . \quad (1)$$

$$\Delta\theta = \frac{\lambda D}{R\theta k}\,(Hp_\theta - p') \quad . \quad . \quad . \quad . \quad (2)$$

$$r\frac{dr}{dt} = \frac{MD}{R\theta}(Hp_\theta - p') \quad . \quad . \quad . \quad . \quad (3)$$

Provided $\Delta\theta$ is small we easily get from these equations

$$r\frac{dr}{dt} = A(H - \psi) \quad . \quad . \quad . \quad . \quad (4)$$

$$A = \frac{MDkp_\theta}{R\theta k + \lambda D\psi\, dp_\theta/d\theta} \quad . \quad . \quad . \quad (5)$$

and inserting numerical values appropriate to 283°A we find

$$A = \frac{1{\cdot}49 \times 10^{-6}}{0{\cdot}676 + \psi}$$

Eq. (4) cannot be solved explicitly if H is a function of time. If, however, we assume that the drops are initially in equilibrium with the ambient vapour pressure and that a sudden jump occurs in H to a new steady value we can compute graphically the time required for the drops to grow by a specified fraction of the difference in the two equilibrium volumes. This has been done for six cases and the results are shown in Table 1.

TABLE 1. RATE OF GROWTH OF DROPS FOR A SUDDEN CHANGE IN HUMIDITY AT HUMIDITIES BELOW SATURATION

Initial humidity (per cent)	Final humidity (per cent)	Time (sec) for drop volume to increase by 90 per cent of the difference between the initial and final equilibrium values	
		$m = 10^{-14}$ g	$m = 10^{-12}$ g
75	80	0·0016	0·03
85	90	0·004	0·1
95	100	15	5200

We see that the assumption of instantaneous equilibrium is amply justified except for the largest nuclei in near-saturation conditions and shall accordingly assume such equilibrium in what follows. If the drops are in equilibrium $\Delta\theta$ must be zero and in Eq. (1) we can replace $p_{\theta+\Delta\theta}$ by p_θ.

4. DROP SIZE AS A FUNCTION OF THE NUCLEUS SIZE AND RELATIVE HUMIDITY

With the assumption of equilibrium we now write

$$H = \frac{p'}{p_\theta} = \exp(P/2r) - \frac{m}{6r^3} \quad . \quad . \quad . \quad (6)$$

After inserting numerical values we find $P = 6{\cdot}408 \times 10^{-5}/\theta$. Since r is at least 10^{-4} cm we see that $P/2r$ is small and the exponential term in Eq. (6) may be expanded, second and higher powers of $P/2r$ being neglected. We thus get

$$m = 6r^3(1 - H) + 3r^2 P \quad . \quad . \quad . \quad . \quad (7)$$

If now log r is plotted against log m for specific values of H and using Eq. (7) and a value of P appropriate to a particular temperature, it will be seen that the resulting

curves are almost straight lines. Thus to a very close approximation m can be expressed as a function of r, by an equation of the form

$$m = ar^b \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where a and b are constants over limited ranges of r with values which depend upon H and, to a small extent, upon the temperature θ. This procedure was adopted, with an assumed value of 283°A for the temperature θ giving $P = 2{\cdot}264 \times 10^{-7}$. The values found for a and b are shown in Table 2.

The fact that it was not necessary to subdivide the range of r from 1 to $16{\cdot}8 \times 10^{-5}$ cm results from the closeness of the curve connecting $\log r$ and $\log m$ to a straight line. The values of a and b for $H = 1{\cdot}0$ can, of course, be deduced from Eq. (7). It will be observed that the values of a and b change rapidly with H near saturation. If the drops are not in instantaneous equilibrium with the ambient air the computed drop size may thus be greater than obtains in practice.

TABLE 2. NUCLEUS MASS AS A FUNCTION OF DROP RADIUS. VALUES OF CONSTANTS IN THE FORMULA $m = ar^b$

H	0·8	0·9	0·95	0·99	0·999	1·0
Range of r (cm $\times 10^{-5}$)			Value of a			
0·1 to 1·0	$1{\cdot}52 \times 10^{-1}$	$2{\cdot}04 \times 10^{-3}$	$3{\cdot}02 \times 10^{-3}$	$1{\cdot}70 \times 10^{-5}$	$1{\cdot}35 \times 10^{-6}$	$6{\cdot}79 \times 10^{-7}$
1·0 to 16·8	1·014	$4{\cdot}27 \times 10^{-1}$	$1{\cdot}62 \times 10^{-1}$	$6{\cdot}92 \times 10^{-3}$	$2{\cdot}14 \times 10^{-3}$	$6{\cdot}79 \times 10^{-7}$
> 16·8	1·20	$6{\cdot}03 \times 10^{-1}$	$3{\cdot}02 \times 10^{-1}$	$4{\cdot}57 \times 10^{-3}$	$1{\cdot}58 \times 10^{-3}$	$6{\cdot}79 \times 10^{-7}$
			Value of b			
0·1 to 1·0	2·82	2·70	2·59	2·23	2·05	2·00
1·0 to 16·8	2·98	2·96	2·93	2·75	2·30	2·00
> 16·8	3·00	3·00	3·00	2·96	2·80	2·00

Big changes in drop size occur at near-saturation humidities. At saturation $H = 1$ and from Eq. (7), $r^2 = m/3P$. The critical size of the droplet, i.e., the size at which further increase in H leads to indefinite growth, is obtained by differentiating the right-hand side of Eq. (6) with respect to r and equating to zero. The exponential term in the resulting equation can be expanded and second and higher powers of P/r neglected and we then obtain $r_c^2 = m/P$, the corresponding value of H being $H_c = 1 + P/3r_c$. Putting $P = 2{\cdot}26 \times 10^{-7}$ and $m = 10^{-12}$ g (for example) we get $r_c = 2{\cdot}1 \times 10^{-3}$ cm and $H_c = 1{\cdot}000036$. Thus the equilibrium drop-radius increases in the ratio $3^{\frac{1}{2}}$ as the relative humidity rises from 100 per cent to the critical value 100·0036 per cent. For other nucleus sizes the ratio remains the same but the critical value of H changes.

5. THE SIZE DISTRIBUTION OF NUCLEI AND OF THE RESULTING DROPS

Having related drop size to nucleus size and humidity the next step is to evaluate the drop-size distribution in terms of the nucleus-size distribution. There appear to be no data available about the size distribution of salt nuclei in the atmosphere over land and so some suitable assumption must be made. The type of formula adopted has been found convenient in other connections and gives a distribution curve which

has an intuitively reasonable shape. We assume that the fraction F of the total salt S per unit volume which is comprised by nuclei having mass less than m is given by

$$1 - F = \exp\left[-(m/m_0)^q\right] \quad . \quad . \quad . \quad (9)$$

where m_0 and q are constants. Then it is easily seen that the number of nuclei per unit volume of air with mass between m and $m + dm$ is given by

$$j\,dm = \frac{Sq}{m_0{}^q}\, m^{q-2} \exp\left[-(m/m_0)^q\right] dm \quad . \quad . \quad . \quad (10)$$

The size distribution of drops resulting from this size distribution of nuclei can now be deduced. Since r is a function of m the number of drops with radius between r and $r + dr$ can be written $f\,(dr/dm)\,dm$. This must be equal to $j\,dm$ so that

$$f = j\frac{dm}{dr} \quad . \quad . \quad . \quad . \quad . \quad (11)$$

From Eqs. (7), (10) and (11) it follows that

$$f = \frac{Sq}{m_0{}^q}\, m^{q-2}\, 6\left[3r^2(1-H) + Pr\right] \exp\left[-(m/m_0)^q\right] \quad . \quad . \quad (12)$$

6. The computation of liquid-water content

The liquid-water content W per unit volume of air is given by

$$W = \frac{4\pi}{3}\int f r^3\, dr$$

the integration being taken over all the drops present. In this integral f is given by Eq. (12) in which m can be replaced by ar^b. After these substitutions we get

$$W(r_1, r_2) = \frac{8\pi Sq\, a^{q-2}}{m_0{}^q}\int_{r_1}^{r_2} r^{3+bq-2b}\left[3r^2(1-H) + Pr\right] \exp\left[-(ar^b/m_0)^q\right] dr \quad . \quad (13)$$

for the contribution to W by drops with radius between r_1 and r_2. If r_1 and r_2 are made to coincide with the limits of the ranges of r indicated in Table 1 the quantities a and b (and of course P for a fixed temperature) will be constant over the range of integration. By a suitable change of variable the right-hand side of Eq. (13) can be put in the form of the sum of a number of Incomplete Gamma functions and so can be evaluated when numerical values are assigned to m_0, q, S and H.

It is convenient to quote here, for subsequent use, a formula which easily follows from Eq. (13) after suitable change of variable, viz.,

$$W(0, r) = \frac{8\pi SP}{b}\,\frac{m_0{}^{\alpha q}}{a^{5/2}}\int_0^x y^\alpha e^{-y}\, dy \quad . \quad . \quad . \quad (14)$$

where

$$x = \left(\frac{ar^b}{m_0}\right)^q, \quad \alpha = \frac{1}{q}\left(\frac{5}{b} - 2\right)$$

Before starting the actual computations it was necessary to consider the numerical values of m_0, q, S and H. Fortunately, as can be seen from Eq. (13), W is directly proportional to S and the problem could thus be simplified to the computation of W/S. Wright (1940) has provided a summary of measurements of the size of sea-salt nuclei from which it appears that the mean mass of the nucleus may vary from 10^{-14} g to 10^{-12} g. These two values were adopted for the computations. There was no

guide from published measurements to suitable values for q and accordingly the values 1·1 and 3·0 were selected arbitrarily for the computations. The smaller of these values was selected in preference to unity because $q = 1$ implies that the total mass of salt represented by particles of a specified size increases continuously as the specified size decreases, an implication which seems intuitively unreasonable. As will be seen below (Table 3) four values were selected for H but owing to the labour involved in the computations two values only were used except for one value of q and one value of m_0.

One further point merits attention before we consider the results of the computations. If the nuclei are of uniform size m we easily see that

$$W = \frac{4\pi S r^3}{3m}$$

where the value of r is computed to correspond to m and the appropriate value of H. We saw in Section 4 that at 100 per cent relative humidity $m = 3Pr^2$ so W will vary as the square root of m.

7. Results of computations of liquid-water content

The results of the computations described above are given in Table 3.

TABLE 3. Liquid-water concentration W as a function of size and size distribution of nuclei and of relative humidity

m_0		10^{-14} g			10^{-13} g	
q	1·1	3·0	uniform	1·1	3·0	uniform
H			$W/8\pi S$			
0·80	0·13	0·13	0·13	0·13	0·13	0·14
0·90	—	—	0·26	—	—	0·27
0·95	—	—	0·49	—	—	0·54
0·99	—	—	2·34	2·57	—	2·63
0·999	—	—	12·20	23·0	—	24·1
1·0	26·4	27·6	28·6	264	276	298

From Table 3 we may draw the following conclusions :

(*a*) Variation in q, i.e., in the shape of the nucleus-size distribution curve, has very little effect, particularly at high humidities.

(b) At low humidities (99 per cent or less) the amount of condensed water is influenced only slightly by the mean nucleus size (with S constant) but at saturation the amount of condensed water varies as the square root of the nucleus mass.

(c) With a large number of small nuclei the change in W as the relative humidity increases from 99·9 per cent to 100 per cent is not very great, but with a small number of large nuclei the corresponding change is very great.

It appears (see for example Wright 1940 a and b) that experimental measurements of S vary over a wide range of values but that a reasonable value is about 10^{-11} g cm^{-3}. At 100 per cent relative humidity we may take $W/8\pi S = 28$ or 280 depending upon whether the mean nucleus mass is 10^{-14} g or 10^{-12} g. These values give liquid-water concentrations of 0·007 g m^{-3} and 0·07 g m^{-3} for the small and large nuclei respectively. Such liquid-water concentrations are rather small when compared with measurements but, of course, apply to conditions when the relative humidity has just reached 100 per cent.

8. Drop-size distribution at 100 per cent relative humidity

At saturation we see from Eq. (7) that $a = 3P$ and $b = 2$. Then from Eq. (14) the fraction F_1 of liquid water in the atmosphere comprised by drops with radius less than r is given by

$$F_1 = \frac{\int_0^{u(1+1/2q)^{1/2}} t^{1/2q} \exp(-t)\, dt}{\Gamma(1 + 1/2q)} \quad . \quad . \quad . \quad (15)$$

where

$$u = \frac{(3Pr^2/m_0)^q}{(1 + 1/2q)^{1/2}}$$

Elsewhere (Best 1951b) an expression for F_1 somewhat similar to that given by Eq. (9) has been deduced from published measurements for drop-size distribution in fog. Although these measurements were not made at exactly 100 per cent relative humidity it is of interest to compare the size distribution of fog droplets given by Eq. (15) with that found from these measurements. Now there is in practice little difference between F_1 as given by Eq. (15) and by

$$F_1 = 1 - \exp\left[-(u/b_1)^{1+1/4q}\right] \quad . \quad . \quad . \quad (16)$$

provided b_1 is chosen to satisfy the relation

$$\log_{10} b_1 = \frac{0{\cdot}6}{4q + 1}$$

If q is infinitely large the two expressions for F_1 (Eqs. (15) and (16)) are identical, while Table 4 shows the differences for three other selected values of q and a few values of u.

TABLE 4. Comparison of values of F_1 computed from Eqs. (15) and (16)

q	5		1		0·5	
u	Eq. (15)	Eq. (16)	Eq. (15)	Eq. (16)	Eq. (15)	Eq. (16)
0·1	0·076	0·080	0·030	0·039	0·009	0·016
0·2	0·154	0·158	0·079	0·090	0·033	0·044
0·5	0·361	0·363	0·253	0·258	0·158	0·163
3·0	0·948	0·948	0·938	0·939	0·925	0·926

We see that, provided F_1 exceeds about 0·1, i.e., provided we are not concerned with the 10 per cent of water comprised by the smallest drops, the two formulae are very similar. The computations were repeated for five other values of u between 0·5 and 3·0 and the maximum difference between corresponding values of F_1 was about 4 per cent, for $q = 0{\cdot}5$. For the other values of q the maximum difference was about 2 per cent, the agreement being closest for the largest value of q. We may conclude therefore that, provided F_1 exceeds 0·1, Eq. (16) is a reasonable approximation to Eq. (15). Now after a little rearrangement Eq. (16) can be written

$$F_1 = 1 - \exp\left[-(r/a_1)^n\right] \quad . \quad . \quad . \quad (17)$$

$$n = 2q + 0{\cdot}5$$

$$a_1 = b_1^{1/2q}(1 + 1/2q)^{1/4q}(m_0/3P)^{1/2}$$

When q takes the values 1·1 and 3·0 the values of n are 2·7 and 6·5 respectively. The values of n found experimentally (Best 1951b) vary from 2·82 to 4·23 so that our arbitrary choice of these two values for q seems to have been fortunate. Similarly if we put $\theta = 283°\text{A}$, $P = 2{\cdot}264 \times 10^{-7}$ we find $a_1 = 1{\cdot}48\ \mu$ and $1{\cdot}25\ \mu$ for $q = 1{\cdot}1$ and 3·0 respectively when $m_0 = 10^{-14}$ g. When $m_0 = 10^{-12}$ g these values of a_1 must be multiplied by 10. The values of a_1 appropriate to the larger values of m_0 correspond very closely to the values based upon experimental measurements (Best 1951b) but the comparison must not be pressed since the theoretical results of this section apply to conditions at exactly 100 per cent relative humidity and the measurements may have been made in slightly different conditions.

9. The variation of visibility

We have so far considered only the variation in the liquid-water content of the fog as the relative humidity increases to 100 per cent. A somewhat similar treatment can be applied to the visibility in the fog though some additional uncertainties are introduced. It can be shown (see, e.g., Middleton 1941) that the visibility v is related to the extinction coefficient σ by the equation

$$V = \frac{3{\cdot}9}{\sigma} \quad . \quad . \quad . \quad . \quad . \quad (18)$$

where

$$\sigma = \pi \int Kr^2 f\, dr \quad . \quad . \quad . \quad . \quad (19)$$

and K is the scattering coefficient of a water drop. The precise value of the numerical factor in the right-hand side of Eq. (18) is subject to some uncertainty, particularly at visibilities less than a few hundred metres. In good visibility it is also necessary to take account of a contribution to σ by the air molecules. Wright (1939) quotes a value $1{\cdot}6 \times 10^{-7}\ \text{cm}^{-1}$ for this contribution. Houghton and Chalker (1949) have tabulated K as a function of the ratio of the drop radius to the wavelength of light. From this table values of K were interpolated for each of the wavelengths 4, 5, 6 and 7×10^{-5} cm and for a selection of values of r. The four values were then averaged for each value of r and a formula for K as a function of a power of r devised empirically to cover various ranges of drop size by varying the constants in the formula. Thus the value of σ could be computed in a similar manner to that adopted for computing W. The results of these computations showed that the variations in σ were in some respects similar to the variations in W. The value of σ (or rather that part of σ arising from the water drops) is directly proportional to S and is affected very little by the shape of the nucleus-size distribution curve. As with liquid-water content there is a significant increase in σ as the relative humidity increases from 99·9 per cent to 100 per cent but this is more marked with large nuclei (in the ratio of about 5 to 1 for $m_0 = 10^{-12}$ g) than with small nuclei (in the ratio of about 3 to 2 for $m_0 = 10^{-14}$ g). The reason for this is of course that at 100 per cent relative humidity the droplets with the larger nuclei are nearer to their critical size than are the droplets with small nuclei. In one respect however the variation of σ differs from that of W. At saturation the value of σ (with S constant) depends only slightly upon the mean nucleus mass. This is to be expected since we have seen that at saturation $r^2 = m/3P$ and thus, if the nuclei are of uniform size, we see from Eq. (19) that σ varies as Sr^2/m, i.e. as S/P, and this is independent of m.

The computations showed that at 80 per cent and 100 per cent relative humidity the values of $\sigma/6\pi S$ are about 8×10^3 and 5×10^5 respectively. Substituting $S = 10^{-11}$ g cm^{-3} we obtain $1{\cdot}5 \times 10^{-6}$ cm^{-1} and $9{\cdot}4 \times 10^{-5}$ cm^{-1} respectively for σ. Adding the contribution to σ owing to the air molecules, Eq. (18) then gives values of 23·5 km and 415 m for the corresponding visibilities. These are reasonable values but, of course, the visibility will deteriorate appreciably as the relative humidity rises very slightly above saturation and the droplets grow much bigger.

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A comparison of hemispheric circulations with particular reference to the western Pacific

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Summary

Using daily hemispheric synoptic charts for midwinter and midsummer months the distributions of pressure systems in the two hemispheres are compared. From mean pressure charts and charts of 'mean streamlines,' meridional cross-sections of mean zonal wind have been constructed for the western Pacific for the mid-months of the four seasons. The main features of the circulation in the Australian region are discussed and comparisons are made between the circulations of the hemispheres.

1. Introduction

In studying the atmospheric circulation over an area such as the Australian region it is desirable that the manner in which such a circulation fits into the broader pattern of hemispheric or world-wide circulation be known. Also the similarity between circulations in the two hemispheres should be investigated so that the possibility of applying results from the denser observational network of the northern hemisphere to a study of the southern hemisphere may be ascertained.

For a comprehensive picture of current ideas regarding the circulation in the Australian region and references to relevant literature the reader is referred to papers by Court (1951), Gibbs, Gotley and Martin (1952), and Palmer (1951, 1952).

Gibbs *et al.*, after consideration of the findings of other workers and experience with synoptic analysis over the Southern Ocean, concluded that in the vicinity of Australia :

(i) the mid-latitude anticyclones generally exhibit a regular eastward movement . . . ;
(ii) higher latitude "cold" anticyclones move generally to the north-east and may develop into "warm" type systems;
(iii) the great depressions of the Southern Ocean have their origin in latitudes lower than 40°S on the majority of occasions and may form as frontal waves or non-frontal depressions;
(iv) at latitudes lower than 40°S these depressions vary considerably in their direction of movement, but in higher latitudes the direction is mainly to the south-east or east.

Court (1951) concluded from a study of literature and available data for Antarctica :

The planetary pattern causes pressures to be lower around 60°S than at the pole . . . But in the upper air . . . on the average Antarctica is overlain by a deep warm semi-permanent anticyclone in early summer . . . by midwinter . . . the tropopause disappears. Cooling . . . (creates) an upper-level cyclone which deepens as the cooling progresses.

Palmer (1951), on the subject of the circulation in low latitudes in the Pacific, states :

. . . in the central Pacific, the trade winds of the two hemispheres lie side by side . . . forming an extensive easterly current . . . This easterly current is subject to perturbations of the same type as the easterly waves of the Caribbean . . . as the waves move towards the west, some of them at least become unstable, growing in amplitude . . . With the formation (of a small vortex) in the wave pattern, westerly winds near the equator appear for the first time.

2. Some general hemispheric comparisons

Petterssen (1950) has suggested that the general circulation consists essentially of a steady low-latitude circulation, a high-latitude circulation from which vorticity must be exported for steady conditions, and a mid-latitude circulation dominated by travelling anticyclones and cyclones in which a separation of vorticity occurs such that the cyclonic eddies are shed towards higher and anticyclonic eddies shed towards lower latitudes, thus contributing to the maintenance of the semi-permanent sub-polar belt of low pressure and of the sub-tropical belt of high pressure. These features should be equally evident in both hemispheres, but Petterssen points out that these major systems are modified considerably in detail by geographical features. He shows that mountain ranges in the northern hemisphere are regions of cyclogenesis as might be expected from considerations of generation of vorticity, also that the transfer of vorticity across boundaries between land and sea surfaces of different temperatures is responsible for the frequency of occurrence of cyclones and anticyclones over particular areas, the sub-polar semi-permanent lows and the low-latitude heat lows being examples of the effect of the disposition of land and sea on the mean circulation.

The Southern Hemisphere Project of the Department of Meteorology of M.I.T. directed by Willett is preparing daily southern hemisphere maps which will provide an excellent means of comparison of the circulation of the two hemispheres when used in conjunction with northern hemisphere maps. However, the analyses as yet are not available for a sufficiently lengthy period to permit an elaborate comparison of conditions in the two hemispheres. Willett (1949), using mean charts prepared by Serra, computed indices of the strength of the westerly and easterly currents in the two hemispheres from mean hemispheric pressure profiles, with the following results :

		Summer		Autumn		Winter		Spring	
		lat.	m/sec	lat.	m/sec	lat.	m/sec	lat.	m/sec
MZW	N. hemis.	53	1·10	55	2·00	54	2·25	51	1·00
	S. hemis.	50	5·60	50	6·25	50	7·10	50	6·30
MSE	N. hemis.	25	1·25	26	3·30	20	5·60	20	3·50
	S. hemis.	22	3·10	18	5·00	13	8·15	18	4·90

In this table MZW refers to the geostrophic west wind computed from the maximum slope of the pressure profile in middle latitudes and MSE to the geostrophic east wind computed from the maximum slope of the pressure profile in sub-tropical latitudes. The figures apply to the appropriate seasons in the respective hemisphere.

The fact that a current of sinusoidal form will have a smaller zonal component than a zonal stream of the same speed may be partly responsible for the considerable differences in the values of the west wind in the two hemispheres, but there seems no doubt that the southern hemisphere mean surface circulation is stronger than that in the northern hemisphere.

Fig. 1 gives a comparison of the mean pressure profiles for the two hemispheres, data having been obtained from the Extended Forecast Section of the Department of Meteorology, M.I.T. and being based on U.S. Weather Bureau normals for the northern hemisphere and Serra's data for the southern hemisphere. It will be seen that the profiles are similar at latitudes lower than 30°, but that at higher latitudes the southern hemisphere profile departs considerably from that for the northern hemisphere in all seasons. As previously mentioned, this difference is likely to be partly due to the greater amplitude of the perturbations in the mean westerlies in the northern hemisphere.

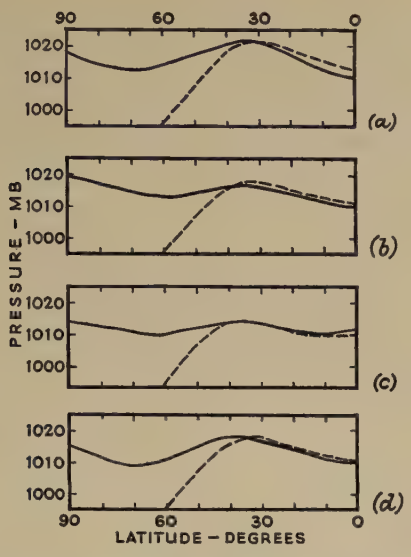

Figure 1. Mean hemispherical pressure profiles. Solid line – northern hemisphere; broken line – southern hemisphere. (*a*) winter, (*b*) spring, (*c*) summer, (*d*) autumn.

In order to investigate the nature of the surface circulations in more detail and to find out more about the hemispherical contrasts and similarities an analysis of the distribution of pressure systems during the midsummer and midwinter months has been made for the two hemispheres. The U.S.W.B. Historical Weather Map series was used for the northern hemisphere and the synoptic analyses of the M.I.T. Southern Hemisphere Project for the southern hemisphere. The charts used were those for January and July 1949 for the northern hemisphere and those for July 1949 and January 1950 for the southern hemisphere. The location and central value of all pressure systems having a closed isobar were noted. Because of the nature of the analyses and the 5-mb pressure interval used this analysis must be considered inadequate for latitudes lower than 20°.

3. The cyclones – midwinter

Figs. 2 and 3 show the locations of the cyclones during the midwinter month for the northern and southern hemispheres, 327 positions appearing for the northern and 344 for the southern hemisphere. A comparison of the two figures emphasizes the greater symmetry of distribution of cyclone centres about the pole in the southern hemisphere. In the northern hemisphere there are definite regions with a high frequency of occurrence but in the southern hemisphere the distribution is relatively independent of longitude. The southern hemisphere pattern, however, does seem to indicate certain favoured cyclone tracks – from the east coast of S. America, from the vicinity of the southern coastlines of Australia and S. Africa and from the central South Pacific. Another remarkable feature is the lack of cyclones at latitudes higher than 70°, in contrast to the distribution in the northern hemisphere. This may be due in part to the lack of observations over the Antarctic continent, but it is believed to represent the true picture. As Antarctica is however believed to have an elevation of about 10,000 ft over most of its area, a discussion of surface (sea level) pressure systems south of 70°S is somewhat hypothetical. Another noteworthy point is the slightly increased density at 60°S in the vicinity of longitudes 80°E and 160°E.

A more quantitative picture of the variation of frequency of occurrence with latitude is given by Fig. 10. The frequency represented here is that for 5° lat. intervals, multiplied by the correction factor $(\cos \phi)^{-1}$ so that frequency per unit area is represented. It will be seen that the winter curves for the two hemispheres are similar, each having a minor maximum at about lat. 38° and a major maximum in higher latitudes. The major maximum however occurs at about 57° in the southern hemisphere but at about 65° in the northern hemisphere.

An analysis of the distribution of the central pressure value of the cyclone and its variation with latitude is shown in Fig. 11. The vertical interval between isobars indicates the percentage of cyclones having central pressures within the indicated range while the interpolated pressure obtained on the 50 per cent axis gives the median value of the central pressure of the systems at a particular latitude. It will be seen that the distributions are quite similar in both hemispheres and that the lowest median value occurs at the latitude of greatest frequency, so that, in winter, the central values of depressions in

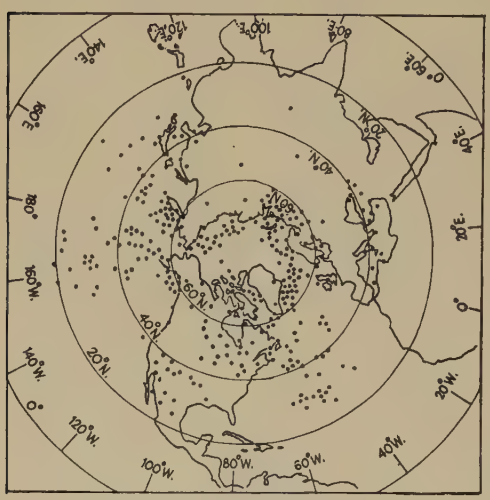

Figure 2. Distribution of cyclone centres on daily N.H. charts during January 1949.

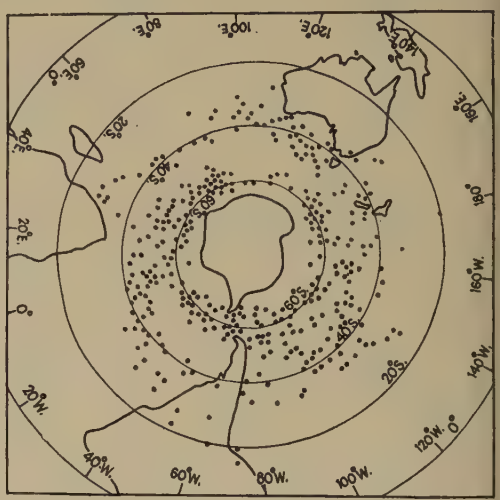

Figure 3. Distribution of cyclone centres on daily S.H. charts during July 1949.

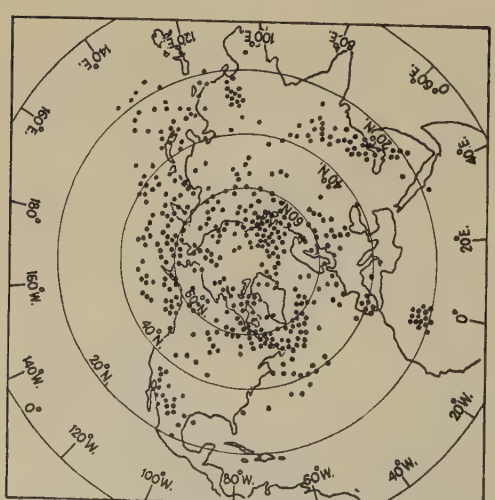

Figure 4. Distribution of cyclone centres on daily N.H. charts during July 1949.

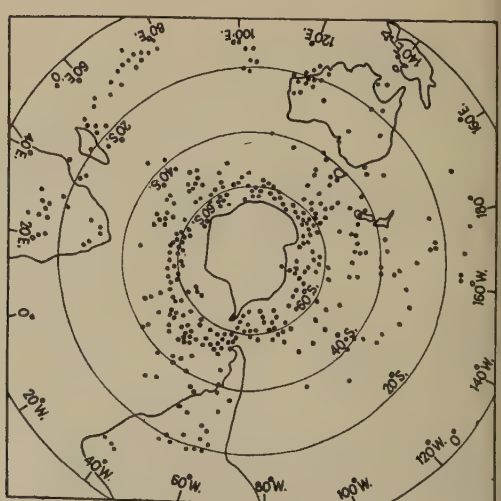

Figure 5. Distribution of cyclone centres on daily S.H. charts during January 1950.

the two hemispheres appear to be comparable, from which it may be inferred that the individual vortices have circulations of comparable intensity in both hemispheres in winter.

4. The cyclones – midsummer

During the midsummer months analysed, the contrast between the hemispheres is still evident in that the southern hemisphere distribution of cyclones is more symmetrical than that for the northern hemisphere (Figs. 4 and 5). However, in this case the symmetry in the southern hemisphere is confined to the zone in the vicinity of 60°. A concentration of heat lows is apparent in the vicinity of Lat. 20° over Australia, S. Africa and S. America, and at slightly lower latitudes, lows of the 'easterly wave' type are noticed over the Indian and Pacific oceans. Undoubtedly the majority of the low-latitude oceanic systems have not been detected and therefore do not appear in this analysis. There is some indication of the same southern hemisphere cyclone tracks noticed in Fig. 3.

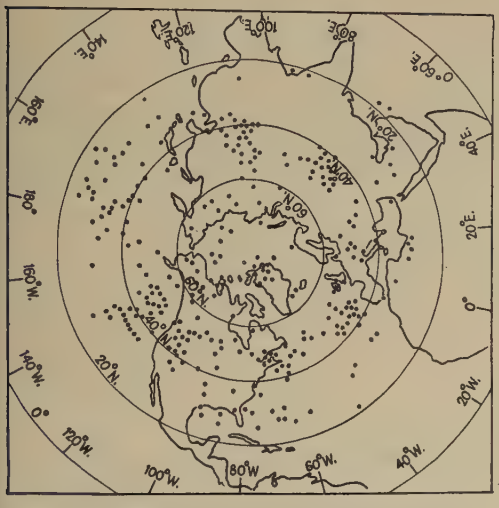

Figure 6. Distribution of anticyclone centres on N.H. daily charts during January 1949.

Figure 7. Distribution of anticyclone centres on S.H. daily charts during July 1949.

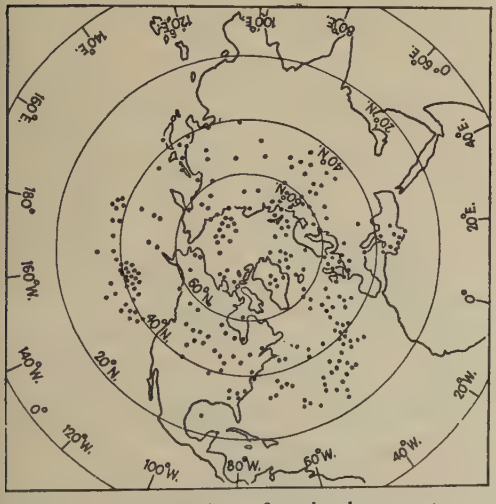

Figure 8. Distribution of anticyclone centres on N.H. daily charts during July 1949.

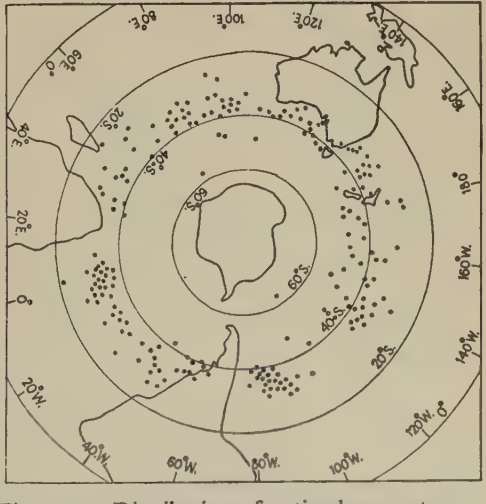

Figure 9. Distribution of anticyclone centres on S.H. daily charts during January 1950.

Fig. 4 contains 461 centres and Fig. 5 contains 368. In the northern hemisphere, low-latitude heat lows are apparent over N. America, Africa, Arabia, India and Indo-china. There is also some evidence of tropical circulations in the western Pacific and undoubtedly there are many more undetected in this and other areas.

Referring now to Fig. 10 it may be seen that the midsummer distribution of cyclone frequency is somewhat similar in the two hemispheres, the main difference being between the double maximum in the northern hemisphere at about 55° and 78° and the single maximum in the southern hemisphere at 65°. As before, the frequencies have been computed for five-degree latitude strips and corrected for area.

Although the midsummer latitude distribution of cyclone frequency is not markedly dissimilar in the two hemispheres, Fig. 11 shows that the median value of the central pressure is considerably lower in the southern hemisphere for latitudes higher than 40°, indicating that the individual vortices in high latitudes in midsummer have a considerably stronger circulation in the southern than in the northern hemisphere.

5. The anticyclones – midwinter

The location of anticyclone centres in the midwinter month in the two hemispheres is shown in Figs. 6 and 7, 276 centres being plotted for the northern and 263 for the southern hemisphere. Once again the greater symmetry in the southern hemisphere is immediately apparent. Fig. 7 shows the regularity of distribution with longitude, the lack of concentration in any one particular area suggesting that the so-called semi-permanent high-pressure cells of the Indian and Pacific Oceans are largely mythical.

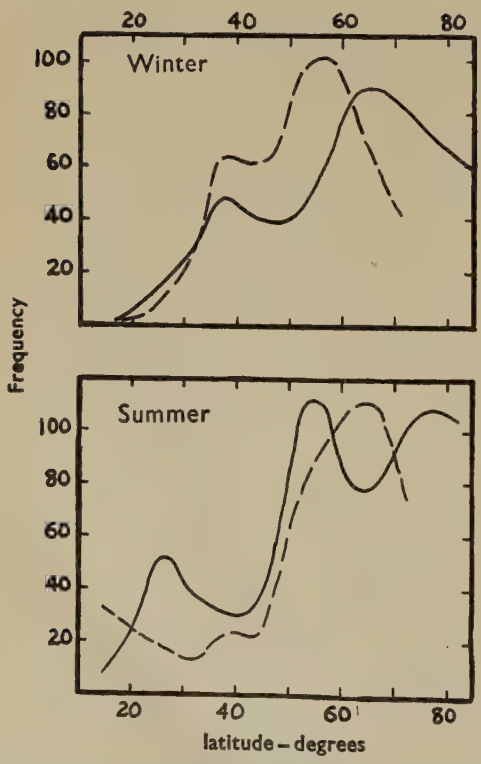

Figure 10. Latitudinal distribution of cyclone frequency. Solid line – northern hemisphere; broken line – southern hemisphere.

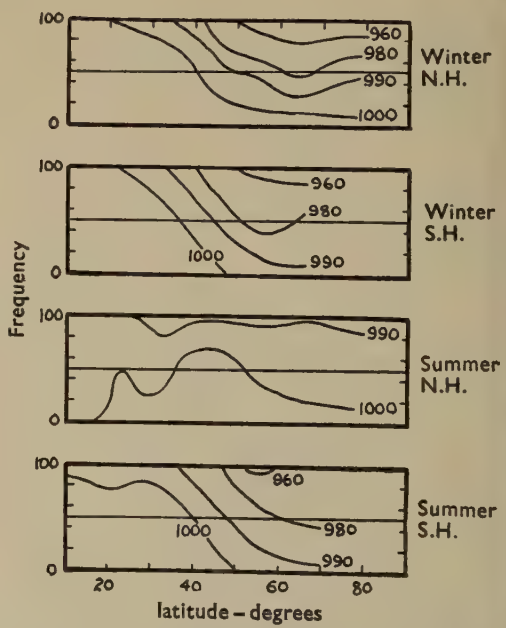

Figure 11. Variation of central pressure value of cyclones with latitude.

By contrast in the northern hemisphere there are zones which show marked concentration of centres and the general lack of symmetry is especially pronounced.

Fig. 12, prepared like Fig. 10, emphasizes that there is a considerable contrast in the distribution of anticyclones with latitude in the two hemispheres in the midwinter month. The majority in the southern hemisphere occur between 20° and 40° with a maximum at about 27°, whereas in the northern hemisphere the distribution is broader with the maximum at about 47°.

The strong contrast between hemispheres is emphasized also by Fig. 13, which shows that the median value of the central pressure of the anticyclones is considerably higher in the northern than in the southern hemisphere.

6. The anticyclones – midsummer

For the midsummer month 249 anticyclone centres were plotted for the northern and 263 for the southern hemisphere (see Figs. 8 and 9). Once again, the greater symmetry of the southern hemisphere distribution is noticeable, although there is more evidence of concentrations at certain longitudes than in the case of the midwinter month in the southern hemisphere. Concentrations are evident off the west coasts of S. Africa and S. America and to a lesser, extent, Australia.

Fig. 12 shows that there is a pronounced difference between the latitudinal distribution of the midsummer anticyclones in the two hemispheres. As in the case of the midwinter

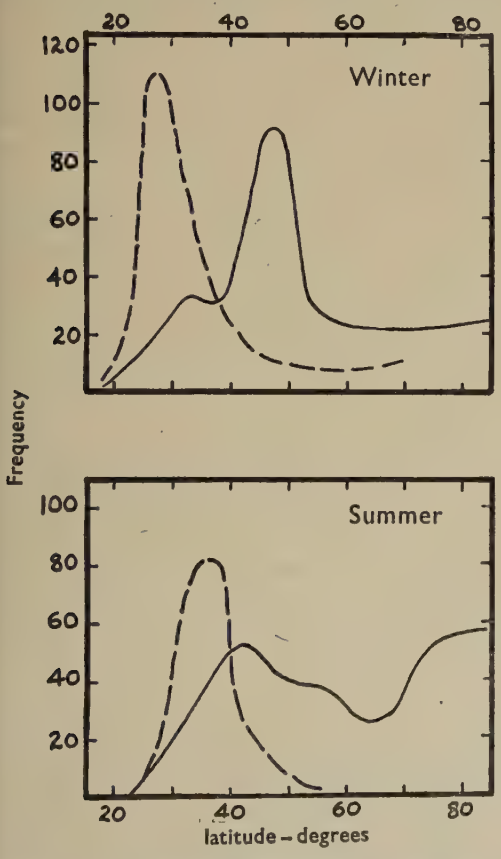

Figure 12. Latitudinal distribution of anticyclone frequency. Solid line – northern hemisphere; broken line – southern hemisphere.

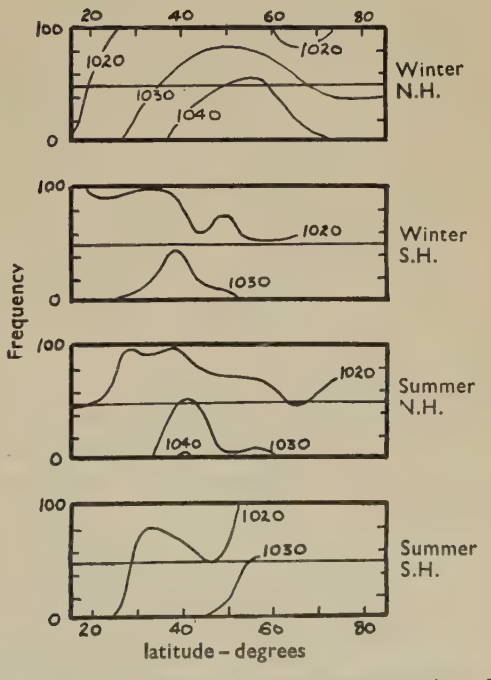

Figure 13. Variation of central pressure value of anticyclones with latitude.

anticyclones the majority of southern hemisphere centres occur in a fairly narrow latitude band whereas the northern hemisphere has a fairly uniform distribution from 35° to the pole. In the southern hemisphere a sharp maximum occurs at about 37° while in the northern hemisphere less definite maxima occur at about 42° and 80°.

Fig. 13 shows that in both hemispheres the midsummer anticyclones do not have a very great range in central pressure. The southern hemisphere winter anticyclones also have only a relatively small variation in central value, but it will be remembered that the northern hemisphere anticyclones showed considerable variation of the value of the central pressure.

Before summarizing and discussing the results of the analysis of pressure profiles and the distribution of the centres of pressure systems, some features of the mean circulation of the upper air in the vicinity of Australia will be examined.

7. General features of the mean circulation aloft

Studies of the mean circulation aloft in the vicinity of Australia have been limited in number and scope mainly because of lack of observational material. Some statistical analyses of pilot-balloon observations have been made, mainly in the form of wind roses for selected points, an example being ' Weather on the Australia Station,' Meteorological Branch, Commonwealth of Australia. Such studies have three main shortcomings : pilot-balloon observations are almost always biased towards cloudless, light wind conditions; the number of observations above 20,000 ft is comparatively limited; in the lowest layers, where the record is reasonably complete, there is often a strong diurnal influence.

Upper (geostrophic) wind data are also available from charts of the topography of constant-pressure surfaces. Thus Loewe and Radok (1950), Hutchings (1950) and Gibbs (1952) have presented meridional cross-sections of the geostrophic west wind for the Australian region. Loewe and Radok and Hutchings used means from radiosonde stations to compute their cross-sections and Gibbs has pointed out some sampling errors inherent in their method. Gibbs used mean constant-pressure charts for the construction of his cross-sections and showed that there was some variation of geostrophic west wind with longitude. Brooks *et al.* (1950) have produced mean constant-pressure charts for the world, and although these charts are admirable in presenting an overall picture of upper winds over the world for the first time, more data have become available in the Australian region since their construction and the charts used by Gibbs, although provisional, are more accurate in this area.

The methods used for estimating geopotential of constant-pressure surfaces from sea-level data (such as those used in preparation of the U.S.W.B. Normal Upper Air Charts and the charts of Brooks *et al.* (1950)), are likely to give considerable errors in areas remote from radiosonde stations, particularly in sub-tropical latitudes where the frequent occurrence of subsidence inversions makes such estimations difficult. Another possible source of error is the departure, in the mean, from geostrophic wind conditions. It is generally assumed that the geostrophic wind relationship is doubtful in low latitudes but little is known of the magnitude of departure from geostrophic conditions and its dependence on latitude.

Studies to date indicate that the main feature of the mean circulation aloft in the Australian region during the southern winter is a maximum of geostrophic west wind of approximately 100 kt at 200 mb in the vicinity of 27°S. This maximum has typical ' jet ' characteristics with strong horizontal and vertical wind shear in its vicinity. There is evidence for a secondary maximum of geostrophic west wind at high latitudes at the 200-mb level or higher with the minimum between these two maxima occurring at about 45°S and being of the order of 30 kt.

In summer the mean circulation aloft appears to be characterized by a maximum of geostrophic west wind of about 60 kt at 200 mb in the vicinity of 33°S, another maximum at about 300 mb in higher latitudes with the intervening minimum of about 30 kt at 200 mb in the vicinity of 43°S.

From a study of Namias and Clapp (1949) it is evident that there is a considerable variation of the mean circulation aloft with longitude in the northern hemisphere and particularly in the vicinity of N. America. As a result it is difficult to draw comparisons between the mean circulation aloft over the Australian region and the northern hemisphere. The meridional cross-sections of Willett (1944) and Hess (1948) for N. America for the northern winter both show a jet of about 100 kt at 200 mb in the vicinity of 40°N to 35°N and Willett shows a secondary maximum of about 20 kt at 200 mb centred in the vicinity of 67°N, which however does not appear on the cross-section by Hess. In the cross-sections for the northern summer both Willett and Hess show maxima of about 50 kt at 200 mb in the vicinity of latitude 50°N to 55°N over N. America. Thus there appear to be considerable differences in the winter and summer upper mean circulations over the Australian region and N. America, but it seems likely that a considerable amount of this difference is due to local perturbation of the mean circulation of the northern hemisphere over N. America.

In attempt to secure a more representative comparison of southern and northern hemisphere conditions and to secure a further picture of the mean upper circulation in the vicinity of Australia a combined northern and southern hemisphere cross-section centred about 150°E has been constructed and will be described in the succeeding sections.

TABLE 1 (a). MEAN GEOSTROPHIC WEST WIND (kt) FOR THE NORTHERN HEMISPHERE BETWEEN LONGITUDES 120°E AND 180°*

	north latitude										
January	65	60	55	50	45	40	35	30	25	20	15
100 mb		17	19	22	39	47	66	96	110	105	98
150		19	26	33	43	56	91	106	120	95	77
250		17	28	43	50	56	96	104	88	59	46
450		15	20	30	40	47	57	60	50	38	35
700		8	14	17	21	30	35	30	27	21	17
1,000	0	3	5	7	5	4	4	− 3	− 6	− 11	− 23
April											
100 mb		11	3	− 5	− 7	28	48	66	85	70	55
150		14	19	20	24	40	57	72	90	82	55
250		17	25	31	34	43	57	72	90	82	55
450		13	20	24	28	35	39	43	45	15	0
700		3	7	14	17	22	23	26	15	0	0
1,000	− 4	0	3	3	3	4	3	0	− 6	− 10	− 20
July											
100 mb		6	− 3	− 5	4	24	27	29	32	23	9
150		10	13	17	30	37	37	36	35	21	9
250		16	18	20	23	28	33	29	17	12	0
450		7	13	17	21	23	25	12	6	7	0
700		4	6	14	15	14	14	9	6	0	− 8
1,000	− 2	0	3	3	3	3	− 4	− 5	− 5	− 7	− 8
October											
100 mb		12	14	20	37	43	48	46	37	15	− 23
150		17	28	33	51	66	66	48	37	21	0
250		21	28	43	51	56	52	46	21	10	0
450		22	26	29	37	33	29	25	21	7	0
700		14	18	19	20	19	16	5	5	0	0
1,000	0	3	6	6	4	2	− 1	− 8	− 9	− 9	− 12

* Data for 1,000 mb are means for complete hemisphere.

8. The profiles

The method used in constructing the cross-sections for latitudes above 20° consisted of drawing profiles of mean geopotential for certain constant-pressure surfaces using mean constant-pressure or constant-level charts. From the slope of these profiles the mean geostrophic west wind was derived.

In drawing the profiles means were computed for latitudes at five-degree intervals over a given sector, the purpose in taking a sector rather than a single meridian being to eliminate inaccuracies in the charts and minor local irregularities. For the northern hemisphere the U.S.W.B. Normal Upper Air Maps were used and means computed over the sector between longitudes 120°E and 180°. Charts for 10,000 ft and 20,000 ft and 10, 13 and 16 km were used corresponding to 700, 450, 250, 150 and 100 mb. For the southern hemisphere the mean constant-pressure charts for the Australian region described by Gibbs (1952) were used and means computed over the sector 110°E to 170°E. The values are listed in Tables 1 (a) and (b).

These figures were entered at the appropriate points in the cross-sections in Figs. 14-17 for latitudes 15°N to 60°N and 15°S to 55°S, thus leaving the latitudes between 15°N and 15°S without data. In order to fill this gap means were computed for this latitude interval and between 120°E and 180°, using the equatorial mean 'streamline' charts of Brooks *et al.* (1950), and the figures for mean observed west wind entered on the cross-sections. Certain discrepancies were immediately apparent between these data and the geostrophic west winds, an example being shown in Fig. 18, which compares the values

TABLE 1 (b). Mean geostrophic west wind (kt) for the southern hemisphere between longitudes 110°E and 180°*

	south latitude								
January	60	55	50	45	40	35	30	25	20	15
100 mb		8	16	22	25	32	26	7	5	5
200		22	34	37	40	48	52	28	30	46
300		44	36	32	31	37	32	23	27	35
500		28	24	22	22	23	18	10	7	0
700		22	19	13	10	12	11	6	− 7	− 11
1,000	14	15	15	12	9	0	− 5	− 6	− 7	− 11
April										
100 mb		28	29	32	35	35	41	35	21	28
200		24	26	37	47	57	72	57	44	49
300		28	30	38	40	45	52	43	32	35
500		18	19	21	23	25	24	21	12	13
700		13	14	15	16	18	17	0	− 5	− 9
1,000	8	11	14	15	11	6	0	− 12	− 12	− 9
July										
100 mb		37	39	43	43	45	40	37	32	35
200		37	39	40	51	70	80	77	56	49
300		49	27	25	40	57	72	66	53	46
500		31	22	23	25	30	35	31	30	35
700		37	19	15	17	21	16	12	9	0
1,000	4	11	14	12	13	13	6	− 6	− 16	− 28
October										
100 mb		7	12	19	24	37	46	28	7	0
200		23	20	32	43	57	76	71	46	35
300		37	34	28	30	55	63	57	37	35
500		32	25	20	24	33	38	28	18	14
700		22	16	11	14	18	18	14	7	0
1,000	11	17	12	10	11	6	0	− 6	− 13	− 14

* Data for 1,000 mb are means for complete hemisphere.

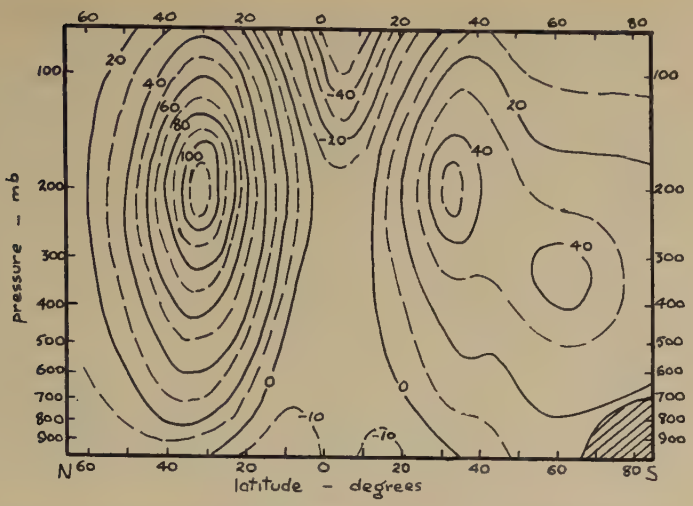

Figure 14. Cross-section of mean zonal wind for sector centred on longitude 150°E – January. Wind speed in knots.

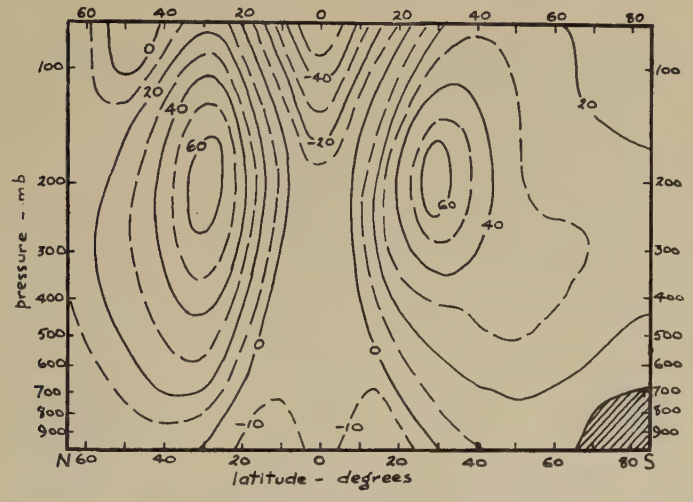

Figure 15. Cross-section of mean zonal wind for sector centred on longitude 150°E – April. Wind speed in knots.

taken from Brooks's data with those from the profiles for 15°. These discrepancies are due to errors in the profiles, errors in Brooks's data, or the non-validity of the geostrophic assumption at these latitudes, or to a combination of these errors.

It seems very likely that at 15° the geostrophic mean wind will be greater than the true mean wind. Also the U.S.W.B. Normal Upper Air maps and the mean charts for the Australian region are based on inadequate data for latitudes lower than 20°, so that part of the discrepancy may well be due to inaccuracies in the profiles in this region. Furthermore, Brooks's data in this region, although based on a not very comprehensive series of upper-wind observations which are most likely selective toward light winds, are probably closer to actual conditions. In drawing isopleths on the cross-sections the probable sources of error were kept in mind and the thermal structure of the cross-section was used in interpolating between the measured values. The cross-section of temperature in the Australian region will be discussed in the section which follows.

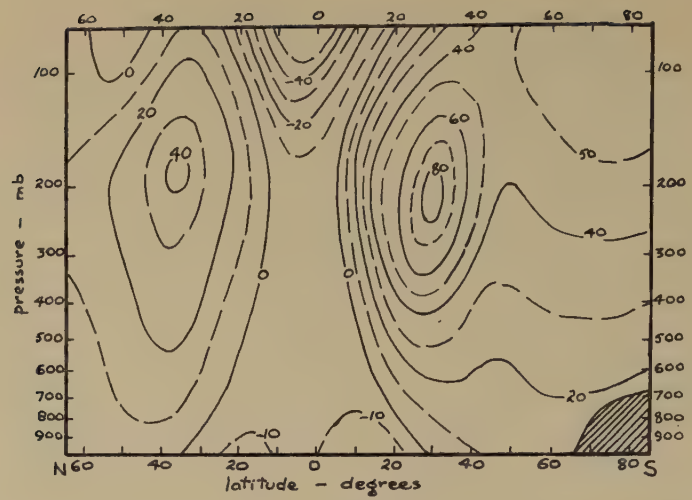

Figure 16. Cross-section of mean zonal wind for sector centred on longitude 150°E – July. Wind speed in knots.

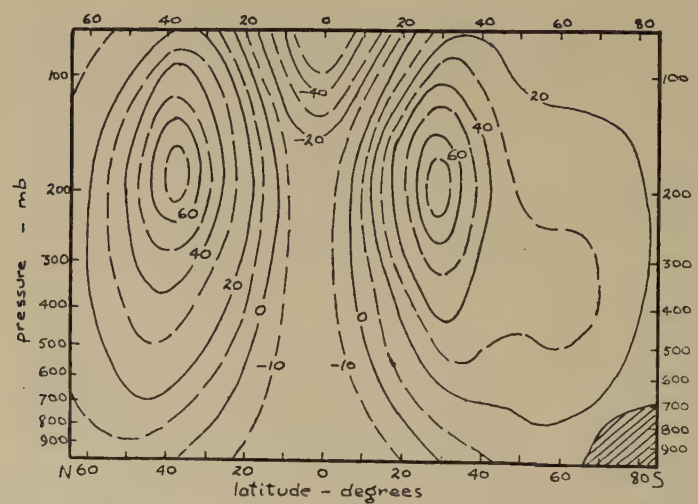

Figure 17. Cross-section of mean zonal wind for sector centred on longitude 150°E – October. Wind speed in knots.

9. The temperature cross-sections

Fig. 19 shows cross-sections of temperature for the midsummer, midautumn, midwinter and midspring months for the Australian region. These cross-sections have been constructed by computing means for each five-degree latitude between 110°E and 180° using charts of mean temperature for the 850, 700, 500, 300, 200, 150 and 100 mb surfaces prepared in Research Section, Weather Bureau, Melbourne from available radiosonde data. The cross-sections correspond quite closely to those of Loewe and Radok (1950) and Hutchings (1950) and show increasing temperature with decreasing latitude throughout the year at levels below 300 mb, and increasing temperature with increasing latitude at levels above 200 mb for all but the midwinter month, in which a maximum of temperature occurs in middle latitudes at these levels.

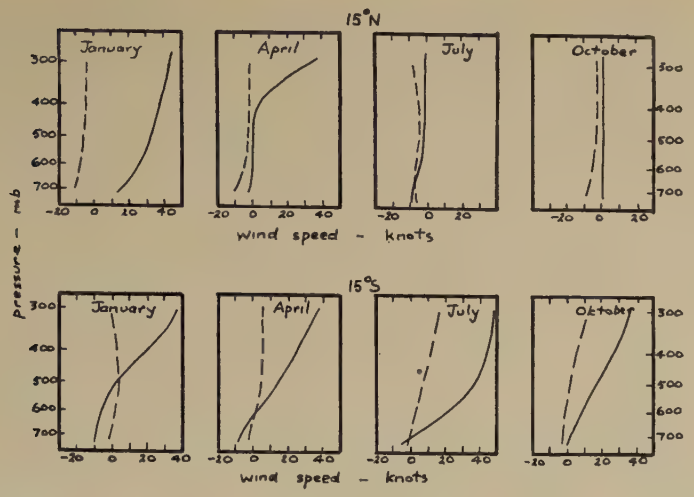

Figure 18. Comparison of values of zonal wind at latitude 15° from Brooks's data (broken line) with those from profiles of constant pressure surface (solid line).

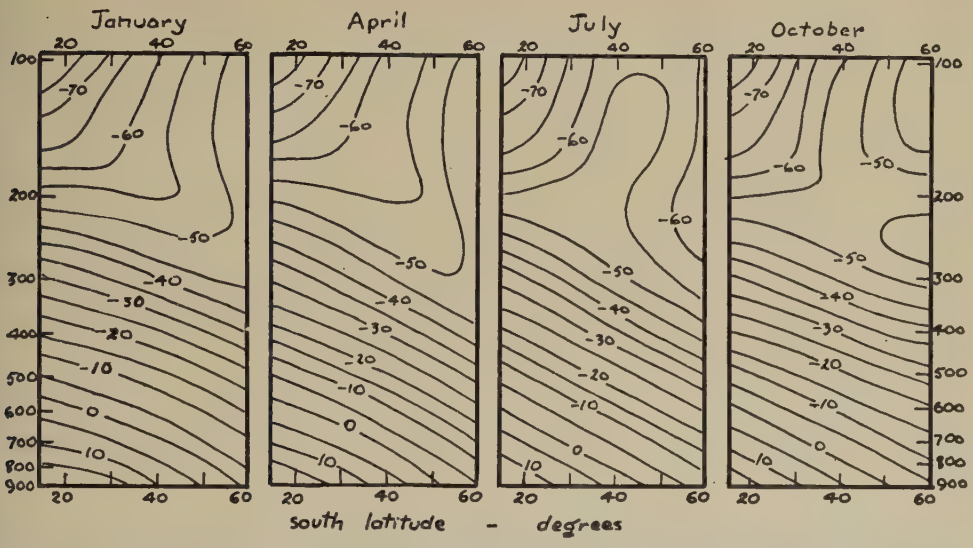

Figure 19. Cross-section of mean temperature for longitude sector centred on longitude 150°E in southern hemisphere. Isotherms are labelled in deg. C.

It was found difficult to compare this southern hemisphere picture with the northern hemisphere because of the lack of readily available, accurate and representative northern hemisphere data. The temperature cross-section of Hess (1948) shows considerable differences, particularly in lower levels, but this is no doubt due in part to the influence of the North American continent. The U.S. Normal Upper Air maps are hardly of sufficient accuracy to use for such a comparison, but temperature cross-sections from this source are available (Petterssen 1950). They show considerable differences from southern hemisphere cross-sections for the same seasons. Below about 6 km the winter cross-sections are very similar but above that height, although there is a warm region in the northern hemisphere cross-section similar to that for the southern hemisphere, the warm region is about 20° latitude poleward of the southern hemisphere position. The

summer cross-section for the northern hemisphere shows much less baroclinity below about 10 km than does that for the southern hemisphere. As previously mentioned, it is difficult to judge how much of the dissimilarity, particularly for the upper levels, is due to incomplete data in the northern hemisphere sections. Undoubtedly, the continents of North America and Eurasia will produce considerable differences in the lower levels.

10. The cross-sections of west wind components

As previously mentioned, the cross-sections contained in Figs. 14-17 are means for the longitudes 120°E to 180° and this sector in the northern hemisphere is mainly oceanic. In drawing the sections the values obtained from the profiles and from Brooks's data were used as a framework and in the southern hemisphere the temperature cross-sections of Fig. 19 were used in a qualitative way to assist in interpolation. Values derived from the profiles for 15° and 20° were reduced so as to fit reasonably well with Brooks's data. Values from the northern hemisphere profiles for levels above 250 mb were regarded as too large in many cases, particularly in jet-stream latitudes. There is some reason to believe that strong easterly winds frequently occur at considerable elevations at low latitudes in the summer hemisphere. Willett (1944) indicated this in his cross-section and from time to time strong easterly winds are observed in low latitudes over 50,000ft. This feature has been included in the sections but should be regarded as a doubtful quantity.

The *January cross-section* shown in Fig. 14 shows the northern hemisphere dominated by a jet of over 110 kt centred at 200 mb in the vicinity of 30°N. At the same level in the southern hemisphere a maximum of west wind of about 50 kt is located at about 33°S and another maximum in excess of 40 kt occurs near 62°S between 400 and 300 mb. Over 5°S a maximum of east wind of more than 50 kt is located somewhere above the 100 mb level. Neither Loewe and Radok (1950) nor Hutchings (1950) indicated a west wind maximum at high latitudes in summer but it is felt that they did not make due allowance for the strength of the surface westerlies. Loewe and Radok give a west wind component of about 6 kt and Hutchings a value less than 10 kt, but the mean hemispheric profile gives a speed of about 15 kt.

The *July cross-section* shown in Fig. 16 shows the main jet now in the southern hemisphere at the 200 mb level located at about 30°S. Its central value is about 30 kt less than the northern hemisphere winter jet and this may be due to two factors. The deformation of the jet stream in the vicinity of the east coast of Australia will cause a general broadening of the maximum shown on the mean section for the region 110°E to 180° and will reduce its central value. The other factor is that, as pointed out by Namias and Clapp (1950), the western north Pacific is an area where the jet in the northern hemisphere has above-average intensity. Moreover, a part of the difference may be due to errors in the Normal Weather Maps.

The northern hemisphere maximum is at the same level and about the same intensity as the summer maximum of the southern hemisphere shown in Fig. 14, but it is about 5° latitude poleward of the summer position of the southern hemisphere maximum. There is no evidence of a secondary maximum at higher latitudes in the northern hemisphere such as that appearing in the southern hemisphere in Fig. 14.

At high levels over the southern hemisphere a secondary maximum of over 50 kt is shown in Fig. 16. This is based on the winter cross-section of Loewe and Radok (1950) and is a consequence of the reversed horizontal temperature gradient in the stratosphere in winter in high latitudes as shown in Fig. 19.

Once again a maximum of east wind is suggested above 100 mb and on this occasion in the vicinity of 5°N.

Figs. 15 and 17 show the *April and October cross-sections* to be essentially the same. In each case the maxima in both hemispheres are of approximately equal intensity and located at about the same level. The spring maxima are at a slightly lower latitude than the autumn maxima, and in the appropriate season the southern hemisphere maximum is at a slightly lower latitude than the northern hemisphere maximum. The largely hypothetical maximum of east wind is shown above the 100 mb level at the equator. In the southern hemisphere in both seasons there is a suggestion of the secondary maximum at higher latitudes which was apparent in the southern summer (Fig. 14).

11. Recapitulation

From the data examined in the foregoing pages it is possible to arrive at certain tentative conclusions regarding the mean circulation in the vicinity of Australia, its relation to the rest of the southern hemisphere and to certain features of the northern hemisphere circulation. But it is most important to state that the data examined are by no means sufficient in quantity or quality to encourage the belief that these conclusions will not need later amendment.

1. The mean circulation in the vicinity of Australia consists of an easterly circulation in low latitudes and a westerly circulation in higher latitudes. The boundary between the two circulations appears to be at its lowest latitude at about the 200 mb level and its highest latitude at the surface and at some height above the 100 mb level (where it may in some seasons extend to high latitudes). The boundary has a slight seasonal shift, being furthest north at all levels in the southern winter.

2. It is probable that the easterly circulation is strongest above the 100 mb level where an easterly jet stream may occur.

3. Perturbations occur in the easterly circulation and there are certain geographical (monsoon) variations in the region of the easterlies, such that in some areas and at some levels westerlies prevail at some seasons. However, these are submerged when the mean circulation is evaluated over the region between 110°E and 180°E.

4. The mean westerlies have their maximum in sub-tropical latitudes at about 200 mb. The intensity of this maximum varies from about 80 kt in winter to 50 kt in summer although at a particular longitude the intensity of the maximum is likely to be higher. The position of the maximum is at a latitude slightly lower than 30°S in winter and slightly higher than that latitude in summer. The maximum is characterized by strong horizontal and vertical gradients of wind speed.

5. A secondary maximum is suspected at about latitude 60°S between 300 and 400 mb in summer, still being evident but of less intensity in spring and autumn. In winter, with the disappearance of the Antarctic tropopause, a secondary maximum is likely at about latitude 70°S.

6. At lower levels the westerly circulation is characterized by cyclonic vortices, the majority of which are generated in the vicinity of latitude 30°S near the coastlines of S. America, S. Africa or Australia. These vortices move with the westerly stream and gradually drift poleward to an equilibrium position which is between 55°S and 60°S in winter and 60°S and 70°S in summer. The vortices appear to attain their greatest intensity immediately before reaching their equilibrium latitude.

7. Relatively few anticyclonic vortices appear in the westerlies. It is likely that those which do appear move from the coastline of Antarctica or the edge of the pack ice in the westerly stream, drifting equatorward to the sub-tropical high-pressure belt. 'Blocking' has been observed in high latitudes in the vicinity of Australia but it is believed to be rare. Many high-pressure centres are observed in the sub-tropical high-pressure belt, but it is believed that the majority of these are merely the intervals between the troughs extending northward from the cyclonic vortices in the westerly circulation.

8. It seems likely that the mean circulation in the vicinity of Australia is typical of the hemisphere as a whole. This belief is based on the fact that the hemisphere is relatively uninterrupted by land masses and that the symmetry of occurrence of pressure systems previously discussed indicates a symmetrical mean circulation. A possible exception to this is the likely deformation of the jet stream in the vicinity of the South American Andes.

9. It appears that the northern hemisphere has the same basic mean circulation as the southern hemisphere, with broad streams of comparable intensity, and with a similar interruption of these streams by disturbances generated in a similar manner. However, in the northern hemisphere the different disposition of land and sea prevents the cyclonic vortices from drifting with the westerly stream with the result that a comparatively great number of anticyclonic eddies enter the westerly circulation; so that in spite of the fact that the cyclonic vortices are comparable in intensity with those of the southern hemisphere the presence of a greater number of strong anticyclonic vortices results in a weaker mean westerly circulation in the northern hemisphere in the lower levels.

Acknowledgments

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Rainfall at Bidston, 1867-1951

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Summary

The 85 years (1867-1951) of rainfall record at the Liverpool Observatory and Tidal Institute, Bidston, are the longest uninterrupted series in NW. Cheshire. Their salient features are :

(i) The mean annual rainfall in the calendar year is 28·68 in. with one exceptionally wet year, 1872, when 45·66 in. were recorded.

(ii) The mean of the seasonal year is 28·74 in. In this case, although the standard deviation is larger than with the calendar year, the driest and wettest years are not so extreme.

(iii) October, although it has been getting drier, is the wettest month, and April is the driest. The characteristics of the other months are scrutinized and compared following the computation of mean daily rainfall figures.

(iv) The probability of a month being the wettest in any year is directly proportional to the cube of its mean rainfall.

(v) The probability of a month being the driest in any year is inversely proportional to its mean rainfall.

(vi) Spring is the driest season and autumn the wettest, summer having significantly more rain than winter.

(vii) No significant correlation exists between one season's rainfall and any subsequent season's fall.

(viii) There have been very few occasions of heavy rainfall, the maximum recorded in 24 hr being just over 2 in., and only three noteworthy falls. This is due to Bidston's geographical position, as it misses the most violent instability storms and is partially in the rain-shadow of the Welsh mountains, which protect it from the heaviest depression rains.

(ix) On the average there is an absolute drought once every 15 months and it is of 19 days' duration. Only two droughts have lasted longer than 26 days.

(x) Frequency curve graduation of annual and seasonal rainfall proved disappointing, as few absolute conclusions relating to extreme values could be drawn from the figures owing to the large probable errors involved, with what, in a statistical sense, is a relatively short period of observations, only 85 years. Deductions based on such computed curves are, however, the best method of estimating rainfall probabilities.

1. Introduction

Before discussing in detail the rainfall of Bidston, we should review briefly some of the geographical features of the area. Bidston is situated in Wirral, a roughly rectangular peninsula measuring 12 mi by 7 mi, lying between the Dee and Mersey estuaries, and orientated with its longer axis in a NW. to SE. direction. The NW. coast is exposed to the WNW. to N. winds coming across the full fetch of a hundred miles of the Irish Sea. To the SW. of Wirral lie the Welsh mountains and to some extent the low-lying Wirral peninsula must be considered to be in their rain-shadow, although the most pronounced rain-shadow effects are noticed in an area immediately to SW. of Wirral where the River Dee flows into its estuary. Here the mean annual rainfall falls as low as 25 in. To the south and east lie the Cheshire Plain and the Lancashire industrial area, backed by the Pennine Chain, but these have a very small effect on rainfall at our station owing to the main rain-bearing winds having a westerly or southerly trajectory.

Homogeneous rainfall records have been maintained at the Liverpool Observatory, Bidston (200 ft above sea level), since 1867, and this is the longest continuous record available for Wirral. Prior to 1867 the records are not homogeneous and have been omitted from this investigation.

The exposure of the gauge is perfect except in the SE. quadrant where a corner of the Observatory building (30 ft in height) comes to within 26 ft of the gauge. It is a

pity but it must be agreed that this must slightly reduce the catch when rain falls with a moderate to strong SE. wind. It is, of course, difficult to evaluate what the loss would be. Hudleston (1928) found that a gauge the same distance from a wood as the height of the trees caught 70 per cent of the true rainfall of the area when strong winds were blowing off the wood. Lammert (1948), experimenting with raingauges in the lee of a row of poplars, found no loss of catch until the raingauge was nearer to the row than 5/7ths of the height of the trees. It will be noted that in the case of Bidston the gauge distance is approaching Lammert's limiting distance for winds from a south-easterly direction.

2. Annual rainfall

Three previous papers have been read before this Society on particular aspects of the annual rainfall at Bidston, and it is thought best to review these very briefly, before drawing attention to the points at present in mind. Bigelstone (1934), gave the main outlines of Bidston's rainfall for the 60-year period 1871-1930. These data became the basis for the second paper by Doodson and Bigelstone (1934) in which the totals of annual rainfall were examined by graduating with Pearson frequency curves. It was found that the rainfall of 1872, when a total of 45·66 in. was recorded, was definitely exceptional and that such a fall would only occur on the theory of probability once in 14,000 years. With the rainfall of 1872 included in their statistical investigation Doodson and Bigelstone found that the minimum annual rainfall to be expected at Bidston was 21·40 in. This fall, however, had not been attained in 1887 when the total was only 20·82 in. Since their paper was prepared, another annual fall even lower has been recorded. Because the purpose of their paper was to tabulate the probabilities of annual rainfall reaching various values, and in view of the very exceptional nature of 1872, they recomputed their figures omitting this year. Their second set of computations fitted the observed falls of the 60 years better. This time the lower limit of annual rainfall at Bidston was reduced to 19·79 in., lower than any observed value, before or since the paper.

Zoch (1936) showed that a small error had been made in the first of these computations, and he recomputed the probabilities including the year 1872. His curve, however, does not fit the observations quite so well as the second curve due to Doodson and Bigelstone. Much discussion at the readings of these papers centred round the justification, or lack of it, in rejecting one year's observation. For fuller details reference should be made to the original papers.

TABLE 1. Annual rainfall in inches

Decade	No. of year in decade										Decadal mean
	0	1	2	3	4	5	6	7	8	9	
1860	—	—	—	—	—	—	—	25·09	25·05	29·88	—
1870	24·27	27·21	45·66	23·70	25·41	29·36	30·23	39·23	28·55	29·80	30·34
1880	31·34	30·67	34·47	28·93	25·28	29·52	34·62	20·82	24·09	27·37	28·72
1890	26·82	31·64	33·03	24·54	27·91	26·26	26·63	28·58	25·72	27·81	27·89
1900	32·05	25·19	25·57	34·42	25·18	24·01	28·08	26·57	28·90	28·45	27·84
1910	28·58	25·26	30·17	25·89	26·02	26·83	24·87	30·59	31·93	29·12	27·93
1920	33·34	22·47	26·70	30·84	32·52	28·40	30·96	36·18	29·99	26·23	29·76
1930	37·50	32·67	27·72	20·01	25·09	30·42	28·61	27·66	28·90	32·18	29·08
1940	28·54	23·32	26·08	26·70	31·38	25·88	33·70	29·05	26·19	26·85	27·77
1950	32·70	31·76	—	—	—	—	—	—	—	—	—

The observations of annual rainfall for the full period of 85 years, 1867-1951, are abulated in Table 1. The mean annual rainfall total is 28·68 in. and its standard deviation σ), 4·01 in. The mean for the 60 years reviewed by Bigelstone was 28·97 in., and the Meteorological Office average for the 35-year period from 1881-1915 is 27·93 in. As the tandard error of the 85-year mean is 0·44 in. it will be quickly seen that there is little r no significance in these differences. They are almost entirely due to purely random ampling. If the frequency curve of annual rainfall followed the normal curve, 95 per cent f the observations should be within the range of mean $\pm 2\sigma$. To the nearest whole umber this is borne out by the observations. Four years, three wet and one dry, lie utside these limits. Only one year, the extremely wet 1872, is outside the range of nean $\pm 3\sigma$. This year is so exceptional, not only for Bidston but for the most parts f the British Isles, as to merit more detailed scrutiny.

The first five months of 1872 gave 12·14 in. of rain, 2·5 in. above the normal, but othing to warn of the deluge to come. The wet spell began on 1 June with a fall of ·78 in., and 4·32 in. fell in that month on 22 rain-days. July (7·51 in.) was the wettest nonth ever recorded at Bidston. Three days gave more than an inch of rain each. There vere 15 rain-days in the month but almost 80 per cent of the rain fell on 5 days. August's all was below average. September, however, was the wettest on record. There were ·57 in. of rain, which fell on 24 days. October was again wet, 6·53 in. being measured. This was the second wettest October on record, and rain fell on 24 days. The falls of November and December, although above average, were not unusually so. The total all for the year is almost exactly 4 σ in excess of the mean. Assuming the normal frequency distribution, a year such as this would only occur once in 16,500 years, a figure very close o that derived by Doodson and Bigelstone by more precise methods.

At the other end of the scale, the driest year on record is 1933 when only 20·01 in. of rain were measured. In this year there was no exceptionally dry period but the year as a whole yielded a majority of dry months. Many of them were 1 in. below average.

Table 2 shows the wettest and driest consecutive years up to ten years. Apart from he slight changes in the column for the percentage of average fall, there is only one lteration in this table from those quoted by Bigelstone for the 60-year period. One wet period, 1875-1886, dominates the ' wet ' table, appearing in 8 out of 10 periods, and one dry period, 1887-1896, following the wet period in immediate chronological order, dominates the ' dry ' table, appearing 8 times also.

TABLE 2. Wettest and driest consecutive years

No. of years	Wettest years			Driest years		
	Period	Average fall in.	Percentage of average	Period	Average fall in.	Percentage of average
1	1872	45·66	159	1933	20·01	70
2	1871-1872	36·43	127	1887-1888	22·46	78
3	1875-1877	32·94	115	1887-1889	24·09	84
4	1877-1880	32·23	112	1887-1890	24·78	86
5	1877-1881	31·92	111	1887-1891	26·15	91
6	1877-1882	32·34	113	1901-1906	27·07	94
7	1876-1882	32·04	112	1887-1893	26·90	94
8	1875-1882	31·71	110	1887-1894	27·02	94
9	1875-1883	31·39	109	1887-1895	26·93	94
10	1877-1886	31·24	109	1887-1896	26·90	94

3. Monthly rainfall

Summaries of monthly rainfall were extracted and means and standard deviations compiled (Table 3). On the average, October, with a mean of 3·22 in., was the wettest month and April (1·65 in.) was the driest.

TABLE 3. Monthly rainfall summary

Month	Average fall (in.)	Standard deviation (in.)	Standard error of mean (in.)	Mean daily rainfall (in.)	Wettest month		Driest month	
					Amount (in.)	Year	Amount (in.)	Year
Jan.	2·31	1·08	0·12	0·075	5·73	1948	0·64	1911
Feb.	1·79	1·04	0·11	0·063	4·46	1937	0·04	1932
Mar.	1·77	0·94	0·10	0·057	4·91	1919	0·25	1931
Apr.	1·65	0·79	0·09	0·055	3·97	1920	0·20	1912
May	2·06	1·13	0·12	0·066	5·22	1924	0·39	1896
June	2·03	1·12	0·12	0·068	4·71	1907	0·06	1925
July	2·70	1·40	0·15	0·087	7·51	1872	0·24	1868
Aug.	3·08	1·26	0·14	0·100	6·21	1879	0·29	1947
Sept.	2·69	1·45	0·16	0·090	6·58	1872	0·44	1915
Oct.	3·22	1·40	0·15	0·104	7·37	1903	0·31	1922
Nov.	2·70	1·25	0·13	0·090	6·33	1951	0·19	1945
Dec.	2·68	1·22	0·13	0·086	6·14	1918	0·51	1905

In order to compare the months closely and to eliminate the effect of their varying lengths the mean daily rainfall of each month was computed. This gave quite a smooth curve throughout the year with a maximum in October and a minimum in April. The range varied from 0·055 in. to 0·104 in. per day. A subsidiary maximum occurred in August. Bushell (1889) found very similar curves in the mean daily rainfall of Bidston (1867-1888) and of Hinderton (1869-1888), also in Wirral, seven miles south of Bidston, with the curious difference that then, although the minima were April, the subsidiary maximum had shifted to July, and September and October were almost equally wet.

The mean daily rainfall for each month in the 85 years was computed and then decadal means were compiled (Table 4) in an attempt to trace a secular trend. These decadal means do not give a smooth curve nor do means of 20- and 30-year periods confirm any secular trend.

TABLE 4. Decadal means of mean daily rainfall in inches

Period	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1870-9	0·078	0·060	0·052	0·053	0·056	0·071	0·104	0·103	0·126	0·122	0·095	0·075
1880-9	0·062	0·059	0·062	0·053	0·060	0·064	0·106	0·081	0·098	0·112	0·096	0·088
1890-9	0·064	0·054	0·052	0·053	0·061	0·084	0·077	0·104	0·092	0·112	0·079	0·084
1900-9	0·060	0·066	0·069	0·058	0·066	0·075	0·075	0·103	0·062	0·114	0·072	0·082
1910-9	0·066	0·062	0·071	0·049	0·061	0·062	0·077	0·108	0·083	0·090	0·078	0·107
1920-9	0·084	0·074	0·046	0·054	0·071	0·059	0·102	0·115	0·095	0·092	0·095	0·082
1930-9	0·083	0·059	0·045	0·061	0·065	0·085	0·091	0·094	0·079	0·109	0·095	0·086
1940-9	0·098	0·065	0·056	0·052	0·079	0·059	0·088	0·082	0·074	0·088	0·095	0·084

When the means for the four 20-year periods were graphed it was found that the graphs for the periods 1870-89 and 1890-1909 most nearly approached the shape of the graph for the complete period under review. The graph for 1870-1889 has its minimum in April, a subsidiary maximum in July and its maximum in September and October. The graph for 1890-1909 again has an April minimum, but its lesser maximum has advanced to August, September is very much lower and October is the maximum. The 1910-1929 graph is very different for the second half of the year. The minimum is still in April, but the maximum is in August with the last four months having almost equal falls. The last graph (1930-1949) is the most irregular. The minimum has regressed to March. May, July-August, October-November are the maxima in increasing importance. June and September, with almost equal falls, form the valleys between these heights.

Turning to the individual months we find that :

January has been getting wetter since 1919. This shows up well in the decadal averages for mean daily rainfall in Table 4.

February shows no apparent change. The range in decadal means of daily rainfall is the second lowest of the 12 months.

April shows no secular change either. It has remained very consistently the driest month of the year, although in three decades it yields pride of place to March, once by a very small figure and only once by a substantial margin. April is the only month which has never given four inches of rain.

August has always been the wettest month of the summer, and is the second wettest month of the year. Its rainfall rose to a peak about 1927, since when it has fallen off considerably.

September. The secondary minimum, the halting of the increasing rainfall of autumn which is so noticeable in so many long-period records of British rainfall, does not appear in the records until 1888, although scrutiny of earlier local records, not considered homogeneous with these, shows a distinct minimum in September for the period 1846-1866. From 1888, it increased to a maximum between 1900 and 1910, when on average September gave 1·6 in. less than August and 1·9 in. less than October. During this period only one September gave more than three inches of rain.

October. Although it has remained the wettest month throughout the 85 years of records, this month has become drier. Its decadal means of daily rainfall show a steady fall from 0·122 in./day during the first decade to 0·088 in./day in the period 1940-9.

November. Rainfall decreased fairly sharply about 1891 and the mean November rainfall was markedly lower until about 1923. The average fall for the years before the decrease is almost identical with the mean for the years since 1923, 0·097 in./day in the early period and 0·102 in./day later. The drier period yielded 0·070 in./day.

December. The rainfall of December has been very constant except for a wet period between 1909-1919 when the fall was 30 per cent above average.

Table 3 gives the standard deviation of the individual values (σ) and also the standard error of the mean, $\sigma/\sqrt{N}$, where N is the number of degrees of freedom, i.e., one less than the number of years for which figures are available. The author feels most strongly that all climatological summaries should exhibit some indication of the variation of individual values about the mean, such as the standard deviation. In every case our meteorological records are all too short, but σ enables us to ascertain how near our sample mean is likely to be to the true long-period average, as the probability of the true mean lying within the range of sample mean $\pm 2\,\sigma/\sqrt{N}$ is 95 per cent and within the range of sample mean $\pm 3\,\sigma/\sqrt{N}$ is 99·73 per cent.

Only four months gave more rain than the mean for their month plus 3 σ. These are listed in greater detail in Table 5.

TABLE 5. EXTREME MONTHS

Month	Year	Total rain (in.)	No. of rain-days	No. of days with more than 1 in.	No. of days with between ½ in. and 1 in.	Greatest fall in a day (in.)
Wet (fall greater than mean + 3 σ)						
Jan.	1948	5·73	30	0	2	0·68
Mar.	1919	4·91	17	1	3	1·08
July	1872	7·51	15	3	2	1·90
Oct.	1903	7·37	30	1	0	1·75
Dry (fall less than mean − 2 σ)					Remarks	
Aug.	1947	0·29	3	5 Aug.-5 Sept.	Part of 32-day drought.	
Oct.	1922	0·31	4	13 Oct.-30 Oct.	18-day drought	
Nov.	1945	0·19	6	5 Nov.-21 Nov.	17-day drought	

January 1948 was one continuous wet period. The 31st was the only day free from rain; October 1903 was a very similar month, the 29th was the rain-free day but this followed immediately the day which gave 1·75 in.

March 1919 and July 1872 were notable for the number of days on which rainfall was considerable. 90 per cent of the rain of March 1919 fell on 10 days, and the same percentage of the fall of July 1872 on 8 days.

At the other end of the scale, as 3 σ is greater than the mean in all months, it is not possible for a month to deviate by 3 σ from the mean. Three months are drier than their mean − 2 σ, and these are also listed in Table 5.

August and October have each been the wettest month of the year on 17 occasions. November on 13 occasions and September on 12, come next. July (7), December (6), January (4), February and May (3), June (2) and March (1) follow. April has never been the wettest month. The above frequency fits in very closely with the hypothesis that the probability of any month being the wettest is directly proportional to the cube of its mean rainfall.

May and June (12 times) share the honour of having been most often the driest month. February, March and April (11 times) run this very close. September (8), January and July (5), November and December (3), August and October (2) follow, an almost complete reversal of the order of the previous paragraph. In this case the probability of any month being the driest is approximately inversely proportional to its mean rainfall.

It was clear from a first inspection of the observed frequencies that they were approximately proportional, in one case directly and in the other case inversely, to the mean rainfall of the different months.

The expected frequency in 85 years of each month being the wettest or driest was first calculated on the assumption of equal probability (a random sample), and then on the assumption that this probability was proportional to the various rising powers of the months' mean rainfall.

Each set of computations was tested by the formula :

$$\chi^2 = \Sigma \frac{(O - E)^2}{E} \quad . \quad . \quad . \quad . \quad (1)$$

where O is the observed frequency and E the frequency computed on the hypothesis under consideration. It was found on consulting tables of χ^2 that the hypotheses given above fitted the distributions best.

4. Seasonal rainfall

One of the difficulties with the study of monthly rainfall figures is that one can never be sure that the calendar months are the appropriate climatological periods. It may happen that real change in the weather type tends to occur about the middle of a particular calendar month. Using the means for that month will obscure rather than clarify the real climatological trend. The other difficulty in comparing the rainfall totals of the different months is their varying lengths : this, as we saw in the previous section, can be circumvented by using mean daily rainfall figures instead of the total rainfall for the month, but in considering seasonal rainfall it hardly occurs, as the varying length of the seasons only amounts to 2 per cent of the whole. The writer feels that the division of the year into the four seasons, winter (December, January, February), spring (March, April, May), summer (June, July, August), and autumn (September, October, and November), albeit imperfect, is the best natural division available. It may be noted that these seasons as defined are not the same as those astronomically defined, but correspond to the seasons of the ' farmers' year.' In the British Isles, the three winter months undoubtedly stand out as the ones in which most of our coldest weather occurs, in spite of occasional excursions into March. The same reasoning regarding hot spells sets aside June, July and August as the months, the only months, which can claim to be summer. The transitional months, though less easy to classify, are conveniently in threes.

With the above facts in mind the author feels that closer attention in climatological summaries should be paid to the variations of the meteorological elements within the seasons. Table 6 shows the principal features of Bidston's seasonal rainfall.

TABLE 6. Seasonal rainfall

	Spring Mar.-Apr.-May (in.)		Summer June-Jul.-Aug. (in.)		Autumn Sept.-Oct.-Nov. (in.)		Winter Dec.-Jan.-Feb. (in.)	
Three wettest seasons	1947	10·67	1927	14·07	1872	15·96	1876-7	12·16
	1920	10·22	1872	14·00	1903	14·45	1899-1900	10·68
	1908	9·20	1879	12·65	1944	13·82	1868-9	10·67
Mean rainfall		5·48		7·82		8·61		6·77
Standard deviation		1·65		2·31		2·40		1·96
Standard error of mean		0·18		0·25		0·26		0·21
Three driest seasons	1893	2·69	1869	4·29	1948	4·98	1879-80	3·60
	1929	2·68	1870	3·80	1915	4·79	1933-4	2·20
	1883	2·64	1868	2·95	1922	4·24	1890-1	2·10

Spring is very clearly the driest season on the average and autumn the wettest. The effect of the heavy thunderstorm rains, and the greater capacity for the retention of water vapour in a warm atmosphere is shown by summer giving on average an inch more rain than winter, contrary to the popular belief. It is of interest to note that the ratio of the standard deviation to the mean rainfall is almost a constant. The degree of variation is least in autumn, the season of highest rainfall.

It is important to consider the possibility that these differences in means between the seasons may be due to random sampling.

The significance of the difference between sample means may be tested by using the formula :

$$\text{Var } (M_1 - M_2) = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2} \quad . \quad . \quad . \quad (2)$$

when M_1 and M_2 are the means of the two independent random variables, σ_1 and σ_2 are the respective standard deviations, and n_1 and n_2 are the number of items in the samples. In the cases under review $n_1 = n_2 = 85$ years except for winter for which season we have only 84 years' records.

Beginning with the two driest seasons spring (mean rainfall 5·48 in., standard deviation 1·65 in.) and winter (mean rainfall 6·77 in. and standard deviation 1·96 in.) and substituting in the above formula we find the variance between the two means to be 0·078 in.

Now the standard error of the difference is equal to the square root of the variance and in this case equals 0·28 in.

The difference observed between the spring and winter mean rainfall was 1·29 in., 4·6 times the standard error of the difference, so we conclude that without doubt this difference is statistically significant.

Doing a similar computation with the figures for winter and summer rainfall we find the standard error of the difference to be 0·329 in. The observed difference of 1·05 in. is 3·2 times this figure, again proving it be statistically significant. For the two wettest seasons summer and autumn the difference between their mean rainfalls, 0·79 in., is not quite so significant. The standard error of their difference is 0·36 in. and thus the observed difference is only 2·2 times the standard error. The probability of this difference being observed by random sampling when the parent populations are alike is only one chance in 72, very good odds, but for absolute certainty we shall have to wait many more years.

The assumption that the frequency distribution of seasonal rainfall follows the normal curve has been made a basis for the foregoing computations. Table 7 will enable the reader to satisfy himself to what extent this assumption is justifiable. It will be seen that in each case the distribution is unimodal and slightly positively skew.

TABLE 7. FREQUENCY TABLE OF SEASONAL RAINFALL

	Frequency distribution of seasonal rainfall (inches) within fixed limits													
	2·00-2·99	3·00-3·99	4·00-4·99	5·00-5·99	6·00-6·99	7·00-7·99	8·00-8·99	9·00-9·99	10·00-10·99	11·00-11·99	12·00-12·99	13·00-13·99	14·00-14·99	15·00-15·99
Spring	3	11	27	17	11	10	3	1	2	—	—	—	—	—
Summer	1	1	4	13	13	17	14	9	5	2	4	—	2	—
Autumn	—	—	3	9	12	17	10	10	10	8	1	3	1	1
Winter	2	3	10	15	21	11	8	10	3	—	1	—	—	—

Reverting to Table 6 and considering the extreme seasons, we find that spring has reached 10 in. of rain on only two occasions. The other seasons have reached this figure at least four times each. All three of wet seasons of both summer and autumn are wetter than the wettest of the other two seasons. Closer scrutiny of the full figures would show that the ten wettest seasons on record occur in summer or autumn. The wettest season on record, autumn 1872, has been discussed in more detail in Section 2, when considering the whole of this remarkable year.

Turning to consideration of the dry seasons, we note that figures for the three driest springs are almost the same. The three driest summers occur consecutively right at the beginning of the period under review, the summers of 1868, 1869, and 1870. The driest since 1870 has been 1949 when the rainfall was 4·46 in. It would appear, therefore, that

he lowest rainfall that can now be expected in either summer or autumn is in excess of four inches. The winter of 1890-1 with only 2·10 in. of rain is the driest season on record. Another dry winter, the more recent 1933-4, is almost half an inch drier than he next driest season, spring 1883. These two winters seem to be really exceptional, as the next driest winter gave 3·60 in. of rain and several other winters approach this figure.

In view of the widely-held belief among the general public that the weather of one season is reflected in subsequent seasonal weather, e.g., a dry summer following a wet winter, it was decided to investigate the seasonal rainfall and seek correlation coefficients between the rainfall of each season and the subsequent four seasons, involving 16 possible correlations in all.

The formula used was :

$$\text{Correlation coefficient} = \frac{\Sigma r_1 r_2 - m_1 m_2}{\sigma_1 \sigma_2} \quad . \quad . \quad . \quad (3)$$

where r_1 and r_2 are the rainfall totals of the seasons under investigation, $\Sigma r_1 r_2$ being the sum of all the products of each pair of seasons, m_1 and m_2 are the mean rainfalls, and σ_1 and σ_2 the standard deviations of the respective seasons.

The results of this investigation showed clearly that no really significant correlation existed, as all the coefficients found were below 0·20, but it is perhaps worthy of mention that of all the resulting coefficients only two out of sixteen were negative in sign; this suggests that a slight persistency of weather type, so noticeable in many other meteorological elements where the periods under review are shorter, also exists in seasonal rainfall.

Figures for the 84 ' seasonal ' years in the period under review were evaluated. The ' seasonal ' year was taken to be synonymous with the farmers' year, 1 Dec. to 30 Nov. For ease of reference the seasonal years will be denoted by one year's date, although they begin on 1 Dec. of the previous year, e.g., the year 1907 will refer to the period 1 Dec. 1906 to 30 Nov. 1907.

The mean rainfall for the 84 seasonal years under review was 28·74 in., the difference between this and the calendar year's mean being accounted for by the four odd seasons at the beginning and end of the record which could not be mated up into a complete year and were rejected. The standard deviation for the seasonal year was computed as 4·31 in. In the computations of all standard deviations in this paper, Bessel's correction, to convert the standard deviations from sample deviations to best estimates of population standard deviation, has been applied.

This value for σ implies more scatter about the mean with the seasonal years' rainfall than with the calendar years' fall. However, the extreme values have moved towards the mean. The maximum fall in a seasonal year is 43·82 in. during 1872, nearly two inches less rain than the calendar years' maximum. The next heaviest fall was 40·40 in. in 1877.

At the other end of the scale, the lowest values are four very similar years, 1934 (21·54 in.), 1921 (21·72 in.), 1868 (21·74) in., and 1933 (21·75 in.). The year 1887, which, when considered as a calendar year, was below the lower limit of annual rainfall as calculated by Doodson and Bigelstone, has now disappeared from the limelight, its seasonal fall being 22·83 in.

The readings for three seasonal years only fall outside the range of mean $\pm 2\,\sigma$, and one outside the range of mean $\pm 3\,\sigma$. All are ' wet ' years. The distribution is slightly less skew than the calendar-year distribution, 39 years receiving more rain than average and 45 years less than average, as against 37 years more than average and 48 years less than average. The skewness of the calendar year is 0·444 and the seasonal year 0·383.

5. Heavy falls on rainfall days

There have been 106 occasions when more than one inch of rain has been observed in a day, these occurrences being tabulated by monthly frequencies in Table 8.

TABLE 8. Frequency of daily falls greater than 1 in., 1½ in. and 2 in.

Fall greater than (in.)	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1	4	7	2	3	6	8	22	19	14	10	7	4
1½	—	1	—	—	—	3	4	1	—	1	2	—
2	—	—	—	—	—	1	—	—	—	—	—	—

The heaviest fall recorded was 2·03 in. on 25 June 1891. Throughout the day a series of violent thunderstorms raged over Cheshire, Lancashire and adjacent counties. Several buildings around Merseyside were struck by lightning. Of the eleven raingauges in Wirral and the Chester district only two (Hoylake, 1·83 in. and New Brighton, 1·99 in.) recorded less than two inches of rain. The rainfall must have been very evenly distributed throughout the area as the heaviest fall at Chester, 2·40 in., was only just over half an inch greater than the minimum 20 miles away at Hoylake. The wind throughout the day was light easterly or south-easterly.

Another day which is worthy of special mention is that close, sultry Sunday, 19 Sept. 1875. A torrential thunderstorm broke over Heswall (5 miles to SSW. of Bidston) about 19 hr and the run-off of the water down the hill through the village did considerable damage. The Parish Church was struck by lightning during the evening service and the organist killed. The rainfall during this cloud-burst will never be known, as there was no gauge nearer than 3 miles away at Hinderton, where 2·35 in. were recorded. On the same evening 2·00 in. of rain fell in a very heavy storm at Frodsham, just to the east of Wirral, where another man was killed by lightning. Bidston was only on the fringe of these outbreaks, recording 0·78 in.

2·02 in. of rain were recorded at Bidston during the civil day, 19 July 1947, but as 1·46 in. fell before 09 hr, this part of the fall was thrown back to the previous day in accordance with current Meteorological Office practice, preventing this day going down in the records as the second wettest.

One year (1872) gave an inch of rain on 6 days, one year (1877) on 5 days, and one year (1947) on 4 days, 9 years on 3 days, 18 years twice, 28 years once, and 27 years have never reached this total on any one day.

The frequency of heavy falls in a day is directly related to the incidence of the intense convectional type of rain occurring with summer thunderstorms. 52 per cent of the occasions when more than one inch fell in a day and 67 per cent of the occasions when more than one and a half inches fell in a day occurred in the three summer months. October, the month which gives the maximum amount of rainfall, accounts for less than 10 per cent of the days when one inch of rain or more fell.

The low reading for the absolute maximum of rainfall recorded in a day is undoubtedly due to two factors. In the first place the convectional type of rain which, as we have seen, is most likely to produce the heaviest falls at Bidston, often occurs in the polar maritime air behind the cold front of a depression. This arrives over Wirral with a north-westerly trajectory, and passes Bidston, having only travelled two miles over land. The main storm breaks further inland. It is a common sight with this air mass

to observe towering cumulus and cumulonimbus cloud stretching all round the horizon from NE. to SSW. while Bidston is enjoying a clear sky above an altitude of about 15°. Another situation, giving intense rain over the British Isles, is when thundery depressions over France move north. The force of these storms is usually largely spent by the time they arrive as far north as Wirral. However, it seems likely that the events of 25 June 1891, and 19 July 1947, may have been due to this sort of synoptic situation. Depression rains arrive largely from the SW., and Wirral is, as was stated earlier, partly in the rain-shadow of the Welsh mountains, so these falls are not as heavy as might otherwise be expected.

Turning to heavy falls in short periods, we again find that their occurrence is very rare. Adopting the *British Rainfall* classifications of very rare, remarkable, and noteworthy falls, Bidston has only given three noteworthy falls in the full 85 years. These are tabulated below in order of their relative intensity :

(i) 15 July 1945. 0·93 in. fell in 25 min in a thunderstorm.

(ii) 5/6 August 1945. 1·09 in. fell in 70 min around midnight.

(iii) 14 June 1931. 1·05 in. fell in 72 min, in a violent thunderstorm, associated with a line squall.

6. Droughts

In this section, the official Meteorological Office definition of absolute drought as at least fifteen consecutive days without measurable rain has been used exclusively. There have been 66 droughts at Bidston in the whole 85 years of record, the mean length of a drought being 19 days. The longest drought was of 32 days' duration, from 5 August to 5 September 1947. Droughts are most frequent in June, in which month they have occurred on 13 occasions. In computing this frequency, a drought which ran into two months was credited to the month in which it began unless more than two-thirds of its duration was in the second month, in which case it was credited to the latter. The remaining months accounted for droughts in the following order : April (9), May (8), March (7), February (6), July (5), August and October (4 each), January and September (3 each), November and December (2 each). The probability of this type of distribution occurring owing to random sampling and not being attributable to a real difference in the likelihood of a drought arising in the various months was investigated by the χ^2 test of significance, and was 0·025, showing that the above frequencies are almost certainly real.

It was found that only half of the droughts exceeded 17 days and less than one quarter exceeded 20 days' duration. Only two droughts were longer than 26 days, the one of 32 days previously mentioned, and one of 30 days, which began on 4 February 1932, a month which was both the driest on record at Bidston and over the British Isles as a whole.

7. Frequency curve graduation

Rainfall figures appertaining to the seasonal and calendar years, and also to the four seasons, were investigated by the methods introduced by Professor K. Pearson as outlined in *Frequency curves and Correlation* by W. Palin Elderton, and are summarized in Table 9. All the frequency curves examined were positively skew. The seasonal year was of type III with a lower limit, but unlimited high values. The calendar year curve was type IV, without limits. Each of the four seasons gave type I, limited range in both directions. It will be noted on referring to Table 6 that one autumn and two winters were drier than the statistical limits here computed would indicate possible.

TABLE 9. Summary of rainfall frequency curve computations

Rainfall period computed	μ_2	μ_3	μ_4	β_1	β_2	κ	Type of curve	Limits (in.)	Skewness
Calendar year, 1867-1951	15·81	70·74	1459·46	1·266	5·838	·673	IV	No limits	·444
Seasonal year, 1868-1951	18·10	58·95	1276·56	·586	3·896	14·82	III	Lower-16·63	·383
Spring	2·76	3·80	27·14	·687	3·562	— ·647	I	Infinite*	·580
Summer	5·30	7·65	98·48	·394	3·513	— 2·093	I	1·29-74·98	·330
Autumn	5·70	7·08	90·28	·271	2·778	— ·175	I	4·52-18·38	·460
Winter	3·78	2·91	37·05	·156	2·589	— ·017	I	2·96-13·68	·368

* Stated to be infinite, in fact finite, but so far outside normal range of seasonal rainfall as to be effectively infinite.

With a view to ascertaining how much reliance could be placed on the curves and limits derived, the probable error of κ (the criterion for deciding the type of frequency curve) was determined, using tables published by Pearson (1914). It was found that this probable error was in every case, except for autumn, large enough to make even the type of frequency curve to be used uncertain.

The probable error of the skewness was also determined. While the skewness ranges from 0·330 to 0·580, the probable error ranges from 0·104 to 0·186, with, in one case, an infinite probable error.

In view of these large probable errors, while only 85 years of record are available for statistical analysis, any findings derived by these methods, as to the properties of the parent population, are only probabilities, and may be contradicted by future events without any real secular change in the climate. A rough calculation was made to determine with how many years' records the probable errors of κ would become ' manageable,' in the sense that they became small enough not to influence the type of curve determined. Assuming that similar values were derived from the analysis, a record of 320 years would only halve the probable errors, making the type of frequency curve to be adopted reasonably certain in only two of the six cases.

It would seem therefore that this type of computation is only of limited value in a meteorological investigation owing to the sparsity of the number of observations for analysis. Before sufficient numbers of years had passed to make the probable errors small enough to place good reliance on the resultant limits there is good chance of a secular change in the climate rendering the figures inhomogeneous. Signs of secular trend of monthly rainfall are evident even in this short period of 85 years (cf., Section 3). Evidence of secular trend would render invalid many of the assumptions on which the theory of the statistics is based, but it is only fair to add that this applies to all forms of statistical analysis.

Nevertheless, even taking into account all the above objections, frequency-curve graduation is very much better than the crude results for expressing Bidston's rainfall in the period under review and, if future forecasting of rainfall probabilities, for example, for water engineers and agricultural experts, has to be made, provides the data in the most suitable form.

8. Conclusions

The rainfall of Bidston is notable mainly for its lack of extremes. The average annual fall is about four inches below the mean for England as a whole. Its one really

remarkable year, 1872, which produced 159 per cent of average, was in general the wettest year over England and Wales. When the seasonal year instead of the calendar year is considered we find more variation in annual rainfall but less noteworthy extreme values.

The individual months in many cases show signs of secular trend. January has been growing wetter in the last 30 years. August, although fairly consistently a wet month, has begun since 1927 to give less rain. The September anomaly, which began in 1888, now seems on the decline. October, the wettest month of the twelve, has become markedly drier. The fall of November divides into three periods, early and late wet periods with a dry period of 30 years sandwiched between them. In December's rainfall a ten-year wet period from 1909-1919 is to be noted.

No correlation could be found between one season's rainfall and a subsequent season.

The maximum fall in twenty-four hours of only just over two inches is very low. It is concluded that the absence of heavy falls in short periods is due to Bidston's geographical position, sheltered by the Welsh mountains from the worst of depression rains, too near to the sea to the NW. for heavy instability storms to build up sufficiently, and outside the usual zone of influence of the severe thunderstorms drifting across from France. These three circumstances combine to make the district around Bidston the driest area of any size on the west coast of the British Isles; only a small area around Prestatyn in North Wales is drier.

Absolute droughts occur slightly more frequently than once a year, and most often in spring or early summer. No drought has been of longer duration than 32 days, the average being 19 days.

The attempt to derive conclusions from graduated frequency curves of seasonal rainfall showed that the results should be treated with some reserve owing to the large probable errors. The graduated curve is, however, more representative of the parent population than the crude method.

9. Acknowledgments

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The variation of the turbulence spectrum with height under superadiabatic conditions

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Summary

A crude spectral analysis of vertical kinetic energy, heat flux and Reynolds stress is presented. It is shown that the relative contribution of high-frequency eddies to the fluxes and energy decreases with height, whereas the relative importance of long-period eddies increases with height.

1. Site

On 31 August 1951 members of the Meteorological staffs of the Woods Hole Oceanographic Institution and the Brookhaven National Laboratory conducted co-operative measurements of turbulent variations of meteorological parameters along the 410-ft tower of the Brookhaven National Laboratory. This tower is set in a wide clearing among clumps of trees about 30 ft high, with low buildings interspersed.

2. Instruments and observations

Each participating group used instruments from which the vertical kinetic energy, Reynolds stress, and heat flux could be determined. The Woods Hole equipment had been designed originally for the determination of heat flux near the ground, and consisted of instruments with characteristic times and free periods of less than a second; the Brookhaven equipment, on the other hand, was much heavier and more weather-resistant, intended for measurements at about 300 ft, with characteristic times and free periods of about 3 sec at wind speeds of 5 m sec^{-1}. All the Woods Hole equipment was mounted on a single frame about 2 ft high; it consisted of a thermistor thermometer, a temperature-compensated thermistor anemometer, and a small vertical vane. The whole frame was turned about a vertical axis by a servo-mechanism, which was guided by a horizontal vane of the same dimensions as the vertical vane. The frame also contained a vertical-velocity computer. The Brookhaven equipment consisted of a windmill-type recording anemometer (Friez Aerovane), a bi-directional vane of about the same dimensions, and an aspirated thermocouple. Thus, the Woods Hole instruments recorded vertical velocity, three-dimensional wind velocity, and temperature; and the Brookhaven equipment, wind speed, wind direction in the horizontal and vertical planes, and temperature. Wind-tunnel tests indicate that the wind speed recorded by the Aerovane is the three-dimensional wind speed multiplied by the square of the cosine of the angle of attack.

31 August was a typical fair maritime-tropical day, with air blowing onshore from the Atlantic. The wind speed at 410 ft increased during the day from 4 m sec^{-1} at 10 EST to 7 m sec^{-1} at 14 EST, and the wind backed from 220° to 200°. The lapse rate from 37 ft to 410 ft during this period was constant at about 1·5°C/100 m.

Observations were made during two periods a little more than 1 hr in length, 0945-1050 EST and 1250-1410 EST. During the morning period, the Woods Hole instruments were at 75 ft, the Brookhaven instruments at 300 ft. During the afternoon, all instruments were mounted at 300 ft.

3. Analysis and deductions

Ten-second averages were estimated from the Brookhaven records and punched on IBM cards. In the reduction, the assumption was made that the angle of attack of the Aerovane is sufficiently well approximated by the vertical angle. The Woods Hole records were read every second and averages, comparable with the Brookhaven values, were derived from eleven consecutive readings with half weight assigned to the initial and final readings. These ten-second averages were also punched on IBM cards.

Heat flux, kinetic energy and stress were computed from the Woods Hole one-second readings as well as from both sets of ten-second averages. The formulae used in the computation of these quantities were :

for the heat flux, $H = c_p \bar{\rho} \overline{\Delta w \, \Delta T}$ . . . . (1)

for the stress, $\tau = -\bar{\rho} \overline{\Delta w \, \Delta V}$ . . . . (2)

and for the kinetic energy, $\text{K.E.} = \frac{1}{2} \overline{(\Delta w)^2}$ . . . . (3)

Here w is the vertical velocity, T the temperature, V the horizontal velocity component, ρ the density, and c_p the specific heat at constant pressure. A bar over a quantity indicates a time mean taken over the complete observation period and a Δ the deviation from such a mean.

One of the purposes of the computations was to find the difference between fluxes and energies computed from single-second observations and from ten-second average observations. Table 1 shows this comparison, all values based on Woods Hole instruments.

TABLE 1. FLUX, STRESS, AND ENERGY FROM ONE-SECOND OBSERVATIONS AND TEN-SECOND AVERAGES

	Kinetic energy $m^2 sec^{-2}$		Stress dynes cm^{-2}		Heat flux cal $cm^{-2} sec^{-1} \times 10^{-3}$	
	1 sec	10 sec	1 sec	10 sec	1 sec	10 sec
300 ft (afternoon)	·194	·165	2·92	2·88	1·80	1·79
75 ft (morning)	·110	·075	1·07	0·96	1·39	1·29

This table suggests two tentative conclusions :

(*a*) High-frequency eddies contribute relatively more to the energy than to heat flux or stress.

(*b*) High-frequency eddies become comparatively less important with increasing height.

In order to study the spectral characteristics of energy and flux in greater detail, the vertical velocities obtained with both sets of instruments were subjected to a type of crude spectral analysis introduced earlier (Panofsky & Singer 1951). This crude analysis makes use of unweighted moving averages as frequency filters. Such averages filter out high-frequency variations, but retain slower fluctuations. Unfortunately, unweighted moving averages do not have a sharp cut-off period. The best one can do to visualize the action of these filters is to define a ' 50 per cent cut-off period.' This would be a period such that 50 per cent of the energy at that period is cut off, 50 per cent is retained. For unweighted moving averages it turns out that the 50 per cent cut-off period occurs at about twice the period over which the moving averages have been computed. For example, an unweighted moving average over 15 observations spaced at ten-second intervals retains half or more of the energy of eddies with periods greater than 300 sec, but removes more than half the energy of eddies with periods of less than 300 sec.

The ten-second average vertical-velocity deviations, Δw, were divided into three sections by the use of 70- and 490-second moving averages. Let w represent the ten-second average of vertical velocity, w_7 the 70-second moving average, and w_{49} the 490-second moving average. Then $w_s \equiv w - w_7$ contains high frequencies only, $w_m \equiv w_7 - w_{49}$ only medium frequencies, and w_{49} itself, only low frequencies. The quantities w_s, w_m, and w_{49} have, however, several serious shortcomings. One is that they are not uncorrelated with each other, because of the imperfect cut-off properties of moving averages. This property proves particularly troublesome in the discussion of kinetic energy. The second shortcoming is that w_s, w_m, and w_{49} do not average to zero. For this reason, w_s, w_m, and w_{49} were replaced by $w''' \equiv w_s - \bar{w}_s$, $w'' \equiv w_m - \bar{w}_m$, and $w' \equiv w_{49} - \bar{w}_{49}$. The new quantities have the property that $\Delta w = w' + w'' + w'''$, an equation which states that the total eddy vertical velocity is equal to the sum of the effects of large, medium, and small eddies. Roughly speaking, w''' contains the energy between 20 sec and 2·5 min, w'' the energy between 2·5 min and 17 min, and w' the energy of eddies with periods longer than 17 min.

The contributions of different eddy groups to the heat flux can now be described by,

$$H = c_p \bar{\rho} (\overline{w' \Delta T} + \overline{w'' \Delta T} + \overline{w''' \Delta T}) \quad . \quad . \quad . \quad (4)$$

and to the stress by,

$$\tau = -\bar{\rho} (\overline{w' \Delta V} + \overline{w'' \Delta V} + \overline{w''' \Delta V}) \quad . \quad . \quad . \quad (5)$$

The averaging period in Eqs. (4) and (5) lacks 4 min at both ends of the observation period, since 490-second moving averages are not available then. ΔT and ΔV are again 10-second averages of temperature and speed deviations from their respective means. ΔT and ΔV in Eqs. (4) and (5) may be replaced by T and V without affecting the results.

In the case of kinetic energy, the analysis is not so clear. Because of the imperfect cut-off properties of moving averages, we must write :

$$\text{K.E.} = \tfrac{1}{2} (\overline{w'^2} + \overline{w''^2} + \overline{w'''^2}) + X \quad . \quad . \quad . \quad (6)$$

where X measures the contribution of ' mixed ' terms to the total kinetic energy. In other words, the sum of the energies of eddies with high, medium and low frequencies does not quite equal the total eddy kinetic energy.

Table 2 summarizes the contributions of the various eddy sizes to the stress, heat flux and kinetic energy, during the afternoon period of 31 August when all instruments were at 300 ft.

TABLE 2. CONTRIBUTIONS OF DIFFERENT EDDY SIZES TO TOTAL FLUX AND ENERGY, IN PER CENT, 1250-1410 EST, 31 AUGUST, 300 FT

	Small eddies		Medium eddies		Large eddies	
	Woods H.	Brookhaven	Woods H.	Brookhaven	Woods H.	Brookhaven
Kinetic energy	23	24	41	41	20	20
Heat flux	12	5	45	47	43	48
Stress	6	6	52	55	42	39

The most important conclusion one can draw from Table 2 is that the Woods Hole and Brookhaven equipment yield essentially identical spectral distributions of energy, heat flux and stress. This is gratifying, especially as the total energies and stresses (which have not been mentioned in this paper) depend very much on the equipment used.

Table 2 also confirms an earlier result that large eddies contribute relatively more to the stress than to the energy. The spectral distribution of the heat flux appears to have shifted even more toward large eddies than that of the stress. This is due to the gradual decrease of temperature and vertical velocity throughout the period. It may be noted that the energy percentages do not add up to 100 per cent because of the term X in Eq. (6).

The results of Table 2 seem to establish that Woods Hole and Brookhaven instruments give the same spectral analyses when mounted at the same level. Since the Woods Hole instruments were located at 75 ft and the Brookhaven instruments at 300 ft, during the morning of 31 August, the observations at that time permit an estimate of the change of the spectrum with height.

Table 3 shows the spectral distributions at the two levels :

TABLE 3. COMPARISON OF SIMULTANEOUS SPECTRAL PROPERTIES OF KINETIC ENERGY, HEAT FLUX, AND STRESS, AT 75 FT AND 300 FT, IN PER CENT

		Small eddies	Medium eddies	Large eddies
Kinetic energy	75 ft	60	29	7
	300 ft	35	35	19
Heat flux	75 ft	46	26	27
	300 ft	13	26	61
Stress	75 ft	55	48	− 3
	300 ft	15	64	21

Table 3 shows that in all categories, large eddies become increasingly important with height, as postulated by the author's working hypothesis (Panofsky and Singer 1951). Such a shift has also been noticed by other workers in the field of turbulence, notably O. G. Sutton and Andrew F. Bunker. Table 3 also shows the same differences between energy and flux distributions as does Table 2.

Acknowledgments

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A comparison between observed and geostrophic winds in low latitudes

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(Manuscript received 18 August 1952, in revised form 8 November 1952)

Summary

Low-latitude wind and pressure data are used to determine the mean departure of the actual wind speed from the geostrophic speed. The computed departure becomes progressively greater equatorward of 30°N and attains a magnitude of about 10 m/sec at 10°N.

1. Introduction

The mean departure of the actual wind speed from the geostrophic wind speed in low latitudes over the oceans has been determined: (a) by comparing the 2,000-ft wind speed with the surface geostrophic speed at four ocean stations and (b) by comparing the geostrophic speed with a 'corrected' wind speed obtained by adding to the surface speed a factor to compensate for the surface frictional reduction. Only the total wind speed has been considered in the quantitative treatment of the data.

2. Comparison of 2,000-ft wind with the geostrophic

The deviation of the 2,000-ft wind speed from the surface geostrophic at four low-latitude island stations has been obtained from the wind data and surface isobaric spacing in the *Synoptic weather maps* (U.S. Weather Bureau 1949-1950) for the period, Sept. 1949 to Jan. 1950. Admittedly the isobars cannot be drawn over the oceans with much accuracy. However, averages over a

large number of cases should reduce the effect of analytical errors. No selection was made on the type of isobaric curvature but the location of the stations were such that slight anticyclonic curvature prevailed in most cases.

The mean 03 GMT wind speed at the 2,000-ft level (v_2), the geostrophic speed (v_g) and the mean geostrophic departure ($v_g - v_2$) are shown for the four stations in Table 1. This computed departure can, however, only be identified with the true geostrophic departure if it is assumed that the pressure gradient remains unchanged between the surface and 2,000 ft. The validity of this assumption has been checked by computing the zonal thermal wind from the mean latitudinal temperature gradient over the oceans in September and November as determined from the *Atlas of climatic charts of the Oceans* (U.S. Weather Bureau 1938) with the results shown in Table 2. Only at the latitudes north of 20°N, where the geostrophic departure is quite small, is this computed thermal contribution of the same order of magnitude as the geostrophic departure.

TABLE 1

Station	Lat. °N	No. of reports	v_s (m/sec)	v_g (m/sec)	$(v_g - v_s)$ (m/sec)
Bermuda	32	55	8·4	9·6	1·2
Wake	19	72	8·1	10·6	2·5
Johnston	17	88	10·0	14·2	4·2
Guam	13	61	7·9	13·1	5·2

TABLE 2

		Lat. (°N)	10	15	20	25	30
Zonal thermal wind,	Sept.		− 0·4	0·1	0·4	0·4	0·5
surface – 2,000 ft (m/sec)	Nov.		0	0·3	1·2	1·1	1·2

2. Comparison of 'corrected' surface wind with the geostrophic

Wind data are available from only a limited number of low-latitude stations. At many of these the low-level wind field is distorted by thermal and topographic effects. Consequently, this study has been extended by comparing 'corrected' surface winds with the geostrophic.

The mean increase of the wind speed from the surface to 2,000 ft to 3,000 ft computed by Riehl *et al.* (1951) for the trade-wind belt of the east Pacific Ocean was found to be, at most, 40 per cent of the surface wind. Again, the mean ratios of the surface and 1 km wind computed from wind records of low-latitude ship stations for the fall and winter of 1945-46 are shown in Table 3. In addition to showing ratios less than 1·4 these data indicate that there exists no marked latitudinal variation in the ratio.

TABLE 3

	Number of observations	Mean ratio 1 km : surface wind
27°N, 125°W	43	1·1
26°N, 149°W	30	1·3
22°N, 134°E	115	1·2
18°N, 165°E	29	1·2
17°N, 130°E	30	1·1
12°N, 154°E	41	1·2

On the basis of the evidence presented it appears that for low-latitude oceanic regions multiplication of the surface wind by 1·4 should give a value, in the mean, greater than the true wind in the region of 2,000 ft to 3,000 ft. Thus, the comparison of surface geostrophic and actual winds was extended by using a ' corrected ' wind (v_c), which was obtained by multiplying the surface wind speed by 1·4. The geostrophic wind could be determined only at those times when the pressure gradient was well defined. Consequently, the wind values shown in this portion of the study are somewhat above average for the respective latitudes.

The statistical sample for this study was obtained from *Historical weather maps* (Hdqs. U.S. Army Air Forces, Oct. 1945; Hdqs. U.S. Air Weather Service, Nov.-Dec. 1945, 1946) and *Synoptic weather maps* (U.S. Weather Bureau, 1949); a sample of about 1,400 observations being taken from the Atlantic and Pacific Oceans between 7°N and 38°N from the periods Oct. 1945 to Jan. 1946 and Jan. to Apr. 1949. The reports were from moving and stationary ships, and small islands, such as Johnston and Kwajalein, when located in well-defined pressure patterns with slight anticylonic or negligible curvature. No reports were used from the larger island groups such as the Philippines or Antilles.

Mean values of v_c and v_g were derived for belts of 4°-latitude width and plotted in profile form in Fig. 1. Since the ' corrected ' wind is greater in general than the actual, and since the anticyclonic curvature has not been allowed for, the departure, $(v_g - v_c)$, should represent a minimum value of the true geostrophic departure. This departure shown by Fig. 1 approaches zero near 30°N and increases both northward and southward; it reaches about 2 m/sec at 20°N and then increases very rapidly southward attaining a value of about 10 m/sec at 10°N. The increase of thermal stability may account for the increase of geostrophic departure north of 30°N (cf., Gordon 1950).

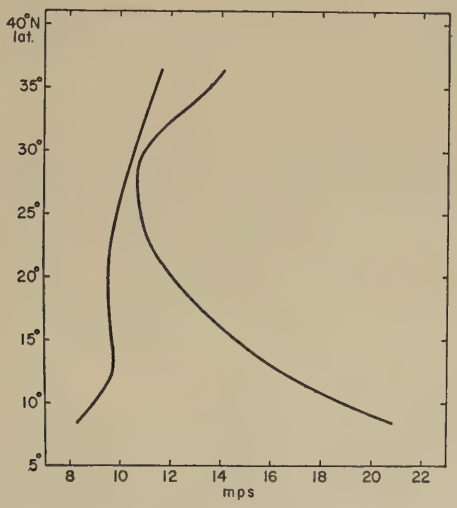

Figure 1. Latitudinal profiles of the mean ' corrected ' surface wind (left curve) and mean geostrophic wind (right curve).

The very large departures south of 15°N could be biased if errors in analysis which tend to crowd the isobars are more likely than those in the opposite sense. This would appear particularly likely since the pressure gradients are weak and since this latitude zone is near the equatorward limit of the analysis. However, the following check failed to reveal any bias in this sense. The mean geostrophic departure was computed for June and July 1946 in the region of the Marshall Islands where, owing to the atomic-bomb tests, the low-latitude data were greatly increased. The mean geostrophic departure computed from 50 reports during these two months at 11°N to 12°N was 7·8 m/sec, compared with about 7·5 m/sec from Fig. 1.

Table 4 shows that the departures read from the curves of Fig. 1 agree closely with those computed for the four stations in the previous section.

TABLE 4

Station	$(v_g - v_s)$ (m/sec)	Lat. (°N)	$(v_g - v_c)$ (m/sec) from Fig. 1
Bermuda	1·2	32	1·0
Wake	2·5	19	2·5
Johnston	4·2	17	4·0
Guam	5·2	13	6·0

The departures shown by Fig. 1 cannot be applied to individual wind reports with any degree of confidence, for the increase of wind speed with height at any time may deviate appreciably from the 40 per cent assumed in this study. However, in light of Fig. 1 and of Table 5, which shows the percentage of occasions on which $v_g > v_c$, some qualitative statements of the following type may be made : equatorward of 15°N the geostrophic speed will be an overestimate of the true speed in over 90 per cent of the cases and the geostrophic departure, in the mean, will be at least as large as that shown by Fig. 1.

TABLE 5 PERCENTAGE OF REPORTS SHOWING $v_g > v_c$

7-10°N	11-14°N	15-18°N	19-22°N	23-26°N	27-30°N	31-34°N	35-38°N
100	94	85	71	69	50	64	67

The results presented above indicate that at the lower latitudes near the ocean surface the geostrophic wind speed becomes progressively greater than the actual wind speed. Other investigations (cf. Sheppard and Omar 1952) have indicated that, in the same regions, the geostrophic direction is strongly veered on the surface wind direction at least as far equatorward as 10°N.

ACKNOWLEDGMENT

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551.501.81 : 551.524.7 : 551.571.7

Radar echoes at vertical incidence from a horizontally stratified atmosphere

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(Manuscript received 14 August 1952, in revised form 11 September 1952)

Summary

An exceptional radar echo of long duration which was observed from the zenith of an apparently clear sky is described, and it is shown that the height of origin of the echo was close to that of an anticyclonic 'inversion' which was present at the time. It is deduced that the echo cannot have been due to a layer of water or dust haze at the inversion, but that it must have been caused by an abrupt discontinuity in the lapse rates of temperature, humidity, or both. If changes in humidity alone are taken into account, then in order to explain the observed intensity of the echo, it is necessary to suppose that the dew point changed by 15°F through a layer not more than 300 ft deep. It is suggested that more use might be made of radar for investigating such tropospheric discontinuities.

1. Introduction

The object of this note is two-fold; first to describe and interpret an observation of an exceptional radar echo from an apparently clear sky and, second, to draw the attention of meteorologists to a powerful but neglected tool for the detection and investigation of sharp atmospheric discontinuities.

2. Apparatus

A radar using a wavelength of 3·2 cm with a peak power of 40 kw and a pulse length of 1 μ sec employs the same aerial for transmitting and receiving. This aerial gives a beam-width of 6·4° measured between the half-power points. The radar echoes are presented on a range-amplitude display, in which the vertical deflection of the spot on a cathode-ray tube is proportional to the amplitude of the echo, the horizontal deflection of the spot being proportional to the range from which the echo is received.

3. Observations

At 1130 GMT on 12 July 1949, during a routine test of the radar, a weak echo was received from a range of 4,560 ft while the beam was directed vertically overhead. When the beam was tilted from the vertical, the range of the echo increased in a way that showed the echo to be coming from an extensive and continuous horizontal layer. The width of the echo pulse was very small; accordingly the depth of the layer causing the echo could not have exceeded about 300 ft. The echo lasted for about 30 min, during which its intensity fluctuated considerably, though it was never much above the level of receiver noise. The power in the echo, given by the average of 4 measurements during the half-hour period of observation, was found to be $(1{\cdot}6 \pm 0{\cdot}8) \times 10^{-10}$ w, the large probable error being due to the low intensity of the echo and its fluctuations. Fig. 1 shows the appearance of the echo at 12 hr.

The Downham Market ascent for 15 GMT on 12 July 1949, is shown in Fig. 2. It will be seen that at the height of the layer which gave the echo (or 'angel' as an echo originating in an apparently clear sky is often called) there is a sharp change in the lapse rates of both temperature and humidity. Winds were very light over SE. England, which lay in a col, and so it seems reasonable to suppose that the Downham Market data fairly represent the conditions over Cambridge some three hours earlier, though of course the ground temperature at the time the angel was observed would be lower than at 15 GMT. It therefore appears probable that the angel was the product of the anticyclonic inversion.

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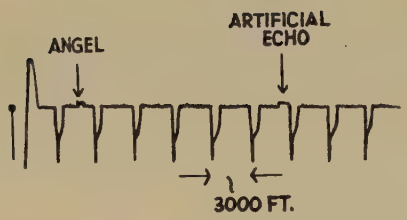

Figure 1. Tracing from a photograph of the range-amplitude display, showing the 'angel' (echo from an atmospheric discontinuity), together with 1,000-yd markers, and the artificial echo from the signal generator which is used as a power calibration.

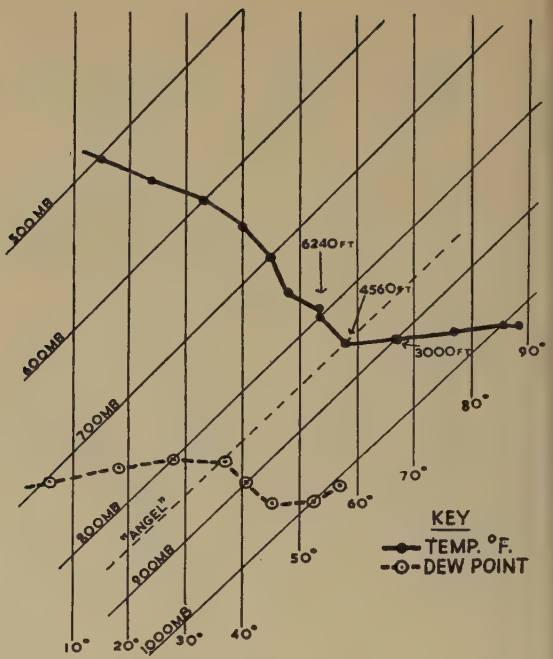

Figure 2. Tephigram, showing the upper-air conditions over Downham Market at 15 GMT on 12 July 1949. Plotted from data in the Upper Air Section of the *Daily Weather Report*.

4. Interpretation

Let us consider first whether the echo could have been caused by a concentration of haze particles at the inversion. The following argument rules out this possibility.

In view of the narrowness of the observed echo pulse it will be assumed that the thickness of the layer is less than half the radar pulse length. Suppose that the layer contains N' particles, all of diameter D, in a vertical column of unit cross-section. We have then (Ryde 1946), if P_r be the power in the echo :

$$P_r = B\Omega\pi^4 N' D^6 [(\epsilon - 1)/(\epsilon + 2)]^2 (4R^2 \lambda^4)^{-1} \quad . \quad . \quad . \quad (1)$$

where ϵ is the dielectric constant of the particles, R the range of the layer, λ the wavelength, Ω the solid angle filled by the beam, and B a constant of the radar equipment which can be determined experimentally, and which has the value $1{\cdot}3 \times 10^{11}$ w cm^2.

For particles of diameter exceeding about 5 μ the optical transmission of the layer is given by exp $(-\alpha)$, where

$$\alpha = \tfrac{1}{2}\pi k N' D^2$$

and where k is approximately unity (Wright 1939) for spherical particles of water or dust. The obscuring power of the layer was small, as is evidenced by the observation of blue sky with clearly defined cirrus clouds. Assuming that the transmission was at least 50 per cent for visible light we have

$$N' D^2 \leqslant 0{\cdot}45 \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and by combining this with Eq. (1), we can deduce a lower limit for D. For water drops, where $\epsilon \simeq 70$ (Ryde 1946), $D \simeq 0{\cdot}12$ mm, while for dust particles where ϵ lies between 2 and 4 (Ford and Oliver 1946), $D \simeq 0{\cdot}20$ mm.

Particles as large as this have velocities of fall of the order of 50 cm sec^{-1} and it seems inconceivable that such a layer could persist at constant height for the 30 min during which the angel was observed. We are therefore driven to attempt to explain the echo as due to a discontinuity in refractive index at the inversion.

Echoes from lapse-rate discontinuities have been reported previously by Friend (1948, 1949), who has given a detailed theoretical discussion of them, together with some qualitative interpretations of observed echoes; Crawford (1949) on the other hand, has shown conclusively that the transitory angels which have occasionally been reported (with durations of the order of a few seconds) are due mainly to insects. In the present instance, the duration of the echo would seem to rule out the latter possibility.

Consider a simple idealized picture of a sharp discontinuity across a horizontal plane of infinite extent which separates media of different refractive indices n_1 and n_2. Then the amplitude reflection coefficient, ρ, for electromagnetic waves at normal incidence is given by

$$\rho = (n_1 - n_2)/(n_1 + n_2)$$

Writing $n_1 = n_2 + \Delta n$, where Δn is small and when n is nearly equal to unity (as is the case in the atmosphere), we can write

$$\rho = \tfrac{1}{2}\Delta n \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

It is readily shown that the power P_r received back at the radar in such circumstances is related to ρ and R by the equation

$$P_r = B\rho^2/(2R)^2 \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Thus the measurement of the intensity of the angel leads immediately to a value of the reflectivity of the plane of discontinuity. Using the values already quoted we find

$$\rho^2 = (9 \pm 4{\cdot}5) \times 10^{-11}$$

and $$|\Delta n| = (2 \pm 0{\cdot}5) \times 10^{-5}$$

It is known empirically that the refractive index of the troposphere is given, for radio waves, by the formula (Sheppard 1946) :

$$(n - 1) \times 10^6 = [p - (e/7) + (4800\, e/T)]\,79/T$$

where T = °A

p = total pressure in mb

e = water-vapour pressure in mb

Putting in appropriate values from the Downham Market ascent we have approximately, for the present occasion :

$$\Delta n = (4{\cdot}80\,\Delta e - 1{\cdot}32\,\Delta T) \times 10^{-6}$$

where Δn, ΔT, and Δe are small changes in the quantities n, T and e. The reflectivity is more sensitive to changes in humidity, and is greatest when the changes in humidity and temperature across the discontinuity have opposite sign.

If it be assumed that $\Delta T = 0$, the change of humidity required to give the observed reflectivity ρ is $\Delta e = 4 \pm 1$ mb. This corresponds to a dew-point variation across the boundary of about 15°F, which is some 5°F greater than the change in dew point between 4,560 ft and 6,240 ft measured from Fig. 2. Thus, unless the humidity measurements during the Downham Market ascent were greatly in error, we are forced to conclude that changes of T as well as of e must be taken into account to explain the echo. It is not possible to deduce numerical values for both ΔT and Δe from the given data. In this simple picture, it would be necessary for the changes in e and T to occur in a range of height less than the radar wavelength of 3 cm : but it is easy to see that this picture is oversimplified, for from it we should expect specular reflection, an expectation which is inconsistent with the observation of echoes returned to the radar at oblique incidence. This observation, however, suggests the presence of a layer with a depth of some tens of metres in which small-scale turbulence was bringing into close juxtaposition parcels of air differing in temperature and humidity by amounts of the order deduced above. A very abrupt change in physical properties across the inversion layer is still necessary, though how such discontinuities can be maintained for many minutes remains to be solved.

A more sophisticated approach to the problem, which can be applied to gradual transitions across a boundary, would use the methods and formulae developed by Friend (1948, 1949); these have not been applied here, because they require more detailed information about the inversion layer than is available.

Although it has not been practicable to maintain a regular watch, similar phenomena have been looked for on numerous occasions when the synoptic conditions seemed favourable, but the effect has not been found again in several years' observations. It seems probable, therefore, that the discontinuity discussed in this note was an exceptionally abrupt one.

5. Discussion

It has been shown that an observed radar echo could have been produced by a sharp humidity change of the order of 40 per cent at an anticyclonic inversion. Friend (1949) emphasizes the fact that transitions in the troposphere are often far steeper than balloon soundings have been designed to indicate, a statement that is corroborated by observations made with a frost-point hygrometer (Dobson 1946).

Thus it seems that radar can be a powerful tool to the meteorologist who is interested in the fine structure of the atmosphere. The observation discussed in this note was made with a radar of comparatively low power. Radars working on two different wavelengths (say 3 cm and 30 cm) would reveal much information on the depth and nature of the transition region at a discontinuity, since the reflectivity of a layer of given rate of change of dielectric constant depends on the wavelength (Friend 1948, 1949.)

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Correspondence

532.527 : 536.252

Some experiments on thermal convection in a rotating liquid

By R. Hide

It is now generally accepted that thermally driven motions in the earth's liquid core, the radius of which is about one half of that of the whole earth, play an important role in the production of the earth's magnetic field. Observations of the geomagnetic secular variation suggest that these motions have speeds of about 0·1 cm/sec and characteristic length scales of about 1,000 km. An accurate estimation of the damping effects is difficult to make but it can be said that electromagnetic forces produce the most powerful influence and further that they are unlikely to be greater than Coriolis forces and are probably a few orders of magnitude less. Hence geostrophic balance is expected to obtain in the core.

An understanding of thermal convection in a rotating liquid is important for the study of the origin of the earth's magnetism and it was for this reason that this work was begun at the suggestion of Dr. S. K. Runcorn. It is impossible to make an exact model of the motions in the core as it is not possible to simulate the field of force of a gravitating sphere in the laboratory. As a first attempt a cylindrical model was constructed.

Two cylinders (of radii 2 cm and 5 cm and length 12 cm) were located in concentric grooves in a bakelite slab by means of a watertight gasket. They were rotated about their axis, which was vertical and the water contained between them was kept in relative motion by maintaining one cylinder at a higher temperature than the other. The flow on the top surface (which was free) was indicated by aluminium particles suspended in the water. There are different types of flow according to whether the Rossby number R is greater or less than a critical value R_c.

(a) $R > R_c$. A meridional vortex fills the space between the two cylinders, liquid rising near the hot cylinder and falling near the cold. The radial motion at the top and bottom is deflected by Coriolis force and the picture presented by the top surface is a spiral.

(b) $R < R_c$. The subcritical flow where geostrophic conditions nearly obtain bears no resemblance to the supercritical flow. The similarity between these motions and some of the main features of the general atmospheric circulation is striking.

Motion on the top surface is mainly zonal and is almost entirely confined to a thin continuous stream, about 0·7 cm wide, which touches first one cylinder, then the other. The general appearance is that of a number of waves. This pattern drifts slowly relative to the cylinders at what seems to be a constant rate in the same sense as the flow in the ' jet ' stream, i.e., eastward or westward depending on whether the outer cylinder is warmer or cooler than the inner and extends throughout the depth of the fluid (except, of course, very close to the bottom), the flow being strongly, though not entirely, two dimensional.

The pattern is not entirely symmetrical and shows a tendency to waver, the orientation of the nearly stagnant cyclonic and anticyclonic regions outside the ' jet ' tilting first one way then the other out of the radial (north-south) direction. When the amplitude of the wave (for one particular wavelength) reaches a certain critical value the wavering develops into a more pronounced oscillation.

One cycle of this phenomenon, which has been termed ' vacillation ' begins (say) with a symmetrical wave pattern with its continuous ' jet.' Some seconds later there is a distinct leaning backward of the troughs and a decrease of their width. This is followed by the troughs returning to N.-S. orientation and then leaning forward in preparation for the stage when the ' jet ' stream is actually interrupted and intense cyclones are formed in the position of the wave troughs. The cyclones decay and the ' jet ' is re-established; the wave pattern returns to the initial stage and the cycle starts again. The period corresponds to a few ' weeks.'

Measurements of the speed in the ' jet ' stream and rate of drift at different angular speeds of rotation of the apparatus and depths of water have been made, from which a value of R_c and the form of the dependence of m, the wave number, on R are deduced.

Heat-flow measurements were made and a technique for investigating the temperature distribution in the convecting liquid has been devised and used to obtain some preliminary results.

The details of this work together with a geophysical discussion will be reported elsewhere.

Department of Geodesy and Geophysics,
Cambridge University.
11 December 1952.

551.551 : 551.557 : 656.7

Clear-air turbulence near the jet stream

By H. Arakawa

Hislop (1951) reported upon the clear-air turbulence which occurs at altitudes above 20,000 ft, and suggested that the chief cause was shear in the vertical which led to a breakdown of smooth flow into eddies in the manner envisaged by Richardson (1920). Bannon (1952) related such turbulence when severe with the broad synoptic picture, and especially with jet streams and discontinuities in the tropopause. His interesting figure has already been reproduced in the *Quarterly Journal* (July 1952, p. 441). He concluded that most of the observations of severe turbulence near a jet stream were probably associated with large wind shear in the vertical, but that it seems likely that a few cases were not associated with large vertical shear but with large horizontal shear.

Clear-air turbulence related to an instability set up by a local excess of horizontal shear, anticyclonic in sense (see for example H. Solberg (1939)), might be suggested on theoretical grounds. In many cases of high-altitude turbulence, however, the horizontal shear has been cyclonic in sense, and this especially so in cases associated with jet streams. This explanation cannot therefore be valid in most cases.

As early as 1941, the present writer published a paper (Arakawa 1941, 1951) containing a criterion for turbulent motion in zonal flow. This criterion differs widely from the criterion for dynamic instability so often quoted in recent meteorological literature.

The critical *cyclonic* shear according to the author is given by

$$-\frac{\partial u}{\partial y} = \omega \sin\phi + \frac{2u}{R}\tan\phi \qquad . \quad . \quad . \quad . \quad (1)$$

where y is the meridional coordinate (positive to the north), u the zonal wind, ϕ the latitude, ω the angular velocity of the earth's rotation and R the radius of the earth. If the shear is cyclonic ($-\partial u/\partial y > 0$) and greater than the value given by (1), turbulence results. In contrast, the criterion for dynamic instability after Solberg and others is that the *anticyclonic* shear, $\partial u/\partial y$ should be greater than $2\omega \sin\phi + u \tan\phi/R$.

Assuming a maximum west wind of 115 kt (the mean maximum wind speed as given in Bannon's diagram) or 59 m sec^{-1}, the critical cyclonic wind shears computed from Eq. (1) for various latitudes are shown in Table 1.

TABLE 1. Critical cyclonic wind shears in 10^{-5} sec^{-1} (or m sec^{-1}/100 km)

	Wind speed as percentage of maximum (115 kt)							
Latitude	20	30	40	50	60	70	80	90
40	5·0	5·1	5·3	5·5	5·6	5·8	5·9	6·1
45	5·5	5·7	5·9	6·1	6·3	6·4	6·6	6·8
50	6·0	6·2	6·5	6·7	6·9	7·1	7·3	7·6
55	6·5	6·8	7·0	7·3	7·5	7·8	8·1	8·3
60	6·9	7·3	7·6	7·9	8·2	8·5	8·9	9·2
65	7·4	7·8	8·2	8·6	9·0	9·4	9·8	10·2

Reference to Bannon's diagram shows that the wind shear in the horizontal reaches the critical cyclonic values $8 \simeq 12 \times 10^{-5}$ sec^{-1} in a narrow zone just north of the jet stream or in the zone near which the tropopause is usually discontinuous. Bannon's figure shows moreover that in almost every case of turbulence the horizontal or isentropic shear was large in the gusty region, i.e., the majority (75 per cent) of cases of severe turbulence occured on the low-pressure side of the jet stream where the horizontal shear was cyclonic in sense and excessive. Many

of the cases of severe turbulence in Bannon's figure were near and at the same level as the discontinuous tropopause surface through which heavy turbulent exchange of air masses between the extra-tropical tropospheric air and the polar stratospheric air may occur.

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Mabashi, Suginami-ku, Tokyo.
22 October 1952.

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551.554

Note on the derivation of the logarithmic velocity profile

By K. L. Calder

In a recent article Björgum (1952) has shown how the well-established logarithmic velocity distribution for fully-developed turbulent flow near a plane boundary may be deduced quite rigorously from dimensional analysis. Being independent of special physical hypotheses, such a derivation has distinct advantages over the historical methods involving the Kármán similarity hypothesis or the Prandtl momentum-transfer theory. In view of the considerable importance for micro-meteorology of the logarithmic law of wind structure in the lower atmosphere, I would like to draw attention to the fact that this velocity-distribution law for wall turbulence has previously been derived from dimensional analysis by Squire (1948), who illustrates the application of the dimensional method to a wide class of turbulence problems.

I believe also that while his final analysis is correct, Björgum's paper contains a serious misconception in that he considers that in the region of fully turbulent flow near a boundary the velocity should be independent of the kinematic viscosity of the fluid. This condition is certainly not implied by the concept of fully turbulent flow. By definition the latter is that domain in which the transfer of momentum by turbulent eddy motion is of such magnitude that momentum transfer by molecular processes can be neglected in comparison, i.e., the total shearing stress is given by the well-known Reynolds expression and is therefore independent of the viscosity of the medium. With this definition it follows immediately from dimensional considerations, that the velocity gradient du/dz in the region of fully turbulent flow over an infinite plane surface (i.e., for z greater than some limiting value z_1) and the distance z from the surface are related by the universal equation

$$\frac{du}{dz} = \frac{v_*}{kz} \qquad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where v_* denotes the shearing stress velocity defined by $v_* = (\tau/\rho)^{\frac{1}{2}}$, τ being the turbulent shearing stress and ρ the fluid density, and k is a non-dimensional constant. Making the customary assumption that v_* is independent of z, integration of (1) gives

$$\frac{u}{v_*} = \frac{1}{k} \log \frac{z}{c} \qquad . \quad . \quad . \quad . \quad . \quad (2)$$

where c is a constant of the dimensions of length. So far nothing has been said about the nature of the boundary surface. The value of the constant c, however, will be determined by the latter and the nature of the final momentum-transfer process between the region of fully turbulent flow, i.e., $z > z_1$, and this surface.

Defining now an aerodynamically smooth surface as one in which the final transfer of momentum at the boundary is by purely molecular or viscous forces (operating through the thickness of the laminar layer) it follows from dimensional considerations that

$$c = \text{const.} \times \frac{\nu}{v_*}$$

where ν is the kinematic viscosity of the fluid, so that from Eq. (2), in the region of fully turbulent flow over such a surface

$$\frac{u}{v_*} = \frac{1}{k} \log \frac{v_* z}{\nu} + \text{const.} \quad . \quad . \quad . \quad . \quad (3)$$

For an aerodynamically rough surface, however, by definition even the momentum transfer at the boundary is independent of molecular processes, the drag at the boundary being the form drag of the roughness elements of which it is composed. It follows again from dimensional considerations that for such a surface $c = z_0$ where z_0 is a length parameter dependent only on the geometry of the surface, i.e., determined by the size, shape and distribution of the roughness elements. Thus from Eq. (2), in the region of fully turbulent flow over such a surface

$$\frac{u}{v_*} = \frac{1}{k} \log \frac{z}{z_0} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Only in this case will the velocity distribution be independent of the viscosity of the fluid.

The single constant k occurring in the velocity-profile expressions (3) and (4) is, of course, identical with the so-called Kármán's constant ($k \simeq 0{\cdot}4$) used by Kármán in his original derivation of these profile laws, although in the above dimensional considerations, k must be regarded as defined by Eq. (1).

Chemical Corps Biological Laboratories,
Camp Detrick,
Frederick, Maryland, U.S.A.
3 December 1952.

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551.554

Wind profiles and the shearing stress

By E. L. Deacon

In a recent contribution to this journal Panofsky (1952) remarked that, by itself, the relationship

$$u = \frac{u_*}{k(1-\beta)} [(z/z_0)^{1-\beta} - 1] \quad . \quad . \quad . \quad . \quad (1)$$

when applied to wind profiles at Brookhaven gives unrealistic estimates of the roughness parameter (z_0) of the surface and of the shearing stress τ_0 from $u_* = (\tau_0/\rho)^{\frac{1}{2}}$ under non-neutral conditions. This confirms the conclusion from the wind profile work at Porton (Deacon 1948) that it was not possible to evaluate shearing stresses (or K values for diffusion studies) on the basis of Eq. (1) and the non-neutral profile alone. It was, in fact, proposed (Deacon 1949) to mitigate this defect of the formulation by using the z_0 value obtained under neutral conditions (logarithmic profile) together with the wind velocities at two heights under the given stability conditions in the equation

$$\frac{u_2}{u_1} = \frac{(z_2/z_0)^{1-\beta} - 1}{(z_1/z_0)^{1-\beta} - 1} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The value of β so derived gives u_* from Eq. (1) but it does not in most cases give the best fit to the wind velocities measured at three or more heights. The departures are, however, usually fairly small.

This point was not very fully treated in the 1949 paper but the final paragraph of Section 4 shows that the direct application of Eq. (1) was known to be unsatisfactory. The matter was also briefly but clearly explained by Pasquill (1949) who applied the β-profile method in evaporation studies.

Eqs. (1) and (2) applied to Panofsky's data of wind velocities at 11·4 m and 125 m, using the quoted value of $z_0 = 100$ cm derived from the neutral profile over this height range, gives the results in Table 1.

TABLE 1. COMPARISON OF SHEARING STRESS COMPUTED FROM EQS. (1) AND (2) WITH THE MEASURED VALUE BY THE EDDY VISCOSITY TECHNIQUE

Date (1950-1)	4 Oct.	1 Nov.	15 Nov.	15 Feb.	27 Mar.
Stress from Eqs. (1) and (2) (dyne/cm^2)	—	1·7	5·7	12·4	10·8
Measured stress	4·7	1·7	6·0	8·0	4·3

The value for 4 Oct. is not given in Table 1 as the wind velocities from Panofsky's Table 1 inserted in his Eq. (7) do not yield the computed wind-profile stress given in his Table 2. A misprint in the wind velocities seems probable.

Clearly the β-profile method applied in the manner actually proposed does not lead to very large errors; in fact the agreement is much the same, or a little better, than obtained by Panofsky with his equation derived from a quadratic expression for K, the eddy viscosity.

The two methods have also been applied to some data of Frankenberger and Rudloff (1949). From a year's observations at Quickborn they give the following wind velocities :

	Velocity (m/sec) at heights of		
	10 m	30 m	70 m
Summer midday hours	3·6	4·9	5·9
Overcast conditions	4·4	6·3	7·5

The velocities under overcast conditions give a roughness parameter of about 75 cm so these observations extend up to about the same value of z/z_0 as at Brookhaven and errors due to neglecting the variation of shearing stress with height should be similar in each case. The wind velocities for radiation nights are not considered here because the height variation of shearing stress is not likely then to be even approximately negligible. The calculated resistance coefficients of the surface (referred to 10 m-velocities) are as follows :

	Unstable conditions	Neutral
Panofsky's formula	0·035	0·049
Eqs. (1) and (2) applied to 10 m and 30 m velocities	0·056	

In this instance Panofsky's formula leads to the improbable result that the resistance coefficient is smaller under unstable conditions than the neutral value.

A great difficulty in deducing shearing stresses from the velocity profiles, particularly when the observations are at considerably heights, is the extent to which the profiles at any one point are distorted from the form characteristic of a uniform surface owing to the effect of hedges, clumps of trees, change in type of crop, etc. Rossby long ago remarked in relation to the roughness parameter (Rossby and Montgomery 1936) :

> It is probable that the z_0-value determining the velocity profile in the first few metres above the surface depends upon the character of the ground within, say, the nearest hundred metres, whereas the z_0-value determining the velocity profile higher up . . . . presumably depends on the character of the landscape over a distance of many kilometres.

At Brookhaven, where the tower is located in a clearing several hundred feet long surrounded by parkland consisting of trees 30 ft high interspersed with grass patches and low buildings, this difficulty is likely to be rather acute and generalization on the basis of the results there perhaps somewhat unsafe.

Panofsky finds that a good estimate of the shearing stress at Brookhaven is obtained using only the 10 m-velocity in the logarithmic law with the value of z_0 from the neutral profile. This is, of course, a result of the very rough surface there ($z_0 = 100$ cm) and it would not be possible to neglect stability effects up to such heights over smoother surfaces such as open grassland ($z_0 \simeq 2$ cm). The corresponding height for grassland to which Panofsky's result might be expected to apply is of the order of 10 z_0, i.e., about 20 cm. This accords with experience of such surfaces.

Section of Meteorological Physics,
Commonwealth Scientific and Industrial Research Organization,
Highett, Australia.
3 October 1952.

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Reply

By H. A. Panofsky

I should like to thank Mr. E. L. Deacon for pointing out an error in Table 1 of my paper. The mean wind at 45 m, 4 Oct., 1028 hr-1129 hr, at Brookhaven was 6·5 m sec,$^{-1}$ not 5·6 m sec^{-1} as printed. This change, however, does not affect the calculation of stress by Deacon's 'two-wind' method as outlined in his letter. Recomputing the stress by his method for 4 Oct., I find a value of 13·8 dynes cm^{-2}, which should be inserted into his Table 1.

In completed form, Deacon's table still shows agreement in order of magnitude between Reynolds stresses at 109 m and stresses computed by his method, but the numerical agreement is not good in two of the five individual cases.

Since no result obtained from five pairs of observations is in any sense statistically significant, six additional Reynolds stresses were computed, also covering periods of approximately an hour. These observations were made at 91 m rather than at 109 m, a fact which should influence the discussion very little.

Table 1 below summarizes the computations for all eleven periods. It shows the date of each observation, the Reynolds stress, the stress computed from Prandtl's logarithmic law applied to the wind at 11·4 m, the stress computed by the method indicated in Deacon's letter, and the 'three-wind' stress of Eq. (7) in my paper.

TABLE 1. Comparison of Reynolds stress with wind stress

Date (1950-2)	Reynolds stress (dynes cm^{-2})	Prandtl one-wind stress (dynes cm^{-2})	Deacon two-wind stress (dynes cm^{-2})	Panofsky three-wind stress (dynes cm^{-2})
4 Oct.	4·7	6·6	13·8	5·3
1 Nov.	1·7	2·2	1·6	6·1
15 Nov.	6·0	4·1	5·8	9·6
15 Feb.	8·0	5·8	12·4	2·5
27 Mar.	4·3	4·8	10·7	2·4
31 Aug. (a.m.)	1·9	2·1	4·8	1·2
31 Aug. (p.m.)	6·5	5·3	7·7	6·0
3 Mar.	1·1	2·3	4·0	1·6
4 Mar.	2·8	2·8	2·8	1·1
14 Mar.	16·6	9·6	11·3	7·4
24 July	11·4	7·1	9·0	14·4

The linear correlation coefficient between Reynolds stress and Prandtl one-wind stress is 0·92, that between Reynolds stress and Deacon two-wind stress is 0·59, and that between Reynolds stress and Panofsky three-wind stress is 0·60. Unfortunately, the sample is still too small to yield definite conclusions. Nevertheless the coefficients strongly suggest that in heterogeneous air the ground stress is best determined from the single wind at the lowest available level. Such a conclusion is, indeed, in accord with ideas expressed in the last two paragraphs of Deacon's letter, as well as the tentative conclusion reached in my paper.

Numerically, the agreement between the Reynolds stress and the one-wind stress is not entirely satisfactory. Thus, the standard deviation of the Reynolds stresses is almost twice the standard deviation of the one-wind stresses. Since this difference can hardly be explained by physical theory, it is probably due to one of two causes, or a combination of them : (1) the roughness length may actually be considerably larger than 1 m, or (2) the Reynolds stresses are systematically overestimated numerically. The second possibility is also indicated by a recent comparison of Reynolds stresses obtained simultaneously by two sets of instruments at the same level (unpublished).

It might be of interest to compare the observed winds at 45 m with those computed from Deacon's formula fitted to the observations at 125 m and 11·4 m. The results are given in Table 2.

TABLE 2. OBSERVED AND COMPUTED WINDS AT 45 M

Date (1950-52)	Wind observed (m sec^{-1})	Wind computed (m sec^{-1})
4 Oct.	6·5	6·3
1 Nov.	5·0	4·7
15 Nov.	6·0	5·4
15 Feb.	5·8	5·8
27 Mar.	5·3	5·3
31 Aug. (a.m.)	3·4	3·3
31 Aug. (p.m.)	6·1	5·8
3 Mar.	3·6	3·5
4 Mar.	4·2	4·4
14 Mar.	7·9	7·9
24 July	7·6	6·8

The agreement is fairly good. However, on 15 Nov. and 24 July the difference between observed and computed winds is almost certainly greater than the inaccuracy of the wind observation. These discrepancies again indicate that it may not be advisable to fit an equation with a constant roughness length to the whole wind distribution from the surface to 125 m. This conclusion is further confirmed by the fact that the curvature of the wind profile on a logarithmic height scale agrees with the stability of the air only in the mean. In many individual situations, the curvature has a sign opposite to that expected from the stability.

The central part of Deacon's letter deals with the calculation of some resistance coefficients. He shows that the resistance coefficient determined as suggested in my paper from the wind profile under unstable conditions is smaller than under stable conditions. This is indeed rather discouraging. However, the wind profile was fitted at all three Quickborn levels in order to arrive at the result. Deacon fits his formula at 10 m and 30 m only, and uses a given roughness length. If the wind profile suggested in my note is treated in the same way, the resistance coefficient under unstable conditions comes out as 0·051, or slightly larger than the neutral coefficient, 0·049. Qualitatively, then the two-wind profiles have similar properties; quantitatively, Deacon's profile resembles the neutral profile less at low levels.

In conclusion, I believe that any formula fitted to individual wind profiles up to 125 m in terrain of varying roughness can at best be an interpolation formula, and that reliable wind stresses cannot be expected from such an equation. Under such conditions, the best values of surface stress are found when Prandtl's logarithmic law is applied to the wind at the lowest level only. On the other hand, if the roughness is uniform, and wind observations are made closer to the ground, an equation fitted to winds at two or more heights should give reliable ground stresses. I should like to admit that Deacon's equation, applied in the manner described in his letter and the references quoted therein, is likely to be more useful in this respect than I had thought.

School of Mineral Industries,
The Pennsylvania State College, U.S.A.
27 October 1952.

551.578.42 (410)

The Snow Survey of Great Britain

(1) By E. L. Hawke
Director, Snow Survey, British Glaciological Society

In Vol. 78 of this *Journal*, p. 652, the reviewer of the 1949-50 and 1950-1 issues of the British Glaciological Society's *Report on the Snow Survey of Great Britain* gives it as his opinion that 'the Survey is accumulating far more lowland observations than are now necessary for the furtherance of knowledge;' that 'in the light of knowledge already gained, our official stations now provide an adequate framework of data up to the 1,000-ft level for practically every part of Britain with the exception, perhaps, of central Sutherlandshire;' and that 'it may be doubted whether much is now gained by adding so extensively to the official summaries in the *Monthly Weather Report*.' It has been fortunate for several generations of Britain's engineers concerned with water undertakings and land drainage schemes that G. J. Symons did not hold similar views, 90 years ago, on the number of observation points needed to provide an adequate network for a rainfall survey of our country. In the second (1861) annual volume of *British Rainfall* Symons was able to publish records from 472 gauges; this approximates to the number of stations supplying snowfall data to the combined Meteorological Office and British Glaciological Society organizations at the present time. Was Symons satisfied when he had accumulated some 500 observers? The answer to the question may be found in the subsequent volumes of *British Rainfall*. Records included therein numbered 1,504 by 1871, 2,145 by 1881, 2,799 by 1891 and 3,501 by 1899, the year preceding Symons's death. Since then there had been a further increase to more than 5,000. All along, let it be noted, a large majority of the records have related to altitudes below 1,000 ft. Symons never rejected the offer of data from a newly-established gauge because he already had a contributing observer in the near vicinity. He looked on two adjacent sets of measurements as usefully counter-checking one another. His constant aim was to draw the mesh of the observational network ever closer. The need for this was particularly impressed on him when, in 1893, consideration of the havoc created by a 'cloudburst' at the Yorkshire village of Langtoft during the evening of 3 July 1892 led him to the conclusion that not less than 100 in. of rain might have fallen on a hillside there within a very short interval of time (*British Rainfall* 1892, p. 105). Subsequent study of earlier meteorological archives has brought to light strong evidence that another cloudburst, on 31 May 1682 (O.S.), deposited at least 20 in. of rain in less than 7½ min over a part of Oxford (*Nature*, Vol. 169, p. 204). Since the 5,000-odd raingauges now at work in the British Isles have an aggregate receiving area of only about 100 square yards and since the majority of them have a capacity not exceeding 10 in., our present chances of obtaining an instrumental measurement of such deluges as those just mentioned are extremely small. The ideal observational network would consist not of 5,000 but of 5,000,000 gauges, evenly spaced over the country so as to allow one for every ten acres or so, each instrument capable of holding 200 in. and each in the care of a competent and attentive observer. If this Utopian dream could be realised we should not only learn much about the micro-structure of depressions, fronts and thunderstorm systems; we should probably find that what we now regard as very exceptional rates of rainfall for the British Isles – 3½ in. in an hour or 9 in. in 24 hr – are actually common and are not rarely put into the shade in one or another corner of the land by catastrophic downpours such as those of the Langtoft and Oxford cloudbursts cited above.

British snowfall is admittedly less capricious in its distribution than British rainfall, for the obvious reason that intense convectional precipitation here is essentially a summer phenomenon and thus nearly always takes the form of rain or hail. Even so, the vicissitudes of snowfall at low levels in an island country like ours can at times produce complete blocking for several days on stretches of roads that remain open to traffic a few miles away. Vagaries of this kind result not only from ordinary wind-drifting and orographic influence but also from the stalling of small depressions by ranges of hills and from convergence of oceanic and continental air streams. The last-mentioned process is capable of causing severe localized snowfalls of some hours' duration at sea level. A notable example occurred on the morning of 15 December 1925 (Vol. 52 of this *Journal*, pp. 108-9). On that occasion a drift of humid air from the North Sea and a colder, drier current from the European mainland converged in the Dover Straits, engendering an exceptionally heavy and continuous snowstorm which was confined to the English

Channel. Now and again, for reasons not always clear, the normal orographic effect of hills in increasing the amount of precipitation apparently goes into reverse and there is less snow on high ground than in the lowlands nearby. Insufficient attention has been given as yet to the questions of how far in distance and azimuth the snow shadows of the major British mountain groups reach and of the extent to which measured precipitation in the lee of these groups is increased by snow blown from the mountain summits and flanks. Mr. L. C. W. Bonacina has rightly stated (*British Rainfall* 1936, pp. 282-3) that ' in our climate snowfall is peculiarly sensitive to local influences, so that considerable differences occur in places not far apart.' In the same article he goes on to express his belief that London is more immune from heavy snow than any other great city in Britain and that ' the hilly country between a line joining Ashford and Canterbury on the west and the coast is probably more snowy, if a balance is struck between quantity and frequency of snow, than any other part of southern England.'

It is with matters such as all these that the Snow Survey is primarily concerned. Our aim is to study the incidence and distribution of British snowfall in space and time as closely as may be. Hence the more observations the better, whether at high levels or low. There cannot be too many records : each one of them has something instructive to tell us sometimes. Valuable though the *Monthly Weather Report* summaries mentioned by the reviewer are, their very nature restricts them to listing the frequency of days with snowfall and snow-cover, plus a few supplementary notes on outstanding measurements. Recognizing these necessary limitations, the Director of the Meteorological Office has given every encouragement and all available support to the British Glaciological Society's Snow Survey year by year since its post-war reinstitution in 1946.

British Glaciological Society,
c/o Royal Geographical Society, Kensington Gore, London, S.W.7.
15 November 1952.

(2) By L. C. W. Bonacina

On page 652 of Vol. 78 of this *Journal* the reviewer of the *Reports of the Snow Survey of Great Britain for the seasons* 1949-50, 1950-51 expresses the opinion, with which I venture to disagree, that no more lowland stations are necessary for the furtherance of knowledge. Surely the very fact to which the reviewer himself alludes that statistical data about snowfall have a degree of uncertainty is a very good argument for the extension of stations, not for the limitation thereof. For my own part I should never regard small differences between neighbouring stations of similar altitude in annual number of snow days as significant unless one station were definitely better equipped than the other. No doubt the chief source of uncertainty is greater vigilance on the part of some observers than others for slight occurrences of snowfall : but another source of discrepancies lies in difficult border-line cases between snow and hail. The distinction would not, perhaps, matter very much if it were not that snow is associated with colder types of weather than any form of hail.

The late Dr. H. R. Mill used to insist that we needed as many raingauges as possible if severe local storms were not to escape measurement, and likewise I feel sure that he would have supported the directors of the Snowfall Survey in securing as many observation posts as possible for snow which in the climate of Britain is so closely related to local relief and exposure.

I suggest that in some coastal regions, notably the south-west of England, judging from the list of stations in the *Monthly Weather Report*, more inland places are required to supplement those on the sea coast itself. In Devon and Cornwall there may be no snow at all in the immediate vicinity of the sea on occasions when it may be lying quite thick on the country roads a few miles inland. Hence a map showing isopleths of ' snow-lying ' whose trend was too much influenced by coastal situations might not give a correct picture of the liability to snow-cover over the region as a whole.

13 Christchurch Hill,
London, N.W.3.
27 October 1952.

Our Reviewer replies :

I am glad to have the opportunity of replying to Mr. Hawke, and also to the comments by Mr. Bonacina whose past papers have contributed much to our understanding of the incidence of exceptional snowfalls. From them it becomes very clear that the orographic factors governing the amount falling are exactly the same as for rain. Hence in regard to quantity in exceptional lowland falls : surely the existence of the several thousand rainfall observers emphasized by Mr. Hawke provides if needed a far wider network of measurements of either depth or water equivalent, than that arising from a few score stations taking account of snow alone ? What we want is some patient work on our existing rainfall records, for example with regard to the great northern snowfall of 18-21 Feb. 1941. With regard to local convergence-effects and the like, if only Mr. Hawke knew how commonly such things occur and how easily and frequently they can be studied in the more northern parts of our island he would no doubt allow that the widespread provision of airfield meteorologists ready to take advantage of such opportunities is one of the welcome gains since 1939. Such places as East Kent are not peculiar : one can recognize the like effects on the Cromer moraine; and all that Mr. Hawke says about the Straits of Dover can be paralleled along the Lancashire coastline.

Locally of course there are drift-effects; and any Post Office driver or rural surveyor quickly learns where and why trouble can be found and what to do about it. Local regional forecasters should find little difficulty in deciding in what parts of their region the wind is likely to be more effective ; take, for example, the familiar difficulties at Askerswell Down on A.35 in Dorset. However dramatic the local results of drifting in narrow lanes may be, the volume of drifted snow is not great relative to the quantity of precipitation; a brief consideration of the relative areas of gathering ground and of deposition – field and lane, mountain tops and neighbouring gullies – or of the probable water equivalent of snowfall on ice-capped summits and neighbouring glaciers, will reveal much ; a few drives over Stainmore on winter nights will reveal more.

Nevertheless I have been at pains to make it clear that we do need some well-designed *mountain* observations and some promising work is now under way in the Cairngorms. Otherwise more can be learnt from a study of the Fanaråken observations in Norway than from any British upland station since the closure of the Ben Nevis record in 1904. Anyone who has been on Ben Nevis in winter knows that the really interesting question is how the snow stays on, not how it blows off.

In regard to frequency of snow and sleet, allowances for observers' vigilance at lowland stations can now be satisfactorily provided following Mr. Ashmore's recent paper (*Q.J.*, 1952, p. 629). All over England, the south-west not excepted, we now have ' first-class ' stations, from which to adjust the snow-frequencies observed by the others at levels up to about 1,000 ft. Similar conclusions can be reached from past records; long ago when I was examining the Sheepstor data (1912-1921) this became evident. For higher levels we have moreover some first-class observations coming in, from the N. Pennines and N. Wales for example, which will enable us to apply Mr. Ashmore's factors more widely, and up to the 1,500 ft level, if not beyond.

I should never dispute that coastal stations in Devon and Cornwall provide an unsatisfactory basis for discussion of upland snow and snow-cover. But we have three decades of observations from Cullompton, Tavistock, and Princetown, with which the inland shorter-period data from Arlington, Ashburton, Newton Abbot, Moreton Hampstead, Sheepstor, Redruth and Widecombe can be integrated, not to speak of Killerton, South Molton, Meshaw and the Society's past records at Brampford Speke, Rousdon and Taunton. With these we can put the more recent airfields and Shaftesbury to the eastward; and I would reiterate that this sort of work has already been done. Further, the range of monthly frequencies of snowfall can be assessed not merely from the more recent records; Devonshire has pretty continuous observations in one place or another back to 1725. How many now take the trouble to look at Glaisher's *Quarterly Returns* ?

Mr. Bonacina's fear with regard to the trend which may be given to isopleths through lack of adequate judgment is one that I share. Drawing isopleths is essentially a geographer's job; at least they may be expected to know where hills are, what is the allowance to be made for their aspect, and what degree of accuracy the scale of the map permits. With this in mind I see no meteorological reason whatever for assuming that we cannot now give a very good assessment of the likelihood of a 9 hr cover in Devonshire at all levels up to 1,400 ft from the existing forty years of data.

My argument I think stands; up to 1,000 ft or thereabouts we have such abundance of observational material with regard to snow and snow-cover that a little processing of what we have will give useful results with less labour and expense than are involved in compiling still

more *ad hoc* observations. Before 1940 this was not so easy; since then there has been a big increase in the number of first-class stations at which observers are trained to distinguish the first hint of sleet or soft hail. With regard to snow-cover I am confident that, up to 1,000ft., a very fair estimate of the average annual expectation can now be given for almost all parts of Britain. Should there be an exceptional snowfall in any area where there are residents, the assemblage of data is not difficult ; hence I remain unconvinced that there is now the need to multiply our lowland observations on any extended scale for publication.

Impactor technique for measurement of cloud particles

By A. W. Brewer

I was glad to note that Dr. R. Frith had written (*Q.J.*, **78**, p. 425) to you to correct any impression that might have been given by his paper, that the impactor he used was due to Diem. I would like, however, to add that the first use of the impactor on an aircraft was by Dr. J. Mazur whose contribution should be acknowledged. His impactor was one developed by C.D.E.E., Porton and his results have been described (1943), but unfortunately not published.

The particular impactor described by Dr. Frith appears to be the instrument designed by the present writer as a modification to an original model devised by Mr. K. R. May. The instrument was handed to Dr. Frith and remained with the Meteorological Research Flight when the writer left in September 1946.

The Clarendon Laboratory,
Oxford.
18 September 1952.

Reference

Mazur, J. 1943 *Air Ministry, Met. Res. Ctee., M.R.P.*, No. 109.

Proceedings at meetings of the Society

Ordinary Meeting, 19 November 1952

The following candidates were declared to have been elected by the Council during the summer recess :

FELLOWS

DOUGLAS, Robert Samuel, 5 Avondale Road, East Preston, Melbourne, Australia.
FITTON, Joseph, M.Sc.Tech., A.T.I., A.M.C.T., 72 Fallinge Park Crescent, Sherriff Street, Rochdale, Lancs.
GARDINER, Geoffrey William, 48 Clarendon Road, Ashford, Middlesex.
HAMMER, Kenneth Arthur, A.S.T. Club, Air Training Service, Hamble, Southampton.
HOLT, Instr. Lt. Matthew Armstrong, R.N.R., 14 Woodlands, Beverley, Yorks.
LIMBERT, David W. S., 29 Rosemary Avenue, Finchley, London, N.3.
MOSS, Flt. Lt. Cyril Edward, 11 Cliffe Avenue, Hamble, Southampton.
NEUMARK, Otto Walter, c/o London Gliding Club, Tring Road, Dunstable, Beds.
NORRIS, William J. T., 52 Millway, Duston, Northampton.
ROTHWELL, James Keith, 10 Cromwell Road, Queens Park, Bedford (*transfer from Student Membership*).
SIMS-WHITE, Harold Phillip Cecil, 44 Lansdowne Crescent, Holland Park, London, W.11.
TAYLOR, James Eric, A.R.Ae.S., 28 Dunstall Road, Barton, Beds.
THOMAS, Thomas George, 13 Keith Road, Hayes, Middlesex.
WINTER, Derek Francis, 15 Dorville Crescent, Ravenscourt Park, London, W.6.
YEO, Douglas, B.Sc., Senior Scientific Officer, Colonial Insecticides Research, Box 204, Arusha, Tanganyika.

FOREIGN MEMBERS

CARAPIPERIS, Leonidas, D.Sc., 29 Koletti Street, Athens, Greece.
CHMELA, Albert C., 29 Kendall Park, Waltham 54, Mass., U.S.A.
KLEIN, William H., B.S., M.S., 2 J, Northway Greenbelt, Md., U.S.A.
LONDON, Julius, Ph.D., Asst. Prof., Dept. of Meteorology and Oceanography, New York University, New York 53, N.Y., U.S.A.
MACCREADY, Paul Beattie, Jr., Aeronautics Dept., California Inst. of Technology, Pasadena, California, U.S.A.

ASSOCIATES

BROSCH, John Charles, Greenwood, N.S., Canada.
JACKSON, Allen Wilfred, Vancouver, B.C., Canada.
MISSIO, Aldo, Macdonald, Manitoba, Canada.
ROBINSON, Ronald Herbert, Moncton, N.B., Canada.
TESCH, Douglas Gordon, Frobisher, N.W.T., Canada.
TURKO, N. C., Fort Churchill, Manitoba, Canada.

STUDENT MEMBER

PENMAN, Roger David, 25 Illawarra Road, Hawthorn, Victoria, Australia.

The following candidates were declared to have been elected by the Council at their meeting on 15 October :

FELLOWS

ARCHER, Michael C. Scott, M.A., ' Gwynant,' Llangollen, Denbighshire.
BIRKETT, Francis Karl, B.Sc., Foxwood, Legh Road, Disley, Cheshire.
BODEM, Joseph, ' Hazelbank,' Blackness Lane, Keston, Kent.
BROWN, Bernard Marshall, B.Sc., 8 North Devon Road, Fishponds, Bristol.
BRYAN, David Walter, ' Blenheim Lodge,' 14 Waterden Road, Guildford, Surrey.
CUTHBERT, Cyril, B.Sc., A.R.I.C., ' Braemar,' Woodplumpton Road, Broughton, Preston, Lancs.
DANBY, Maud, B.Sc., 36a Market Street, Kirkby Stephen, Westmorland.
FORBES, Flt. Lt. Sydney Murdo, Royal Air Force Station, Fayid, M.E.A.F. 15.
HELLIWELL, Norris Cedric, B.Sc., 6 Spring Cottage, Sowerby Bridge, Yorkshire.
HOUSEAGO, Neville George William, ' Gilwell,' Allen's Lane, Newmarket Road, Norwich.
LAYCOCK, Peter Allan, 9a Brookdene Avenue, Oxhey, Watford, Hertfordshire (*transfer from Student Membership*).
MAIN, Percy Cromwell, M.P.S., 75a High Street, Walthamstow, E.17.
RIDDELL, Leonard Harper, B.A., 3 Kent Road North, Harrogate.
RICHES, James Peter, Ph.D., 13 Kimberley Road, Cambridge.
WEST, William Clayton, 28 Kirkfield Road, Withernsea, Yorkshire.
WESTWOOD, Raymond, 36 Howard Road, Handsworth, Birmingham 20.

Foreign Members

Cambiaso, V., Juan Bautista, Avenida Jose Trujillo Valdes No. 28-Altos, Ciudad Trujillo, Distritto de Santo Domingo, Rep. Dom. W.I.

Plass, Gilbert N., Ph.D., Rowland Hall, Johns Hopkins University, Baltimore 18, Maryland, U.S.A.

Thompson, Major Philip D., Room 24-512, Massachusetts Institute of Technology, Cambridge 39, Mass., U.S.A.

Student Members

Hardy, Joan Evelyn, 99 Castellain Mansions, Maida Vale, W.9.

Lang, David M., 9 Park Avenue, Chippenham, Wiltshire.

Spicer, Margaret Grace, 39 William Road, Queens Park, Bournemouth, Hampshire.

Wilson, Joan Frances, 17 First Avenue, Walton-on-Thames, Surrey.

The following candidates were then declared to have been elected that day by the Council:

Fellows

Bigg, E. K., M.Sc., Department of Meteorology, Imperial College, S.W.7.

Goodey, R. L., B.A., M.R.C.S., L.R.C.P., 140a Haverstock Hill, Hampstead, N.W.3.

Howorth, B. P., M.Sc., D.I.C., 14 Waverley Road, Enfield, Middlesex.

Jones, R. A., B.Sc., 34 Roxborough Road, Harrow, Middlesex.

Milne, C. R., B.Sc., A.F.R.Ae.S., A.M.I.Mech.E., Ministry of Supply, Officers' Mess, Royal Air Force, Farnborough, Hants.

Sidebottom, E. H., Meteorological Office, Royal Air Force Station, Changi, Singapore.

Summers, P. W., B.Sc., D.I.C., Highfield, Belbroughton, Nr. Stourbridge, Worcs. (*transfer from Student Membership*).

Watts, A. J., B.Sc., 2 Hambrook Villas, Southbourne, Emsworth, Hants.

Foreign Members

Arakawa, H., D.Sc., Meteorological Research Institute, Mabashi, Suginami-ku, Tokyo, Japan.

Coyle, J. R., Panair do Brazil, Caixa Postal 694, Rio de Janeiro, Brazil.

Hekking, Capt. Alan M., Box 175, Moody Air Force Base, Valdosta, Georgia, U.S.A.

Kleis, Capt. W. D., Det. 28-24, 28th Weather Sqn., A.P.O. 129, c/o P.M. New York, N.Y., U.S.A.

Salinger, W. A., B.A., Meteorological Office, Dublin Airport, Dublin, Eire.

Thom, H. C. S., U.S. Weather Bureau, Washington 25, D.C., U.S.A.

Veranneman, N. L., c/o World Meteorological Organization, Campagne Rigot 1, Avenue de la Paix, Geneva, Switzerland.

Student Members

Flower, D. J., A.R.C.S., B.Sc., 65 Church Drive, Kingsbury, N.W.9.

Pakes, H. W., 2 Ranelagh Avenue, S.W.6.

Savidge, J. P., 'Milwood,' Spital, Wirral, Cheshire.

Thomas, D. C., B.Sc., 14 Gordon Square, W.C.1.

The following papers were read and discussed:

(a) 'Orographic cirrus clouds' by F. H. Ludlam (*Q.J.*, **78**, p. 554).

(b) 'A study of the rainfall of two synoptic situations' by J. S. Sawyer (*Q.J.*, **78**, p. 231).

Second Annual Dinner, 27 November 1952

The second Annual Dinner of the Society was held on 27 November 1952 at the Hotel Rubens, London. Some 80 Members and guests were present, the President, Sir Charles Normand C.I.E., being in the chair. The guest of honour was Professor Sir James Scott Watson, C.B.E., LL.D., M.C., Chief Scientific and Agricultural Adviser of the Ministry of Agriculture and Fisheries, and Director-General of the National Agricultural Advisory Service, who proposed the toast of the Society; the response being made by Professor Emeritus Sir David Brunt, Sec. R.S., Vice-President. Other organizations also represented at the Dinner were the Royal Agricultural Society, Agricultural Research Council, National Farmers' Union, Farmers' Club, Rothamsted Experimental Station, Anti-Locust Research Centre, and the Meteorological Office.

A full account of the proceedings was published in the January 1953 issue of *Weather*.

Reviews

Compendium of Meteorology. Prepared under the direction of a committee, H. G. Houghton, Chairman, and edited by T. F. Malone. Published by American Meteorological Society, 1951. Pp. (11 in. x 8½ in.) ix, 1334; Figs., tables. 12$.

'The purpose of the *Compendium of Meteorology* is to take stock of the present position of meteorology . . . and to indicate the avenues of further study and research which need to be explored in order to extend the frontiers of our knowledge.' These words introduce about the largest single enterprise in meteorological publication ever to be undertaken in the English language. The concept first took shape in 1948 in the mind of an unnamed member of the Geophysical Research Division of the Air Force Cambridge Research Centre, was handed to the American Meteorological Society to work upon, and has been realized through the labours of a supervising committee, over one hundred authors drawn from the four corners of the northern hemisphere (ten from the United Kingdom), and a specially appointed editorial staff. To review such a volume calls almost for another committee! For certainly no one person could bring to it the knowledge and judgment which would be required to assess the quality of all its many parts. In fact no such review is needed and so will not be attempted. The reviewer will be satisfied if he can manage to convey the scope and flavour of the work as a whole and will be excused if he picks out some parts which have particularly excited his interest.

The work is in twenty-five sections of which a few contain only one or two individual contributions and some, e.g., The upper atmosphere, Dynamics of the atmosphere, Weather forecasting, up to ten or more articles. No branch of meteorology of any consequence fails to receive notice and a refreshing spirit of adventure is displayed in sections dealing with Cosmical meteorology, Observations and analysis (world weather network), Biological and chemical meteorology (some problems of atmospheric chemistry) and Laboratory investigations. However much the supervising committee may have attempted to obtain articles written to some general plan, it was inevitable that so many authors would throw up some very different sorts of fare. Thus articles range from true reviews of their subject, to something in the nature of monographs; some may be read as an introduction to their subject matter, many more are for the specialist in the field or at least for the meteorologist bringing the appropriate background and sense of proportion to the matter discussed. Such variety in mode of treatment is not to be regretted, for no meteorologist, however young and enterprising, is going to work steadily through this book. It is one to be fairly fully read in those sections dealing with one's own well-developed interests, and to be sampled here and there elsewhere to see the way in which researchers' minds are moving in other fields and to pick up useful ideas from them for one's own ends.

It is interesting to turn over these pages to see where new knowledge has appreciably changed meteorology since the pre-war years, and equally, where new knowledge has not yet led to much improved understanding. To mention some personal impressions under the latter heading first. Radiation would appear to be in the doldrums, though not for lack of attention, and perhaps the wind has freshened slightly since the *Compendium* went to print. Atmospheric electricity has made little advance in understanding of its major problem, the generation of charge in cloud, and, to this reader, the whole subject as here presented seems lacking in vitality. The subject of the general circulation, while very alive, is proving difficult to bring out of its Keplerian stage (the phrase is Professor Starr's who contributes two of three articles). The difficulty even of completing a description of this problem is incidentally brought out in the third article in a map of the radiosonde and upper-wind stations of the northern hemisphere – large blank spaces appear in the Pacific and the Atlantic and in all the low-latitude regions except India; and what of the southern hemisphere? Another subject, climatology, appears to remain rather detached in method and outlook from the rest of meteorology, though so evidently calling for integration with the general circulation problem itself. C. S. Durst, in one article, calls for the physical-synoptic approach in place of means and (sometimes) standard deviations, while H. E. Landsberg and W. C. Jacobs show a healthy awareness of the scope of application of climatology to human needs.

And what of those parts of meteorology which are being transformed? Cloud physics and dynamical meteorology seem most on the move; the former in asking the relevant physical questions and, by experimental methods in part, reaching towards the answers, the latter by *solving* equations thought to represent the heart of the particular matter, and not leaving them in effectively indeterminate, differential form as was so much the habit in earlier time. The physical nature of the models which are implied by the dynamical meteorologists' simplified equations could well be clearer in some cases as could the energy transformations associated with the motions dealt with. Here the use of model techniques in the laboratory may in the future provide clarification – the *Compendium* would be an admirable jumping-off ground for anyone thinking of undertaking such research. Tropical meteorology also shows liveliness after a long quiescent period, and this largely because attention was forced to the subject by war requirements. Meteorology can have no balance while the behaviour of half the earth's atmosphere is not understood – or even known with some precision – and the recrudescence of interest is therefore specially to be welcomed. The subject has not yet progressed far, being still mainly in the descriptive stage, but C. E. Palmer has done admirable service in his article by exposing inadequacies in earlier thought and preparing the ground for the future. The preparation has already shown profit as evidenced by Palmer's recent contribution to our own *Journal's* Reviews of Modern Meteorology.

In a volume of such length and from so many hands there must be unevenness of quality, and disappointments are inevitable. But few meteorologists can fail to be stimulated by this work and to find themselves returning again and again to its pages, for information and for suggestion. We can take pride that our

subject has reached a stage which merits such treatment as the *Compendium* provides and should feel profound gratitude to the American Meteorological Society for undertaking it. Evidently the book must be readily accessible if it is to serve its purpose and accessibility is often related to cost. This is very reasonable considering the amount and quality of the printing, and the reviewer would recommend it as a first-class personal investment to anyone taking the subject seriously. But, if reprinting is ever called for, would the American Meteorological Society please consider publishing the work in two volumes so that each can be comfortably carried by one person and held by him for reading.

P.A.S.

Climatological Atlas of the British Isles. London (H.M.S.O.), 1952. Royal 4to. Pp. iv, 139; plates, Figs., tables. £2. 12s. 6d.

It is significant that this volume, as its introduction points out, is ' the first reasonably complete climatological atlas of the British Isles to be published.' In these islands we talk and think more of weather than of climate and our climatic types are not readily pinned down to calendar dates. Because of this the need for a climatological atlas is not so obvious as in some larger countries, for example, Russia, India, U.S.A., where climate so markedly changes with location and the seasonal changes too are more striking. There the people are more climate-minded. If the lack of a climatic atlas here has not impressed itself so forcibly as it would elsewhere the verdict now that one has been published must be that it is a ' good thing,' is justified, will be useful and will whet our appetite for more.

The Atlas is in ten sections dealing with pressure, wind, temperature, rainfall, snow, thunder, humidity, fog and visibility, sunshine, cloud. All except cloud have maps and diagrams of normal conditions by the month or by the season and many of extreme conditions as well. Each section has its own introduction about the source of the data and about the kind of adjustments and corrections that have been applied for height above sea level or systematic errors or to reduce records to the same standard period. The colour-shading on the maps is helpful and realistic, the redder the warmer, the greener the more humid and so on. But is not there an anomaly in the colour-shading for visibility ? In order to correlate directly with the neighbouring fog maps, it would seem better to reverse the order of shading for visibility and associate the lighter shades of brown with the clearer air. In general, however, the shading successfully conveys the meaning.

Different points will appeal to different readers. Some will note how closely the rainfall-contour patterns are repeated month by month and how the numbers of rainy and snowy days are opposed – in the sense that the rainy days increase from east to west and the snow-days from west to east. Snowfall is clearly continental, rainfall oceanic. The reviewer, back in this country after many years in the east, feels that we malign our climate by having so low a limit as 0·01 in. to define a rain-day. Umbrellas are not wanted as often as the rain-day maps would seem to signify.

The regime of the winds is conveyed by monthly wind-roses of nine representative stations. As is to be expected, most of them have their own local peculiarities of topographic origin, for example, a partiality for the E-W direction at Abbotsinch near Renfrew and to a slighter extent for a N-S direction at Tiree. The seasonal wind change is best seen in a comparison of the May and December diagrams with their north-east and south-west components respectively suggesting the existence of a European monsoon. From the pressure charts we note that Lerwick in the north has a simple kind of annual fluctuations of the Arctic type with low pressure in winter and high in summer, but the pressure in S. England is less easily classified and subject to a variety of influences, since there seem to be three maxima in the year : in January or February, in June and September.

In his preface Sir Nelson Johnson gives credit deservedly to the numerous voluntary observers throughout the British Isles who have maintained meteorological stations or raingauges and whose accurate work has made it possible to produce the Atlas in its present detail. The Atlas should be of use to people in many walks of life. We are told in particular that the National Agricultural Advisory Council in 1945 emphasized the importance of an atlas of this kind for the development of agriculture in Great Britain; let us hope that its appearance coincides with an increased liaison between weather men and farmers.

C.W.B.N.

Notos. South African Weather Bureau, **1**, No. 1, 1952. Pp. 64; Figs., charts. 10s. Annual subscription 35s.

In introducing this new periodical we cannot do better than quote the editorial introduction :

' *Notos* is primarily intended as a medium for the dissemination of basic data assembled under the Southern Hemisphere Analysis Project of the South African Weather Bureau. Provision is also made for the publication of original papers on any aspect of the southern hemisphere circulation, for correspondence arising out of such published contributions and for reviews of literature pertaining to this field.'

" Circulation " is used here in its widest sense, the underlying idea of this journal being the creation of a common forum through which workers on southern hemisphere problems can come into closer contact. Contributions from other meteorological services, institutes and interested individuals are accordingly invited. It is hoped that the journal will serve as a further means of fostering international co-operation in matters meteorological, especially in regard to the southern hemisphere.'

There can be no doubt that meteorologists everywhere, especially those interested in synoptic and large-scale circulation problems, will welcome this new venture and will be grateful to the Editors and to the Director of the Weather Bureau, Dr. Schumann, for this further evidence of a determination to do as much for our science as limited resources permit. In his preface Dr. Schumann very rightly draws our attention

to the vastness of the area, and to the practical difficulties of adequate observations, which can hardly be overcome if left to the almost unaided efforts of the few bordering countries. But it will be a pleasant surprise to some to learn that daily synoptic charts can be and are being drawn for most of the hemisphere ' with a reasonable degree of confidence ' and that from these charts interpolated values of sea-level pressure at intervals of five degrees of latitude and ten degrees of longitude can be read off sufficiently certainly to justify their publication for three months in this first number. It is understood that the intention is to maintain this practice with a time lag of about one year. Some 35 large pages of tabulation are needed to present the three months' data and it is understandable that so far the resources have not been found to publish each quarter the hundred charts from which these grid values were extracted – much as everyone would delight to see them. Monthly mean isobaric charts for the hemisphere do however appear and will provide much of interest, as the series grows, to workers everywhere.

Upper-air charts are more difficult as is readily appreciated from an inspection of the distribution of sounding stations shown in the key map; western longitudes in the hemisphere are almost untouched. Nevertheless Dr. W. Schmitt, in an interesting article, describes how a daily 500-mb chart may be inferred with some confidence and presents mean monthly charts for Dec. 1951, Jan. and Feb. 1952 with an interesting discussion.

It is not an exaggeration to say that this enterprise marks an important step forward in the dissemination of information about the southern hemisphere and certainly no meteorological library with any pretentions can afford not to subscribe to the periodical.

In its present form the periodical is apparently reproduced photographically from typescript and line drawings and cannot be called handsome. Also perhaps a reviewer would be shirking his duty if he did not protest at the reproduction of diagrams which a competent draughtsman could improve enormously. But this duty done he hastens to an apology : a better appearance might have meant no appearance at present and that would have been tragic.

R.C.S.

(a) *Relations entre les phénomènes solaires et géophysiques* (Colloque Internationale, Lyon 1947). Paris (Revue d'Optique), 1949. Demy 4to. Pp. 312; Figs. tables. N.p.

(b) *Problèmes de Géomagnétisme* (Congrès de l'Association Française pour l'avancement des Sciences, Geneva 1948). Paris (Bibl. Cosmophysique), 1950. Medium 8vo. Pp. 114; Figs., tables. N.p.

These publications from France deal mainly with solar and geophysical subjects which ordinarily receive little detailed attention in the *Journal*.

The first volume contains 44 formal contributions to the international symposium on solar-geophysical relationships which was convened in 1947 by the French National Centre for Scientific Research. The papers, about half of which are of French authorship, vary in length from short summaries of a few lines to a 34-page memoir by M. Nicolet on solar radiation and the origin of the ionosphere. Discussion on a number of the topics is briefly reported. A few papers on ultra-violet emission and other solar processes pave the way for various studies of the electrical and luminescent phenomena of the high atmosphere which are caused by the action of radiations from the sun. These studies include theoretical discussion of excitation and ionization processes, consideration of ionospheric observations and of evidence obtained by spectroscopic measurements on the aurora and the light of the night-sky and at twilight. Three contributions, one a summary, deal with the association between solar emissions and geomagnetic events. While much of the matter in this collection calls for specialist understanding, other readers will obtain an appreciation of the intricacy of the problems discussed and of the diversity of effort involved in their investigation.

Problèmes de Géomagnetisme is of rather more general appeal. It consists of five reports, on selected aspects of geomagnetism, which were prepared for the meeting of the French Association for the Advancement of Science held in 1948. In the longest report J. P. Rothé discusses, with several examples, the relationship between local and regional anomalies in the distribution of the earth's magnetic field and the geological structure. Practical applications are mentioned, as also are the effects of earthquake disturbance on magnetic anomalies. Professor E. Thellier writes on the fascinating topic of geomagnetism in past ages : the attempts, attended by many pitfalls and uncertainties, to determine the earth's field in past epochs by examining the present magnetisation of eruptive rocks and sedimentary deposits. Thellier has contributed considerably to the technique of these investigations, in particular to the study of the magnetism of clay vases and bricks of known age. The characteristics of the ionosphere and linked geomagnetic phenomena are clearly summarized by the late R. Jouast. Professor A. Dauvillier describes geomagnetic effects on the incidence of cosmic rays on the earth, and the evidence of correlations between variations in cosmic ray intensity and variations in the magnetic field. The fifth report, also by Prof. Dauvillier, reviews various theories on the origin of the earth's magnetic field.

H.W.L.A.

Observer's Handbook. (M.O. 554). Meteorological Office, Air Ministry, London (H.M.S.O.), 1952. Small royal 8vo. Pp. viii, 216; 30 Figs., 42 plates, 15 tables. 12s. 6d.

This is not just another edition of the well known *Meteorological Observer's Handbook* which has appeared in many editions from 1909 to 1942, it is an entirely new book to replace the old. The change is most decidedly for the better, for the new book is very much more mature and attractive. The author, unfortunately, must remain anonymous, but whoever he may be, he is to be congratulated on his work.

The *Observer's Handbook* deals strictly with observing routines and only the minimum of attention is given to descriptions of the construction and care of instruments. These matters will be dealt with in another new book which will be called the *Handbook of Meteorological Instruments*, which it is understood will be published in about one year's time. Until it appears the present work is somewhat incomplete.

Throughout the book the requirements of the international synoptic code are constantly in mind, and climatological observers, who are only concerned with codes in a minor degree, may feel that the code aspect is more prominent than they would wish. In this matter the preface asks that they should pass those parts of the book which apply only to synoptic stations.

The information is of course authoritative and the instructions are plainly and clearly given. For a first edition the book has very few defects. It is hoped, however, that the first possible opportunity will be taken to reset the type of the index. In almost any technical book the index is important, but in a reference work of this kind it is more than usually so. It is stated on the first page of the index that ' Principle entries are given in bold type ' but this valuable help is only used nine times in an index of about 600 items, roughly half of which refer to more than one page and would have justified its use. The indexing itself is thorough.

This book should be on every meteorologist's book shelf (or desk).

A.W.B.

Conference on Water Resources (Oct. 1951). Ed. Ross Hanson. Bull. No. 41, Division of the State Water Supply, Urbana, Illinios. 1952. Demy 4to. Pp. 336; 160 Figs. N.p.

The opening of a new Water Resources Building at Urbana was made the occasion for a conference at which a large number of papers were formally presented and informally discussed. This verbatim account of proceedings is in three sections : Hydrology (120 pages), Treatment (40 pages), and Radar-Weather (130 pages).

Part I deals with hydrological measurements, sedimentation, ground water, surface water, and ' Engineering meteorology,' a paper concerned solely with use of rainfall records and which provoked the just criticism that hydrologists do not take a sufficiently broad view of what meteorology can do for them.

Part II has no meteorology. The papers are on corrosion, composition of water substance, ion-exchange materials and boiler-feed water conditioning.

Part III abounds in mysterious initials. After a census of 32 radar-weather projects, almost exclusively American, there are copiously illustrated papers on cloud detection, relation of signal strength to rainfall rate, measurement of point and areal rainfall, fluctuations in signal strength, measurement of raindrop size, and hurricane tracking. A final discussion surveys outstanding problems in the use of radar.

The bulletin has something useful for all interested in rain – its source, its arrival and its subsequent fate.

H.L.P.

Climate and the British Scene. By Gordon Manley. London (Collins), 1952. 8vo. Pp. xviii, 314; 81 photographs (41 in colour), 75 maps and diagrams, 10 tables. 25s.

In this delightful book Prof. Manley portrays the weather and climate of the British Isles throughout the 24 hours, throughout the year and through the centuries. Whether the reader inclines to art or science or prefers hunting, shooting and fishing, he will find much of interest put before him in a most attractive manner and the book is a valuable and important addition to the New Naturalist series of which 22 volumes have now appeared. It is beautifully produced and good value at 25s.

So much trouble has gone into the selection and production of the illustrations and their quality is so high that one is tempted to deal with the book in two parts, the artistic and the scientific, but there must be no suggestion that they are separate; this is not the case and, almost without exception, every picture has its technical note of explanation. There are delightful colour photographs of landscape and skies from all parts of the country and among such a collection of fine pictures it is difficult to pick out individuals, but we should mention one or two. ' Birch trees following an ice storm ' by F. Goldring is a rare study of great delicacy and charm showing the many shades of blue associated with sunshine on snow. In contrast is ' January morning in Derby ' by Cyril Newberry, a study rich in reds showing the coppery smoke-haze of a frosty morning in town : it is so good that one can almost hear the crunching of wheels on the frozen snow – a triumph of colour photography. In black and white is an excellent air view through a hole in stratocumulus cloud showing Liverpool in the dense industrial haze below the inversion. This is by C. H. Wood who also contributes ' Snow in Victorian Bradford,' a night scene of great beauty. It is evident that there is much here for the artist.

For the scientist the book is an excellent introduction to meteorology in the wider sense, and the serious student who makes good use of the comprehensive and up-to-date bibliography which is provided at the end of each of the fourteen chapters will find a planned and extensive course of study at his disposal. The author sets out to demonstrate ' the myriad ways in which the British Scene in all its diversity is affected by the vicissitudes, regular and irregular, past and present, of our atmospheric environment ' and, drawing upon his own wealth of qualitative and quantitative recording of weather phenomena which he supplements by well-documented quotations from recognized amateur and professional meteorologists, he succeeds in producing a volume which a wide section of the population of these Islands and beyond would welcome as a present at any season of the year. It is certainly meteorology without a headache.

Local climatological peculiarities in all parts of the British Isles receive treatment : we are reminded of the contrast between the treeless windy uplands of Cornwall and the lush vegetation wherever there is protection, as along the Helford River, and in Devon where Dartmouth and Salcombe are similarly favoured. There is the breezy exposure of Peacehaven contrasted with the friendly shelter of Telscombe in the South Downs. On other coasts there is the distortion of trees due to the dominant wind from the sea as distinct from the prevalent south-wester; here wartime memories of the North Norfolk coast linger and we remember the incursions of North Sea stratus and stratocumulus which characterize the scene there according to season. We must mention too the frost hollows which menace the work of the horticulturist unless he is forewarned and the reviewer can personally confirm the cool current, if only about 2 mi/hr, which affects one's ankles even before the sun has quite disappeared behind the picturesque slopes of Rickmansworth. In the opposite direction, cities like London have less frost than suburban and country districts; the additional warmth is just sufficient to prevent the densest fogs at street level in the City, but towards the suburbs wet clay, industrial pollution and temperatures a degree or so lower combine to produce the worst visibilities. Thus the reader will be impressed by the author's well-known enthusiasm and capacity for detailed weather study. It is fascinating to watch the pen of a Dines anemograph as it records the gusts and lulls of the wind and traces atmospheric turbulence with its concurrent periods of seconds, minutes and hours, but it remained for Gordon Manley to rise to the elegant description of 'a Dines anemograph in full cry during a gale.'

R.M.P.

Physical Geography. By Philip Lake. Cambridge University Press. 3rd edition, 1952. Demy 8vo. Pp. xxviii, 424; 210 Figs., 7 maps, tables; 17s. 6d.

This work is a standard text-book for late school and early university students, and the fact that since first publication in 1915 it has had 13 reprints and two new editions is evidence that it fulfils a continuing need. The work is divided into three parts, dealing with the air, the waters and the land, and the revision of these for the second edition (published in 1947 under the editorship of J. A. Steers) has been by separate authors, respectively Gordon Manley, J. A. Steers and W. V. Lewis.

The meteorological section is unfortunately inclined to offer facile explanations for matters which are really very difficult and controversial, and in outlook as well as minor matters such as notation, is somewhat out of date. The new knowledge which has accumulated since the war, largely due to the development on a continental and hemispherical scale of three-dimensional air-mass analysis, has modified our outlook on subjects as important as the general atmospheric circulation and the frontal depressions of middle latitudes in ways which could hardly have been envisaged in 1947, when the second edition was published, and we may hope that a further revision will be made for the next edition to bring the general outlook and emphasis into line with modern thought.

The remainder of the work seems comparatively free from this failing, but in the final chapter, on river regimes, more emphasis might have been given to the principle illustrated by the Lynmouth disaster, that the effects of one extreme occurrence can outweigh those of centuries of normality.

Notwithstanding these criticisms, the work as a whole reaches a high standard of scholarly lucidity which should commend it to a scientific public much wider than that for which it was originally intended.

J.M.C.

Annual Report of the Director of the Meteorological Office, April 1951-*March* 1952. H.M.S.O. Small royal 8vo. Pp. 54; tables. 2s.

Among the many items mentioned in this report three seem to call for special comment : the supply of forecasts to the general public, forecasting research, and gliding. None looms large in the budget, and perhaps for this reason it is worth making suggestions. The great body of the work will continue, as it must, with only slow changes.

The first is not to be confused with the much more expensive work of forecasting for aviation, public utilities, government services, and other concerns whose needs are vital. The general public consists of those who obtain their weather forecasts by radio, through the press, or from the milkman. In the report, we find within eight lines the following : 'It was not found possible to resume the radio-telephone AIRMET broadcasts, as no medium- or long-wave band was available . . . . Daily weather bulletins (B.B.C. Sound) for the general public . . . . were maintained.' It has often been argued that AIRMET was not intended for the general public : but that did not prevent them from using it, and the great increase in public knowledge of meteorology gained by the expansion of the Meteorological Office and the Royal Air Force during the war increased the public use of it enormously. It was not advertised and many people only came upon it by chance : had it been publicized there is no knowing what use would have been made of it. Now it is dead, and the B.B.C., who monopolize British radio, have not seen fit to make their weather service more frequent or helpful. The B.B.C. remains very smug, in the knowledge that the AIRMET bid failed through lack of numerical support : if it can keep the public ignorant of the value of a first-rate forecast service it will never have to provide one. In spite of the enterprise of the Third Programme even the most socialistic meteorologist will be driven to support commercial radio simply because he knows it will give him the forecasts. In Canada advertisers are charged extra for the programmes next to the weather forecasts. It

is irrelevant whether others may be criticized for not being insistent enough, for the B.B.C. alone stands between the public and the service it would never forgo if once it could taste it. Let the B.B.C. experiment and see !

But this is not all : what forecaster can give of his best when his words are edited, and sometimes misleadingly summarized, by someone who does not know the situation or the care with which the forecast has been phrased. This happens when the B.B.C. inserts its own précis of the forecast in a news bulletin. No other consultant would tolerate such interference with his advice to his client.

Finally, could we not hear the forecaster on the air and obtain our forecasts at first-hand and perhaps a bit less formally ? In many other countries he is heard – and seen.

The second subject, forecasting research, is described in less than a page. Forecasters seem as reticent in describing their research as in expounding their techniques. We read : ' Mathematical research led to a system of equations which might lead to a mathematical forecast and some progress was made in planning to use an electronic calculating machine. The main difficulty remains, however, that the mathematics are only approximate and there is still no evidence that the computor will do as well as the conventional forecaster.' We are thus prepared for success or failure ! Yet who can see any other way to improve forecasts ? By ' conventional ' methods progress has become ever more painful and slow. It is wrong to view a new technique simply as competing with the old : new possibilities are made open. The optimism of the twenties has gone but there is no doubt that this is because our knowledge has correspondingly increased, and new techniques must be ready to exploit that new knowledge : above all it is essential that there should be ready some meteorologists who can use these new computing machines.

The work has only just begun, and it may be only after many years that real progress in forecasting will be made. If this ' computing research ' is not pursued we may well find ourselves incapable of making use of the steadily accumulating store of observations on the behaviour of the atmosphere in three dimensions.

The setting up in May 1951 of a separate branch to deal with Upper-Air Climatology is a sign of the times in this connection. It used to be thought that the upper atmosphere was essentially simpler in structure and behaviour than the air near the ground with all its local and daily variations, but, as in most sciences, the simplicity vanishes as our knowledge grows and new ways of presenting and treating it have to be developed. Many methods are being tried, the most useful at present being contours and thickness patterns, and in future it is probable that the most successful representation will be one which exploits the power of electronic computors. Impatience for results is not justified : even if the computor works fast the pace of human thought remains as before.

Climatological information does not enter into forecasting practice because the forecast period is short and the situation is envisaged within the scheme of current charts. The failure of the analogue method of longer-range forecasting was due to the impossibility of providing the analogues. The upper air is too important and climatological information that did not describe both the surface and upper-air conditions could not be helpful except in so far as it reminded the forecaster of the possibilities. It is possible that when enough climatological data have been assembled, since it contains information about the physical processes of energy exchange, it will make longer-range forecasting possible. The forecasting techniques must be worked out in parallel with the assembling of the data in order that they may be assembled in the way found to be required by those techniques.

The British Gliding movement is a national asset in many ways, among which is its stimulation of meteorological research. After its triumph in Spain it is now at its zenith and it is the nation's duty to keep it there. One way in which the Meteorological Office has helped is that ' assistance was given in connection with the National Gliding Contests at Camphill in July 1951.' These contests were the prelude to victory in the international field and the best possible forecasting service is part of the making of good contests. Glider pilots are, of course, also keenly interested in anything the B.B.C. may do.

R.S.S.

Die Meteorologische Tagung in Bad Kissingen vom 12 *bis* 15 *Oktober* 1951. Bad Kissingen, Deutschen Wetterdienstes in der US Zone, No. 35, 1952. Pp. 328. N.p.

This official publication of the German Weather Service in the United States Zone of Germany contains about sixty papers which were read at a conference of meteorologists lasting over the four days 12-15 October 1951. The meetings were attended by representatives from most European countries some of whom presented papers or contributed to the discussions, although the great majority of the authors came from Western Germany. To some extent therefore this publication presents a digest of the state of development of the subject in that country.

The subject matter was divided roughly into six groups : synoptic meteorology and forecasting; dynamics of the atmosphere; evaporation; atmospheric electricity, ionosphere; bio-climatology, aerosols, climatology; methods of measurement, instruments. The scope of the meetings was therefore very large and it is impossible to cover the field in any detail in a short review.

During the last war German meteorologists were of course cut off from the rest of the world and afterwards the German Meteorological Service was split up into separate services in each of the occupation zones. Certain fields of research were forbidden, including radar, and no work was allowed in the field of aviation. Although there has been some relaxation recently the effects are immediately apparent in that this collection of papers contains very little information on subjects such as cloud physics and short-period forecasting for aviation. On the main subjects covered however, such as synoptic meteorology, dynamics

of the atmosphere, evaporation, it is evident that the development in Germany has lost very little in comparison with say Great Britain or America and in fact many of the articles contain reference to recent major publications in those countries.

The subject matter of the fourth and fifth sections is covered very sketchily and in some cases only one paper is presented on a particular subject, although its author is often of international repute. Included for example is an article by Dr. Israel on atmospheric electricity, a discussion on the red and blue sun by Dr. Baumbach and a paper by Dr. Junge on aerosols. The final section on instruments includes papers on several subjects of current interest in this country. As well as surface measurements relating to atmospheric composition, dew, rainfall and pressure, improvements in radiosonde technique are considered in two papers.

It is clearly impracticable for most British meteorologists to study all this publication in detail. The specialist in any of the fields covered will however be well advised to read the relevant articles to keep abreast of the most recent German work on his subject.

R.J.M.

Atmospheric pollution – its origins and prevention. By A. R. Meetham. London (Pergamon Press), 1952. Crown 4to. Pp. viii, 268; 81 Figs., 26 tables. 35s.

The evils of atmospheric pollution in London were described in John Evelyn's *Fumifugium* as long ago as the 17th century; the modern scientific study begins with Shaw and Owen's *The Smoke Problem of Great Cities* (1923) and the formation of the Atmospheric Pollution Research Committee. The present book, by a well-known worker in this field, comes opportunely at a time when technological advances make possible the realization of a clean atmosphere, even in an industrial country. How far we have to go towards this ideal is forcibly brought out by the statement that some nine million tons of dirt are poured into the air of this country every year.

The problem is highly complicated and Dr. Meetham has attempted a heroic task in surveying the whole field in a single volume. The first half of the book is concerned with sources of pollution, and passes rapidly over the origin, nature and uses of fuels, industrial boilers and furnaces, the generation of electricity and domestic heating. The present reviewer, being neither engineer nor fuel technologist, cannot assess adequately the merits of this survey, but it seems that occasionally there is a lack of balance. Only a fuel chemist is likely to be interested in the details of the methods of determining the calorific value of fuels, and such a reader will not require to be told how to use the bomb and the continuous-flow calorimeters or need examples of the calculations. Nor should it be necessary to explain the working of a simple vertical reciprocating steam engine, or a two-stroke internal-combustion motor, to convince the lay reader that engines in general are prolific sources of pollution. On the other hand, the chapters on the different types of coal and on the economy of heating the home seem admirably done and should interest everyone.

The second half of the book deals with pollution after it leaves the source and here Dr. Meetham evidently is on more familiar ground. These chapters are of direct interest to the meteorologist. There is an excellent account of methods of measuring pollution, in which it is very properly emphasized that accuracy in individual samples should not be pushed to extremes, because the samples themselves are typical only of the immediate surroundings. The policy of providing simple but effective sampling devices has had a tremendous effect in encouraging local authorities to join in the work. To-day, Great Britain is one of the very few countries for which the ' normals ' of pollution are known with substantial accuracy. The author has drawn on his experiences in the ' Leicester survey ' for his discussion of pollution in and near towns, but very wisely, has not attempted to deal in detail with turbulent diffusion. Instead, he has given an effective summary, in plain language, of the main effects of wind and temperature distribution, and it would be pedantic to insist that every meteorological statement in a book of this type should be meticulously accurate. The two final chapters deal concisely with methods of preventing pollution and the legal aspects of the problem.

In the author's own words, ' This book is meant to be of use to all who are professionally interested in atmospheric pollution ' and ' is also intended to encourage and help the many citizens who . . . are helping to fight against it.' A work designed for two such different types of readers is in danger of pleasing neither, but Dr. Meetham seems to have navigated a middle course with considerable skill. It is a book that all professional meteorologists should read, both for general knowledge and as an introduction to more specialized studies.

The printing (done in Germany) is good, the line diagrams are very clear and the photographs are well reproduced. It is a pity that the book is so costly, since many of those to whom it is addressed will hesitate to pay £1. 15s. for a semi-technical work of less than 300 pages, but the present high price of books is as much a sign of the times as our easy-going toleration of a dirty atmosphere.

O.G.S.

The Canterbury Project. A New Zealand radio-meteorological investigation. Report of factual data. Wellington (Dept. of Scientific and Industrial Research), 1951. Vol. I, 8vo. Pp. ix, 858; 11 Figs., 17 tables. Vol. II. Ob. fo. Pp. vi, 283. Vol. III. Ob. fo. Pp. vi, 187. N.p.

The wartime importance of the atmospheric refraction of short-wave radio signals led the Departments of Scientific and Industrial Research of New Zealand and the United Kingdom to set up a large-scale radio-meteorological experiment in 1946-7. The sea area off the Canterbury Plains in the South Island of New

Zealand was chosen for this experiment to exploit the frequent north-west föhn wind which develops a steep humidity lapse rate and stable temperature stratification as it passes out to sea. These weather conditions are favourable for the bending of short-wave radio signals so that they are propagated beyond the visual horizon (see, for example, Booker, *Q.J.*, **74**, p. 277, 1948).

The report under review describes the experiments and gives in detail the meteorological and radio observations obtained but does not discuss them except in the broadest manner, the aim being to provide data for research by the radio physicist and meteorologist.

The meteorological data mainly consist of detailed measures of temperature and humidity in the first 2,000 ft of the atmosphere along a line over the plain and 60 km to 80 km out to sea, using kites, tethered balloons and aircraft to obtain the observations. In addition the usual surface observations and a few radiosonde, aircraft and pilot-balloon soundings of the upper air are recorded. Most of the information is displayed graphically in Vol. II, the remainder being tabulated in Vol. I. Vol. III is devoted to radio measures. Observations are given for 74 days, including 42 occasions of off-shore winds and 34 of shallow sea-breeze under a general north-west wind.

The radio experiments demanded a knowledge of the meteorological conditions at the instant at which certain radio measurements were made. For this purpose the meteorological observations are admirable. A close time-sequence of observations, however, would have been of much greater value meteorologically, especially if wind information had been adequate. As it is the observations provide a series of interesting ' snapshots ' of the state of the lower atmosphere but in most cases the information necessary to see exactly how these states have arisen is lacking.

The printing, by photo-lithography of typescript, is poor and tiring to read but the diagrams and figures in Vol. II and III are excellent. The author is to be congratulated on his clear presentation of the vast amount of data.

J.K.B.

Reports on Progress in Physics, Vol. 15, 1952, The Physical Society, London. Crown 4to. Pp. iv, 338; Figs., tables. 50s.

Advances in Applied Mechanics, Vol. 2, 1951. Edited by R. von Mises and Th. von Kármán, Academic Press Inc., New York. Pp. x, 233; Figs., tables. $6.50.

We have here under review one very familiar and one less familiar volume of progress reports, the one annual and the other occasional, and both of interest to the meteorologist. To deal with the more familiar publication of the Physical Society first, the subjects dealt with, nine in all, cover the usual wide range of territory, with two deserving some attention and a third, may one say, demanding attention of some of our Fellowship. ' The hydrogen bond ' by L. Kellner will be of interest since the nature of water substance and of association in the liquid state are at issue and the method of investigation discussed is infra-red spectroscopy. Both cloud physicists and experts on atmospheric radiation will probably therefore wish to look at this article. Then there is a review by R. Kompfner, ' Travelling-wave tubes,' a recently developed class of electron tubes operating in the micro-wave region, ' which amplify by virtue of continuous interaction between an electron stream and a slow-travelling electromagnetic wave.' It is likely that meteorologists will sooner or later be concerned with these devices or their outputs in studies of cloud and precipitation echoes and of micro-wave propagation over the earth.

The review which concerns meteorology most is on ' Turbulent motion ' by G. K. Batchelor, of the G. I. Taylor school of fluid dynamicists at the Cavendish Laboratory. Here is a masterly exposition of the new approach to the subject, initiated by Taylor himself in 1935, and followed up or developed independently by such notable workers as von Kármán, Kolmogoroff, von Weizsäcker, Heisenberg, Townsend and Batchelor himself. No thorough analysis of turbulence can be easy reading but the author has succeeded in maintaining a rigorous treatment (within limits laid down) with the physics well to the fore, the mathematics held in subjection, and the whole eminently enlightening. The new methods and theory, of spectral analysis, equilibrium and quasi-equilibrium spectra, may not have taken us very far towards solution of the more important problems posed by turbulent fluids, but at least they point a way and give one confidence that the era of empiricism is waning – it is still far from dead. One section of the review, on the effect of turbulent motion on the distribution of local properties such as temperature and vorticity, is particularly to be commended to meteorologists, and not only those engaged on small-scale problems. The article incidentally provides an admirable background to some sections of the recent review of atmospheric turbulence in our own *Journal*.

Passing to *Advances in Applied Mechanics* we meet first an article by Th. von Kármán and C. C. Lin ' On the statistical theory of isotropic turbulence ' which is part review, part original paper, and which makes a good second course to Batchelor's first. The authors develop the general properties of the energy spectrum $F(k)$ in an infinite field on the basis that it may be divided into three broad ranges of frequency k : (i) of small eddies (large k) containing little energy, with properties dependent on energy dissipation ϵ and kinematic viscosity ν, (ii) of largest eddies characterized by a coefficient D of diffusion of turbulent energy and a constant J_0 ($= F(k)/k^4$ for $k \to 0$) and (iii) of medium-sized eddies also characterized by D, and necessarily by ϵ. If V^* and L^* are the characteristic velocity and length of the largest eddies, and V and L corresponding quantities for the middle range, then $D = V^* L^* = VL$ and the following characteristic quantities follow from dimensional arguments

Frequency range	High	Medium	Low
Basic quantities	ν, ϵ	ϵ, D	D, J_0
Characteristic length	$\eta = (\nu^3/\epsilon)^{1/4}$	$L = (D^3/\epsilon)^{1/4}$	$L^* = (J_0/D^3)^{1/2}$
Velocity	$v = (\nu\epsilon)^{1/4}$	$V = (D\epsilon)^{1/4}$	$V^* = (D^5/J_0)^{1/2}$
Time	$\tau = (\nu/\epsilon)^{1/2}$	$T = (D/\epsilon)^{1/2}$	$T^* = (J_0^2/D^7)^{1/2}$
Form of spectrum	$v^3\eta\,\phi(k\eta)$	$V^2 L f(k L)$	Fixed $V^{*2} L^* f_1(kL^*)$
at lower end	$F \propto \epsilon^{2/3} k^{-5/3}$	$F \propto D^2 k$	$F = J_0 k^4$

The spectral form in the transition ranges (bottom row of table) can exist only when the scales η, L and L^* are well separated, i.e., Reynolds number of turbulence is large. These relations appear to be nearer the truth than those derived on earlier theory.

The second volume also contains an article on the flow of liquids through porous media which should be of interest to hydrologists and others, and, for the more mathematically-minded meteorologist, on inverse and semi-inverse methods in fluid dynamics.

The Physical Society's volume has valuable author and subject indexes for the whole of Vols. 1 to 15.

P.A.S.

Corrigenda to Vol. 78

Sheppard, P. A. and Omar, M. H., p. 587, Table II.

Replace the first two entries under v_J, printed 1·6 and 3·6, by 0·8 and 1·9 respectively. Remaining columns are unaffected by the change, which arises from an error in transcription.

I. C. Browne, p. 593.

For '17,000 ft' in the last line of paragraph 3, read '1,700 ft.'

Obituary Notices

Sir RICHARD GREGORY, Bt., D.Sc., LL.D., F.R.S.

Amongst all the distinguished men who have guided the destinies of this Society from the presidential chair there can have been none better known or more generally esteemed in the scientific world than Sir Richard Gregory. To him belongs much of the credit for the high position now occupied by science not only in our educational system but also in our national life. Born in Bristol on 29 January 1864, Richard Arman Gregory passed his schooldays there and after serving for three years as a laboratory assistant at Clifton College he entered the Royal College of Science in London, taking as his principal subjects astronomy, chemistry and physics. Here he made the acquaintance of men such as Norman Lockyer, H. G. Wells and T. H. Holland, all of whom played a part in shaping his subsequent career. On leaving the College Gregory was appointed demonstrator in H.M. Dockyard School at Portsmouth. A year later he became computer to the Solar Physics Committee and research assistant to Sir Norman Lockyer, by whom his interest in astronomy and astrophysics was greatly encouraged. This period saw the first beginnings, in the form of occasional notes, of his long association with the leading science organ *Nature*, which Lockyer had edited since its foundation, in 1869, by himself with the support of the Macmillans. Gregory's co-operation proved so fruitful that in 1893 he was made assistant editor. As time went on the conduct of the journal was left more and more in his hands, and he had been virtual editor for a number of years before receiving the title nominally on Lockyer's retirement in 1919. Gregory introduced into *Nature* several important innovations, one of which was the regular ' leader ' expressing a point of view on matters scientific and their significance. Under his able control the periodical prospered increasingly. During an epoch of notably rapid progress in numerous branches of science his keen and comprehensive mind kept pace with all advances as they came to light, and many were the outstanding discoveries which were first explained to the non-specialist by one of his lucid articles. He resigned from the editorship of *Nature* early in 1939.

Gregory was always deeply interested in educational problems. Both in print and from the lecture platform he continually urged that every school and university curriculum should include the teaching of basic scientific facts; no member of the community, he maintained, ought to be allowed to embark on any kind of career in ignorance of these. In 1899 he helped to found the *School World*, which was later incorporated in the *Journal of Education*; he retained joint editorship of the publication until 1938. When still a comparatively young man he set out to remedy a defect by which he himself had been incommoded during his student days – the dearth of good elementary scientific literature. His text-books written with the aim of filling this need covered an extensive range, including astronomy, physics, chemistry and experimental science. These served their purpose so admirably that many of them passed through several editions (which he always brought carefully up to date, even in his ninth decade) and are still being widely used, both in Great Britain and abroad. Another valuable work from his pen was ' Discovery : or the Spirit and Service of Science,' published in 1916.

The British Science Guild, a body formed to promote the application of scientific methods in public affairs and later merged into the British Association, owed much to Gregory's guidance and enthusiasm. Just after the cessation of the 1914-18 war he was responsible in large measure for the conspicuous success of a British Scientific Products Exhibition organised under the auspices of the Guild. As a result of this he received the honour of knighthood. A baronetcy followed in 1931. Two years later he was elected a Fellow of the Royal Society under the rule which empowers the admission of persons who have rendered outstanding service to the cause of science. Honorary degrees were conferred on him by the Universities of Bristol, Leeds and St. Andrews.

For upwards of half a century Sir Richard attended almost every meeting of the British Association, at home and overseas, having become an annual member thereof in 1896 and a life member in 1901. He was a leader in the formation of Section L (Education), serving as secretary or recorder to it for ten years and as its president in 1922. Presidency of the Association itself came to him in 1940, and because of the war he continued to hold office until 1946. At a one-day meeting of the Association in the latter year his presidential address contained the now famous protest against atomic warfare. ' There can never be moral sanction for the mass destruction of human life by atom bombs,' he declared. Always a radical in politics, he developed a decided inclination to the left wing. Latterly he was a vice-president of the Association for the Cultivation of Cultural Relations with Soviet Russia.

Sir Richard Gregory was elected to Fellowship in this Society on 17 March 1926. Within two years (18 January 1928) he proceeded to presidency, accepting office ' with much diffidence,' he said, as being ' merely a layman in meteorology.' Nevertheless, his second presidential address (1930) on the subject of ' Weather recurrences and weather cycles,' prepared with the acknowledged help of Dr. C. E. P. Brooks, probably achieved a degree of publicity unique in the Society's history. This was because Gregory took the opportunity to deny the applicability to London in the 20th century of Buchan's cold and warm periods, which were at that time front-page news in the popular press as a result of their recent mention in parliamentary debates on a fixed Easter.

Sir Richard died on 15 September 1952 at his home near the Sussex coast, well on in his 89th year. At a memorial service held in the Church of St. Martin-in-the-Fields, London, on 3 October the Society was represented by one of its vice-presidents, Mr. W. M. Witchell.

E.L.H.

Sir MICHAEL NAIRN, Bt.

Sir Michael Nairn, one of the few remaining members of the Scottish Meteorological Society who were transferred to Fellowship in this Society on the fusion of the two bodies in 1921, died at his home, Elie House, Elie, Fifeshire, on 24 September 1952. A leading business man of Scotland, he had long been well known also for his keen interest in public welfare and charitable institutions. Sir Michael was grandson to the founder of the noted firm which bears his family name. Until January 1952 he had been chairman of the directors for a quarter of a century. He was the second baronet, born in 1874, and succeeded his father, Sir Michael Barker Nairn, in 1915. Despite the many claims on his time and energies, Sir Michael's enthusiasm for meteorology continued unabated to the end of his life. Since the establishment of the Society's monthly magazine *Weather* in 1946 he had read every issue ' almost from cover to cover,' as he put it. Mr. James Paton, Scottish Secretary, was the Society's representative at the funeral.

E.L.H.

Captain T. L. KEEN

Captain Thomas Lawrie Keen who died on 2 April 1952 at the age of 66 had been a Fellow of the Society since 1949, and was a regular attendant at the Society's meetings. Those who met him found him a kindly entertaining companion and he will be much missed by numerous friends in the Society. He was born in Argyll, but his family soon moved south and he spent the remainder of his life in England. He was educated at the University of Durham and subsequently studied art at South Kensington. He served with the Argyll & Sutherland Highlanders through the first world war in France, Gallipoli and on the Vardar, where he was wounded, and thereafter had to give up active service, although he continued to serve until 1922 on General Milne's staff, Intelligence Branch, in Constantinople. Between the wars he worked as an artist with the Daily Sketch, and during the second world war with the News Chronicle. In 1945 heart trouble forced him to retire, and he then devoted himself to apiculture, the study of meteorology and fungi.

N.O.

Mr. E. G. DYMOND, M.A., F.R.S.E.

The news of the sudden death, at the early age of 52, on 26 October 1952 of Edmund Gilbert Dymond, a Life Fellow of the Society since 1946, came as a great shock to his friends and colleagues, for only three days previously he had attended a meeting in London of the Instruments Sub-Committee of the Meteorological Research Committee and only a day before he had been in his laboratory, apparently in excellent health and spirits. He was due to preside on 31 October for the first time at a meeting of the Scottish Centre; he had just succeeded Sir Ernest Wedderburn as Chairman and Vice-president for Scotland.

After a distinguished career both as an undergraduate and research student at Cambridge, he was awarded an International Education Board Fellowship and worked at Göttingen in 1924-5 and at Princeton in 1925-6. He then returned to Cambridge, having been elected Fellow of St. John's College in 1925, and shortly afterwards he became a University teacher. In 1932 he joined the staff of the Department of Natural Philosophy, Edinburgh University, as lecturer and Carnegie Teaching Fellow. In the summer of 1937, he accompanied the Wordie Expedition to Baffin Bay with the purpose of making high-altitude measurements of cosmic radiation near the geomagnetic pole. It was his use of balloons in this work that first aroused his interest in meteorology; with Dr. Hugh Carmichael, he investigated winds on the coast of NW. Greenland to heights of 30 km by large balloons observed by two theodolites and made a sounding by meteorograph to a height of 19·5 km in latitude 76°N. At the outbreak of war, the Director of the Meteorological Office secured his release from University duties and he proceeded to Kew Observatory where he was engaged during the whole of the war period on the development of the British radiosonde. His valuable paper on this work with its discussion of the errors to which this instrument is subject appeared in 1947 in the *Proceedings of the Physical Society*. Soon after returning to Edinburgh in 1946, he was appointed Reader in Natural Philosophy and assisted Professor Feather in the reorganization of the honours course, in particular of the laboratory work. He resumed his high-altitude cosmic ray researches and had prepared for his group of research students an ambitious scheme of work, which was already well under way at the time of his death. His students are continuing the work that he so ably initiated but they sorely miss his guidance.

Gilbert Dymond was quiet, even reserved in manner; indeed, so reticent was he about himself and his activities that one might be associated with him for years and yet know little of him. Perhaps this characteristic derived from his early years, when poor health had precluded his attendance at school and his education had to be given privately. His unfailing courtesy never altogether concealed this shyness from new acquaintances, but when his friendship was gained, the barrier vanished to reveal a great depth of culture and character as well as a sprightly humour. He was no narrow specialist; his interests ranged not only over the whole of physics but into the fields of literature and music. He was a delightful companion in the jaunts among the mountains that he so dearly loved. Only this summer, he spent a happy holiday with a friend in Norway among the Jotunheim Mountains. The writer vividly recalls how, after a few days spent camping and scrambling in the Cairngorms in fearful weather, he secretly prepared a five-course dinner in his tent and to the great delight of his companions produced it course by course, at first with mock decorum and finally with an unfeigned glee and boyish sense of fun, that dispelled the gloom of the enveloping cloud and driving rain.

J.P.

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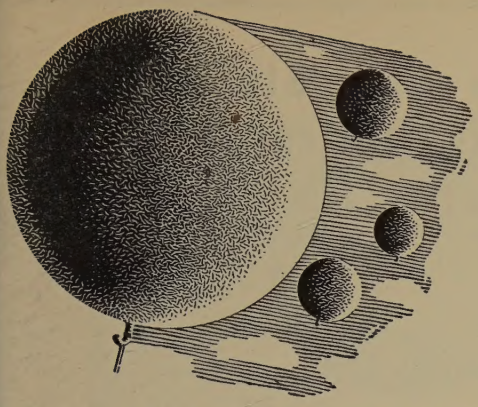

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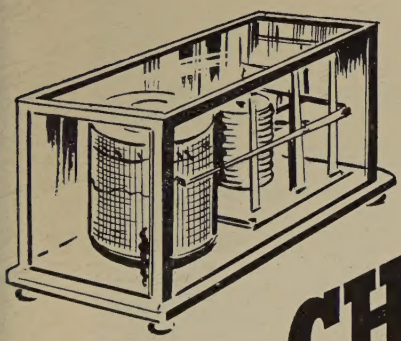

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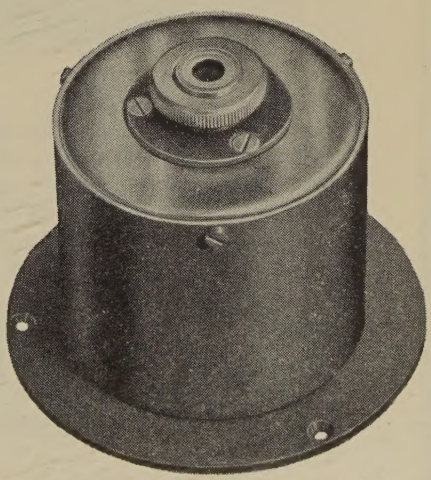

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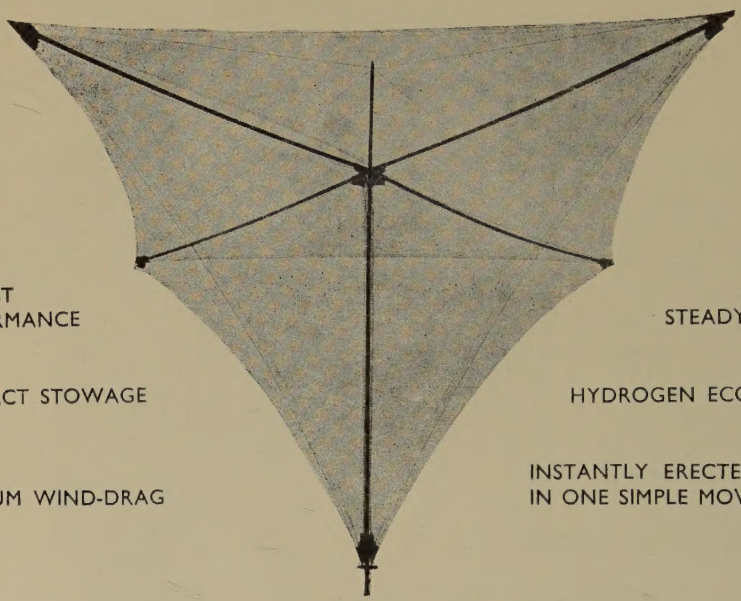

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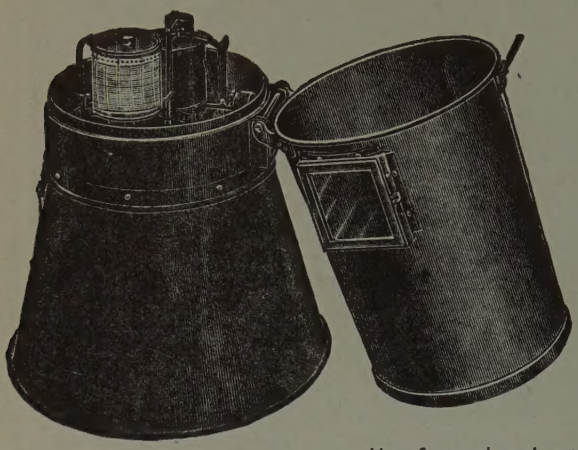

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